Politecnico di Milano
MSc Course - MEMS and Microsensors - 2023/2024

E16
Photocurrent in CMOS image pixels

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Problem

You work as a system architect in an imaging company. You analyze a case study where a smartphone is used to take a picture of a basket of oranges. A lamp with 60-W emitting optical power illuminates the scene, which reflects light as a Lambertian reflector with a 0.2 coefficient. Oranges are 2 m away from the light source, and we are 10 m away from the scene. Compared to the direction orthogonal to the scene surface, we are off by 60°. The smartphone features a sensor with 3260x2480, 1.5 μm-side pixels, with 0.6 quantum efficiency. The photograph is shot with an F-number of 4. Fig. 1 sketches the situation.

- Evaluate the focal length that is suited to fit 1 m^2 of the scene in the final picture.
- Calculate the photon flux per unit area incident on one pixel, in $[ph/s/m^2]$.
- Find the photocurrent generated within a red pixel (transmittance of 0.85).
- Verify whether an anti-alias filter is required.

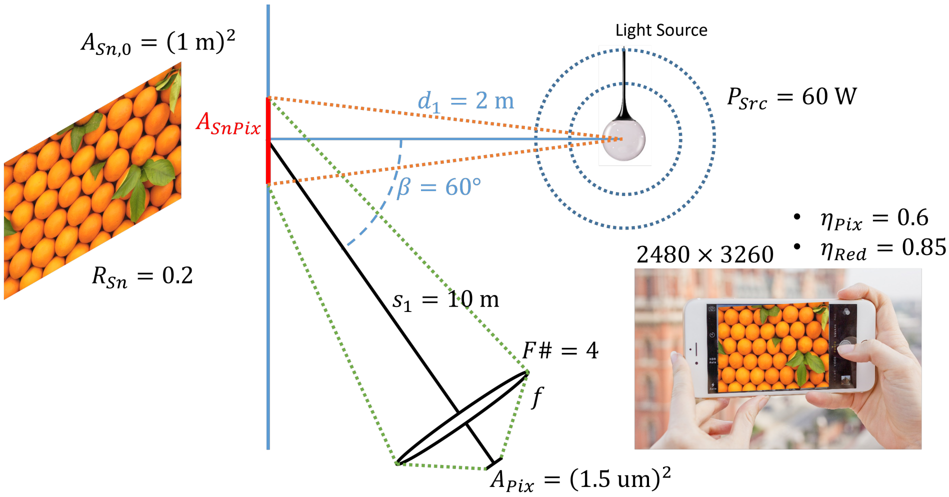

Figure 1: Sketch of the situation.

Part 1: Choice of the focal length

We know that the focal length of an optical system in front of an imaging sensor determines the magnification factor between an object on the scene plane and its image on the sensor plane. The focal length of an acquisition system is usually chosen to fulfill the field of view requirements. The magnification factor of a lens can be evaluated as

$$m = \frac{s_2}{s_1}.$$

where s_1 is the distance between the scene and the lens, and s_2 is the distance between the lens and the sensor. If $s_1 \gg s_2$, the magnification factor can be calculated as

$$m = \frac{f}{s_1},$$

where f is the focal length of the lens. The side of each pixel is equal to 1.5 μm. From the pixels number, we can infer the dimension of the sensor. The height and length are:

$$H_{Sens} = N_{Pix,H} \cdot l_{Pix} = 3.72\,\text{mm}, \qquad L_{Sens} = N_{Pix,L} \cdot l_{Pix} = 4.89\,\text{mm}.$$

As we can see from Fig. 2, in order to fit the whole scene (one square meter), one can chose the magnification factor such that the highest dimension of the scene (1 m) corresponds to the lowest dimension of the sensor (3.72 mm):

$$m = \frac{H_{Sens}}{l_{Sn}} = \frac{3.72\,\text{mm}}{1\text{ m}} = 0.0037.$$

Hence, the focal length of the lens should be $f = s_1 m = 10\,\text{m} \cdot 0.0037 = 37\,\text{mm}$.

