Politecnico di Milano
MSc Course - MEMS and Microsensors - 2023/2024

E14
MEMS Microphone Design

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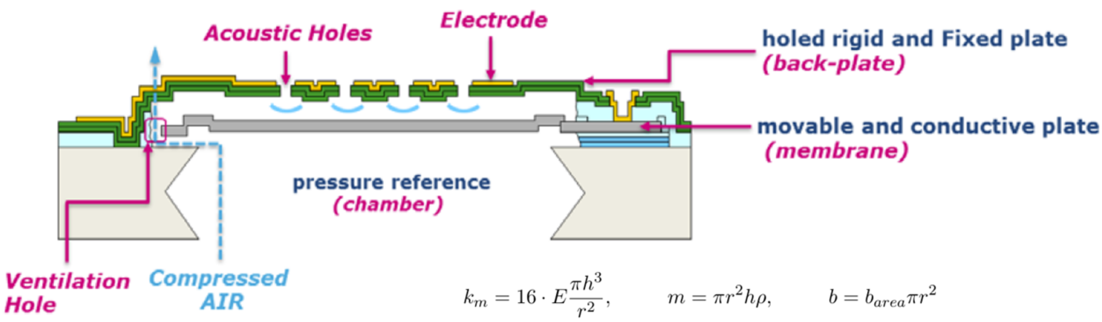
Problem

After a request from a mobile phone company, your audio start-up needs to design a novel MEMS microphone. The schematic of the device is shown in Fig. 1. The system parameters are reported in Table 1. You are asked to...

1. Choose the diameter of the membrane in order to comply with noise requirements, aiming at a well-balanced sensor in terms of noise performance. Calculate then the electromechanical sensitivity [in fF/Pa].
2. Calculate the required bias voltage, V_{DC}, in order to have a well-balanced sensor in terms of noise performance.
3. Suitably dimension the feedback network of the front-end and graph the final transfer function from input pressure to output voltage.

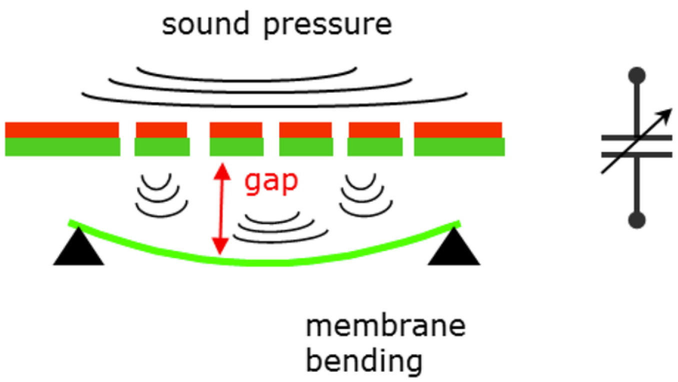
Structure		
Membrane thickness	h	1 μm
Vertical gap	g	1 μm
Poly-Si density	ρ	2390 kg/m^3
Normalized damping coefficient	b_{area}	350 N/(m/s)/m^2
Electronics		
Parasitic capacitance	C_I	5 pF
Op-amp voltage noise	$s_{n,OA,v}$	20 nV/$\sqrt{\text{Hz}}$
Maximum analog voltage swing	V_{max}	$\pm$ 2.5 V
Requirements		
Min. detectable pressure	$p_{a,min,dBSPL}$	33 dBSPL
Max. detectable pressure (acoustic overload point)	$p_{a,max,dBSPL}$	123 dBSPL
Min. detectable frequency	f_{min}	20 Hz
Max. detectable frequency	f_{max}	20 kHz

Table 1: Parameters of the microphone.



$$k_m = 16 \cdot E\frac{\pi h^3}{r^2}, \qquad m = \pi r^2 h\rho, \qquad b = b_{area}\pi r^2$$

Figure 1: MEMS microphone sketch and spring-mass-damper formulas.



$$p_{SPL} = 20\log_{10}\frac{p}{p_{ref}}$$

$$p_{ref} = 20\ \mu Pa$$

Figure 2: MEMS microphone model and SPL definition.

Introduction

A MEMS microphone is, as usual, a dual-die device consisting of two components, the integrated circuit and the sensor. The sensor uses MEMS technology and is basically a single-ended polysilicon capacitor, consisting of two plates/surfaces. One plate is fixed while the other one can deform under the action of an AC pressure (respectively, the green plate and the grey one shown in Fig. 1). The fixed surface is full of acoustic holes, allowing sound to pass through. A ventilation hole allows the air compressed in the back chamber to flow out and consequently allows the membrane to move back, with no deflection under DC pressure. A simplified model is reported in Fig. 2. The integrated circuit converts the change of the MEMS capacitance into an analog output.

The input signal of a microphone is sound pressure, p. Sound pressure (or acoustic pressure) is the local pressure deviation from the ambient (average, or equilibrium) atmospheric pressure. An AC sound pressure wave can be described as

$$p = p_a \sin\left(2\pi f_a t\right),$$

where f_a is the acoustic frequency. The acoustic frequency range is defined from 20 Hz to 20 kHz. Usually, sound pressure is expressed in dBSPL:

$$p_{SPL} = 20\log_{10}\frac{p_a}{p_{ref}},$$

where p_{ref} is the reference pressure,

$$p_{ref} = 20\,\mu\text{Pa}, \qquad p_{ref,SPL} = 0\,\text{dBSPL},$$

which is commonly considered as the threshold of human hearing (roughly the sound of a mosquito flying 3 m away). Fig. 3 reports examples of sound levels.
We can thus easily translate our requirements in pascals:

$$p_{a,min} = p_{ref}\cdot 10^{\left(\frac{p_{a,min,SPL}}{20}\right)} = 893\,\mu\text{Pa},$$

$$p_{a,max} = p_{ref}\cdot 10^{\left(\frac{p_{a,max,SPL}}{20}\right)} = 28.3\,\text{Pa},$$

that corresponds to a quiet whisper and a loud rock concert. Given the frequency range of interest, we can determine the input-referred pressure white noise density, by simply dividing the minimum detectable signal by the bandwidth of our sensor, i.e. the whole audio range:

$$s_{n,p} = \frac{p_{a,min}}{\sqrt{f_{max}-f_{min}}} \simeq \frac{p_{a,min}}{\sqrt{f_{max}}} = 6.31\,\mu\text{Pa}/\sqrt{\text{Hz}}.$$

Given a certain pressure, p, applied to the membrane, the corresponding applied force can be simply evaluated as

$$F = pA = p\pi r^2,$$

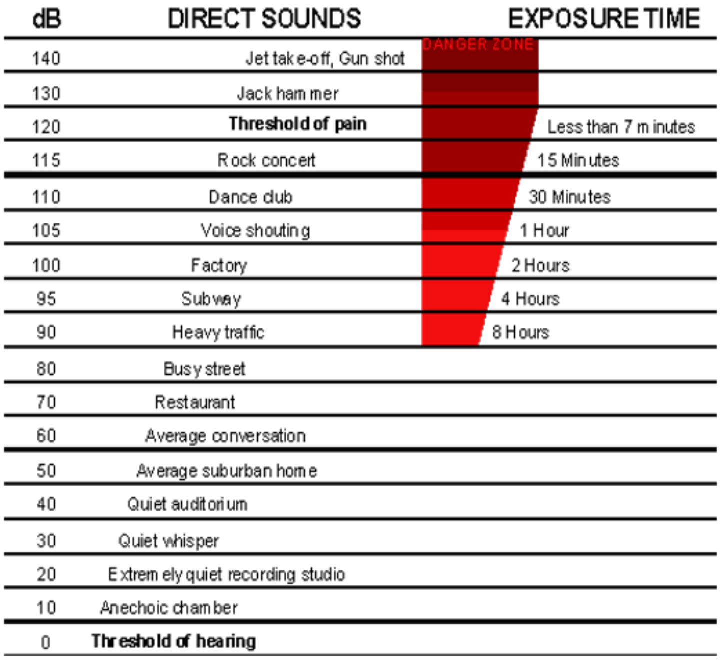


Figure 3: Sound level examples.