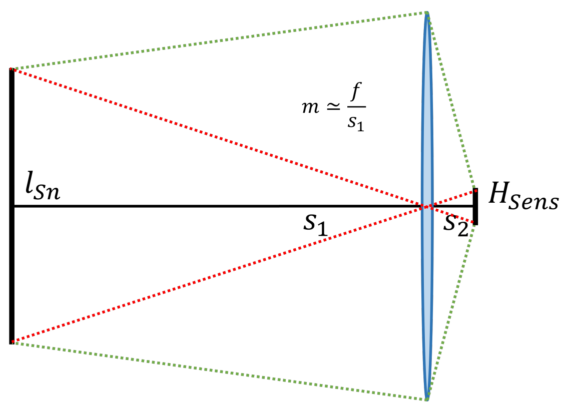

Figure 2: Magnification factor.

Part 2: Optical power and photon flux

The photon flux impinging on a pixel can be evaluated with the following steps.

1. We first calculate the area of the scene that corresponds to one pixel: only photons coming from this portion of the scene are captured by the considered pixel.
2. Starting from the optical power of the source, we evaluate the optical power that is impinging on the just-calculated surface.
3. We evaluate the optical power that is reflected and its intensity.
4. We finally calculate the optical power incident on the pixel. From this number, one can deduce the corresponding photon flux.

As a first step, we need to calculate the area of the scene that corresponds to one pixel, i.e. the area of the scene whose reflected photons, if captured by the lens, are focused on this pixel. Photons reflected from other points of the scene will not reach this pixel. For a given pixel area $A_{Pix} = (1.5\,\mu\text{m})^2$, the corresponding area of the scene $A_{SnPix,0}$ is[1]

$$A_{SnPix} = \frac{A_{pixel}}{m^2} = \frac{(1.5\,\mu\text{m})^2}{(0.0037)^2} = (0.4\,\text{mm})^2\,.$$

Only photons coming from the area denoted as A_{SnPix} will be focused on one considered pixel. Other photons will be focused on other pixels or not focused on the sensor at all.

The next step consists in evaluating the optical power coming from the light source that is impinging on A_{SnPix}. This can be calculated with solid angles.

[1]Note: in our situation, the camera system is tilted, by an angle $\beta = 60^\circ$ and the effective area of the scene, whose reflected photons will be captured by the lens, would be larger. To avoid additional boring geometrical considerations, we ignore this as we know that the error we take is low.

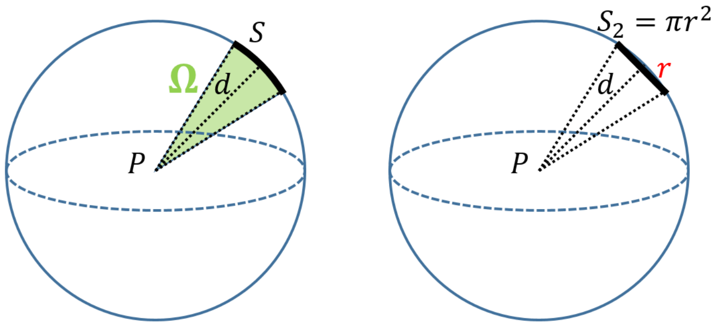

Figure 3: Solid angle.

Few words about solid angles. The solid angle defined by a point P in the space and a portion of spherical surface S belonging to the P-centered sphere with a radius d is defined as (Fig. 3)

$$\Omega = \frac{S}{d^2}.$$

It is immediate to calculate the full solid angle (4π sr, steradians), and the half solid angle (2π). For small angles, the spherical surface portion S can be well approximated by the surface S_2, whose area is easier to calculate since it is a flat 2D area (Fig. 3):

$$\Omega \simeq \frac{S_2}{d^2}.$$

The source is said to emit 60 W of optical power. The optical power is proportional to brightness. The higher the optical power, the higher the source brightness. However, the *perceived* brightness of a source/object is strongly wavelength-dependent as the responsivity of our eye depends on the wavelength (it has a peak around 550 nm). Hence, if the PSD (power spectral density) of two light sources is different, it may happen that a low-power emitter is perceived as brighter than a high-power emitter, if the PSD of the former is centered in the green.
A light source is usually characterized by its power spectral density rather than with its optical power, as the reflectivity of objects or the transmittance of filters is usually wavelength-dependent, as well. The final optical power will depend on the integral of the overall power spectral density integrated on the whole visible range. Just for the sake of simplicity, we will not deal with PSDs, but we will directly use powers, to avoid lengthy integral computations. As the light source is isotropic, its optical intensity (i.e. the optical power per unit solid angle) can be calculated as