where A is the membrane area and r its radius. The force F is a uniformly distributed action that causes a certain displacement of the membrane.
To be more precise, the membrane deflects if there is a pressure difference between its two sides. As one can observe from Fig. 1, both the chambers faced by the membrane are kept at the atmospheric pressure, so that the DC displacements induced by the atmospheric pressure are canceled; the pressure difference applied to the membrane, p_m, is

$$p_m = (p_{atm} + p_{sound}) - p_{atm} = p_{sound}.$$

It can be demonstrated (e.g. in I. O. Wygant, M. Kupnik and B. T. Khuri-Yakub, *Analytically calculating membrane displacement and the equivalent circuit model of a circular CMUT cell*, in 2008 IEEE Ultrasonics Symposium, Beijing, 2008, pp. 2111-2114) that a vibrating membrane can be well-approximated as a 1D mass-spring-damper system (Fig. 4). This 1D model is defined in this way: given a uniformly distributed force applied onto the membrane, the dynamic behavior of the membrane can be equivalently modeled as a 1D piston that uniformly moves in the y-direction, whose mass is m, whose spring constant is k_m, and whose damping coefficient is b. For a circular membrane of radius r, the 1D parameters are

$$k_m = 16 \cdot E\frac{\pi h^3}{r^2}, \qquad m = \pi r^2 h\rho, \qquad b = b_{area}\pi r^2.$$

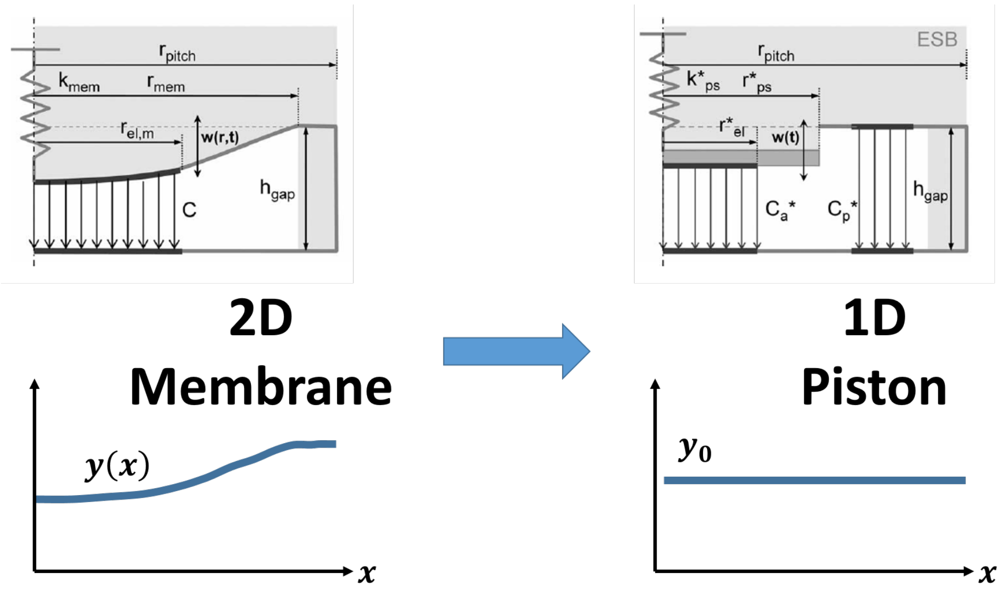


Figure 4: 2D vs 1D model of the membrane.

The resonance frequency of such a membrane can be evaluated as

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k_m}{m}}$$

The model is defined in such a way that, given a certain force F applied to the membrane, the corresponding displacement

$$y = \frac{F}{k_m},$$

represents the average displacement of the membrane (see Fig. 4).

Question 1

The diameter of the membrane can be chosen from noise specifications. Aiming at a well-balanced sensor in terms of noise performance, we get:

$$s_{n,MEMS,p} = s_{n,ELN,p}, \qquad s_{n,TOT,p} = \sqrt{2}s_{n,MEMS,p} = \sqrt{2}s_{n,ELN,p},$$

i.e.

$$s_{n,MEMS,p} = \frac{s_{n,TOT,p}}{\sqrt{2}},$$

As $F = Ap$, input-referring MEMS thermo-mechanical noise, we get

$$s_{n,MEMS,p} = \frac{s_{n,MEMS,F}}{A} = \frac{\sqrt{4k_bTb}}{A} = \frac{\sqrt{4k_bTb_{area}A}}{A} = \frac{\sqrt{4k_bTb_{area}\pi r^2}}{\pi r^2}.$$

And we can hence derive the required radius:

$$r = \frac{\sqrt{\frac{4k_b T b_{area}}{\pi}}}{\frac{s_{n,TOT,p}}{\sqrt{2}}} = 304\,\mu\text{m}.$$

Given the membrane radius, we infer the 1D mass-spring-damper equivalent coefficients:

$$k_m = \frac{16\pi E h^3}{r^2} = 81.5\,\text{N/m},$$

$$m = \pi r^2 h \rho = 0.69\,\text{nkg},$$

$$b = b_{area}\pi r^2 = 101 \cdot 10^{-6}\,\text{N/(m/s)}.$$

So that the resonance frequency and the quality factor become:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k_m}{m}} = 54\,\text{kHz}, \qquad Q = \frac{2\pi f_r m}{b} = 2.3$$

Given the 1D model of the mechanical structure, we can easily report its mechanical transfer function, as

$$G(j\omega) = \frac{Y(j\omega)}{F(j\omega)} = \frac{1}{k_m + j\omega b - \omega^2 m},$$

where F is the applied force. As we are dealing with a microphone, i.e. a (sound) pressure sensor in the acoustic frequency range (20 Hz - 20 kHz), we can more conveniently evaluate the mechanical transfer function of our microphone as the displacement divided by the input pressure. As $F = \pi r^2 p$,

$$T(j\omega) = \frac{Y(j\omega)}{P(j\omega)} = \pi r^2 \frac{1}{k_m + j\omega b - \omega^2 m},$$

whose magnitude is reported in Fig. 5.

Note that this is a good dynamic behavior! Indeed, as we are working with a very large signal bandwidth, the best choice for our structure is to have the resonance frequency outside the frequency range of interest (like in accelerometers), possibly with a low quality-factor, in order to reject under-damped-related phenomena, such as long time constant, overshoots, ... The 54 kHz value of the resonance frequency is thus perfect for bandwidth constraints, as it automatically filters out any force/pressure at frequencies higher than the audio range (e.g. ultrasound).

As, for microphone purposes, we are working at frequencies lower than the resonance frequency, our microphone can be equivalently modeled as the spring constant only. As our input is a pressure, we can more conveniently define the mechanical sensitivity of our sensor, $\partial y/\partial p$, defined as the ratio between the average membrane displacement, y, and the pressure, p:

$$\frac{\partial y}{\partial p} = |T(\omega)|_{\omega \ll \omega_r} = \frac{\pi r^2}{k_m} = \frac{\pi r^2}{\frac{16\pi E h^3}{r^2}} = \frac{r^4}{16 E h^3} = 3.56\,\text{nm/Pa}.$$