$$I_{Src} = \frac{P_{Src}}{4\pi} = \frac{60\ \mathrm{W}}{4\pi\ \mathrm{sr}} = 4.77\ \mathrm{W/sr},$$

since, for a point-like isotropic source, the emitted optical power is spread evenly over a sphere whose solid angle is 4π steradians.

How much of this optical intensity drops on the portion A_{SnPix} of the scene? We should calculate the solid angle seen from the source towards A_{SnPix}:

$$\Omega_{SnPix,Src} = \frac{A_{SnPix}}{d_1^2} = \frac{(0.4\,\mathrm{mm})^2}{(2\,\mathrm{m})^2} = 40\,\mathrm{nsr}.$$

The optical power impinging on $A_{Sn,Pix}$ is simply given by

$$P_{SnPix,I} = I_{Src}\Omega_{SnPix,Src} = 4.77\,\mathrm{W/sr} \cdot 40\,\mathrm{nsr} = 192\,\mathrm{nW}.$$

Not all of the incident optical power is reflected: part of it might be absorbed, as in our case where the reflectivity is 0.2, i.e. only 20% of the incident photons are reflected:

$$P_{SnPix,R} = P_{SnPix,I}R_{Sn} = 192\,\mathrm{nW} \cdot 0.2 = 38.4\,\mathrm{nW}.$$

We should now evaluate *where* this luminous optical power is reflected and how much of this reflected power is captured by the lens. Let us first consider the simplest situation, where the reflection is isotropic on half of the solid angle. In this case, the reflected optical power per unit solid angle, i.e. the optical intensity, would be

$$I_{R,Iso} = \frac{P_R}{2\pi},$$

uniform across all angles of the hemisphere of reflection.
In our case, the scene is said to have a Lambertian reflectance, i.e. the reflected optical intensity is not uniform across all angles, but it follows a cosine law:

$$I_{R,Lamb}(\beta) = I_{R,Lamb,0}\cos(\beta).$$

One can demonstrate that, for a given reflected optical power P_R, the peak of the reflected intensity $I_{R,Lamb,0}$ is twice the intensity we would have with an isotropic emission:

$$I_{R,Lamb,0} = \frac{P_R}{\pi},$$

and

$$I_{R,Lamb}(\beta) = \frac{P_R}{\pi}\cos(\beta).$$

This is due to the fact that the the 2D integral of the intensity over the hemisphere should give the total reflected power. The two intensity profiles (isotropic and Lambertian) are sketched in Fig. 4.
In our situation, we have that the optical intensity $I_{SnPix,R,L}$ reflected by A_{SnPix} is

$$I_{SnPix,R,Lamb}(\beta) = \frac{P_{SnPix,R}}{\pi}\cos(\beta).$$

Since the lens is 60°-tilted with respect to the normal of the surface, the luminous intensity reflected by the portion A_{SnPix} *directed in the direction of the lens* is:

$$I_{SnPix,R,Lamb,\beta} = I_{SnPix,R,Lamb}(60^\circ) = \frac{P_{SnPix,R}}{\pi}\cos 60^\circ = \frac{38.4\,\mathrm{nW}}{\pi\,\mathrm{sr}} \cdot 0.5 = 6.15\,\mathrm{nW/sr}.$$

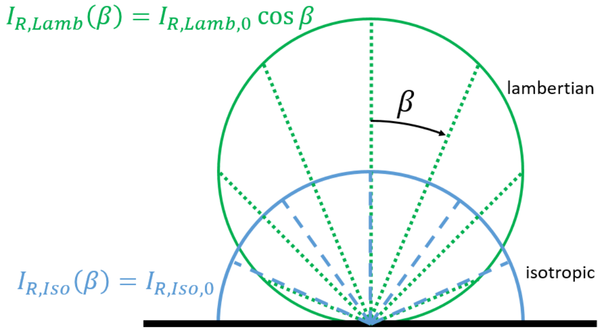

Figure 4: Isotropic vs lambertian reflectance intensity profile.

We now evaluate the optical power impinging on one pixel. This value is equal to the optical power captured by the lens, i.e. all photons coming from A_{SnPix} and captured by the lens are focused on the considered pixel. This can be calculated by multiplying the reflected optical intensity by the solid angle seen from A_{SnPix} towards the lens. Note that this is an approximation: we are assuming that all the lens aperture is seen from the same angle β from every captured point of the scene. This is an acceptable approximation, as (i) generally the distance between the scene and the lens is several times the lens aperture and (ii) the area A_{SnPix} of the scene, corresponding to the area captured by a single pixel on the sensor, is generally small compared to the distance s_1. Since the focal length is 37 mm and the F-number is equal to 4, the lens diameter and area are:

$$D_{Lens} = \frac{f}{F} = \frac{37\,\text{mm}}{4} = 9.3\,\text{mm}, \qquad A_{Lens} = \pi\left(\frac{D_{Lens}}{2}\right)^2 = (8.2\,\text{mm})^2\,,$$

and the solid angle seen from A_{SnPix} towards the lens is

$$\Omega_{Lens,SnPix} = \frac{A_{Lens}}{s_1^2} = 680\,\text{nsr}.$$

The optical power impinging on the pixel is thus

$$P_{Pix} = I_{SnPix,R,Lamb,\beta}\Omega_{Lens,SnPix} = 6.15\,\text{nW/sr} \cdot 680\,\text{nsr} = 4.2\,\text{fW}.$$

The rate of photons incident on the considered pixel can be calculated by dividing the incident optical power by the average photon energy E_{ph} of the incident photons (650 nm corresponds to the orange color). We have

$$E_{Ph} = \frac{hc}{\lambda} = \frac{6.626 \cdot 10^{-34}\,\text{J s} \cdot 3 \cdot 10^{8}\,\text{m/s}}{650\,\text{nm}} = 304 \cdot 10^{-21}\,\text{J} = 1.9\,\text{eV}.$$

and thus

$$N_{Ph} = \frac{P_{Pix}}{E_{ph}} = \frac{8.4\,\text{fJ/s}}{304 \cdot 10^{-21}\,\text{J/photon}} = 13750\,\text{photons/s}.$$

We can calculate the photon flux per unit area, by simply dividing the previous number by the pixel area:

$$\phi_{Ph} = \frac{N_{Ph}}{A_{Pix}} = \frac{13750\,\text{photons/s}}{(1.5\,\mu\text{m})^2} = 6.1 \times 10^{15}\,\text{photons/s/m}^2.$$

Part 3: Calculation of the photocurrent

The quantum efficiency of the pixel $\eta_{Pix} = 0.6$ is defined as the ratio between the number of photogenerated electrons and the number of photons that reach the photodetector. In front of the considered pixel, an optical red filter is placed, whose transmittance coefficient is $T_{Red} = 0.85$. Considering these losses, the photocurrent i_{Ph} can be calculated as

$$i_{ph} = q\eta_{Pix}T_{Red}N_{Ph} = 1.6 \cdot 10^{-19}\,\text{C} \cdot 0.6 \cdot 0.85 \cdot 13750\,\text{photons/s} = 1.12\,\text{fA}.$$

The photocurrent could have also been calculated through the responsivity $\Re$ of the sensor, which relates the photocurrent and the optical power. Assuming an average wavelength of 650 nm,

$$\Re = \eta\frac{q}{hc}\lambda = \frac{0.6 \cdot 1.6 \cdot 10^{-19}\,\text{C}}{6.626 \cdot 10^{-34}\,\text{J s} \cdot 3 \cdot 10^{8}\,\text{m/s}} 650\,\text{nm} = 0.31\,\text{A/W}$$

$$i_{ph} = T_{filter}\Re P_{Pix} = 0.85 \cdot 0.31\,\text{A/W} \cdot 4.2\,\text{fW} = 1.12\,\text{fA}$$

We can write the expression of the photocurrent by expanding all the used equations. We can start from the optical power incident on the pixel.

$$P_{Pix} = I_{SnPix,R,Lamb,\beta}\Omega_{Lens,SnPix} = \frac{P_{SnPix,R}}{\pi}\cos(\beta)\frac{A_{Lens}}{s_1^2} = \frac{P_{SnPix,R}}{\pi}\cos(\beta)\frac{\pi\left(\frac{D_{Lens}}{2}\right)^2}{s_1^2}$$

$$P_{Pix} = \frac{P_{SnPix,I}R_{Sn}}{\pi}\cos(\beta)\frac{\pi\left(\frac{\frac{f}{F}}{2}\right)^2}{s_1^2} = \frac{I_{Src}\Omega_{SnPix,Src}R_{Sn}}{\pi}\cos(\beta)\frac{\pi\left(\frac{\frac{f}{F}}{2}\right)^2}{s_1^2}$$

$$P_{Pix} = \frac{I_{Src}\frac{A_{SnPix}}{d_1^2}R_{Sn}}{\pi}\cos(\beta)\frac{\pi\left(\frac{\frac{f}{F}}{2}\right)^2}{s_1^2} = \frac{\frac{P_{Src}}{4\pi}\frac{\frac{A_{Pix}}{m^2}\cos(\beta)}{d_1^2}R_{Sn}}{\pi}\cos(\beta)\frac{\pi\left(\frac{\frac{f}{F}}{2}\right)^2}{s_1^2}$$

$$P_{Pix} = \frac{P_{Src}A_{Pix}R_{Sn}}{16\pi d_1^2F^2}$$

Hence, the photocurrent is

$$i_{ph} = q\eta_{Pix}T_{Red}N_{Ph} = q\eta_{Pix}T_{Red}\frac{P_{Pix}}{E_{ph}}.$$

$$i_{ph} = q\eta_{Pix}T_{Red}\frac{\lambda}{hc}\frac{P_{Src}A_{Pix}R_{Sn}}{16\pi d_1^2F^2}.$$

Few considerations are reported.

- An increase in the quantum efficiency and in the transmission coefficient of the color filter would be beneficial for the signal.
- The pixel photocurrent is proportional to the squared inverse of the F-number. An F-number change from 4 to 11 decreases the photocurrent by a factor ≈ 8.
- A larger pixel would lead to a higher photocurrent.
- The optical power is independent from the angle β! This is in agreement with the fact that the scene is a lambertian reflector: the incident photon flux is the same, independently on the angle; in fact, when increasing the angle, the intensity decreases, but the projected area increases by the same amount.
- Do not be fooled by the dependence on the λ coefficient: a higher wavelength does not *always* lead to a higher current! We all know that given a flux of photons, the photocurrent is independent from the wavelength: one photon means one EHP. The dependence from the wavelength is *real* when considering a fixed optical power: the same optical power corresponds to different fluxes if the wavelength is different.

Part 4: Resolution

We can calculate the dimension of the first Airy circle, that determines the diffraction-limited resolution:

$$d_{Airy} = 2.44 \cdot \frac{\lambda}{D}f = 2.44 \cdot \lambda F = 2.44 \cdot 650\,\text{nm} \cdot 4 = 6.3\,\mu\text{m}.$$

The diffraction spot turns out to be larger than the pixel size (1.5 μm). The resolution of this sensor is thus diffraction limited: this means that, for this sensor size (and this F-number), increasing the number of pixels does not improve the resolution. Additionally, we can compare the spatial sampling frequency and the maximum frequency of the impinging optical signal, limited by diffraction:

$$f_{sample} = \frac{1}{l_{Pix}} = 0.67\mu m^{-1} \qquad f_{max,signal} = \frac{1}{d_{Airy}} = 0.16\mu m^{-1}$$

It turns out that $f_{sample} > 2 \cdot f_{max,signal}$, which implies that there is no risk of aliasing effects as the sampling theorem is respected. There is in this case no need for an anti-aliasing filter.