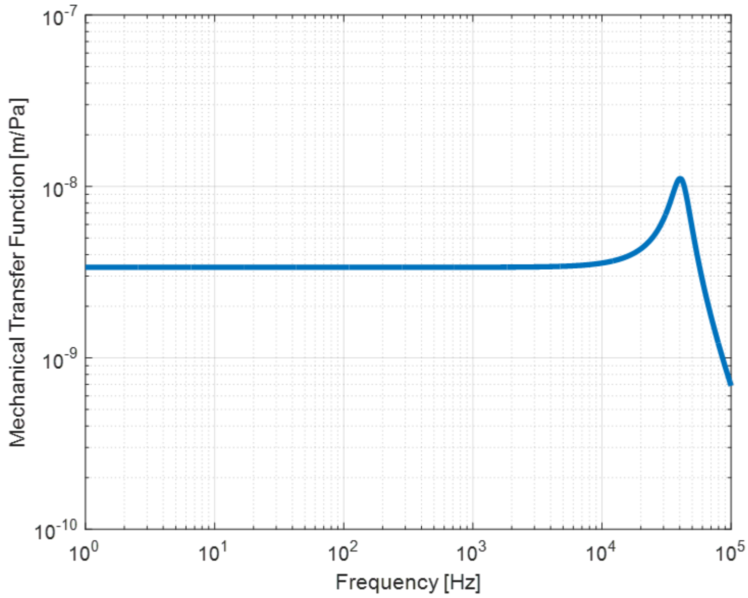


Figure 5: Transfer function modulus of the sensor, from sound pressure to displacement.

How can we read out the pressure-induced displacement? The variable-gap structure is basically a single-ended variable gap capacitor, whose capacitance variation can be easily read-out with a TCA-based front-end. We can calculate the rest capacitance of the structure and its variation per unit y-axis displacement

$$C_0 = \frac{\varepsilon_0 A}{g} = \frac{\varepsilon_0 \pi r^2}{g} = 2.57\,\mathrm{pF},$$

$$\frac{\partial C}{\partial y} = \frac{C_0}{g} = \frac{\varepsilon_0 \pi r^2}{g^2} = 2.57\,\mathrm{fF/nm}.$$

Combining the equations above, we get an overall electromechanical sensitivity of 9.2 fF/Pa. The previously calculated thermo-mechanical noise can be re-calculated as capacitance noise:

$$s_{n,MEMS,C} = s_{n,MEMS,p} \frac{\partial y}{\partial p} \frac{\partial C}{\partial y} = s_{n,MEMS,F} \frac{1}{k_m} \frac{\partial C}{\partial y} = 40.9\,\mathrm{zF}/\sqrt{\mathrm{Hz}},$$

flat within the whole frequency range of interest. Our hypothesis of a well-balanced sensor is satisfied if the input-referred noise of the front-end is equal to 40.9 zF/$\sqrt{\text{Hz}}$.

Question 2

The readout electronics system is reported in Fig. 6. The membrane is biased at V_{DC} with a dedicated voltage supply, while the other electrode is connected with the virtual

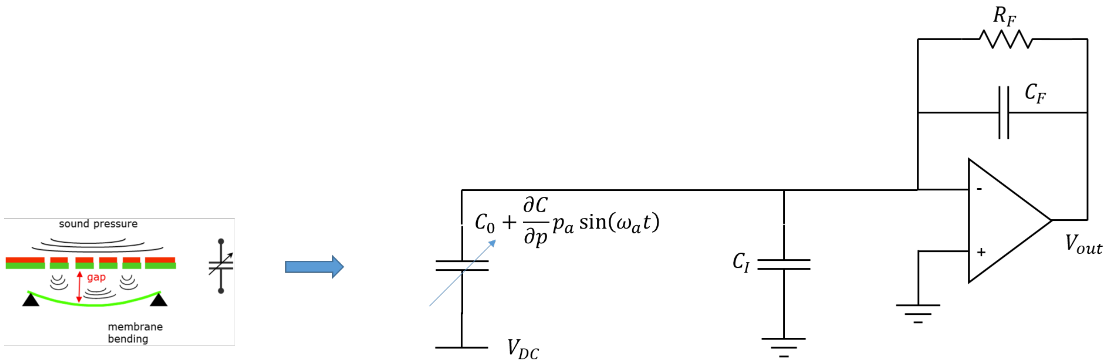


Figure 6: MEMS microphone redout system.

ground of the front-end. The transfer function of the front end is

$$\frac{V_{out}(s)}{I(s)} = -\frac{R_F}{1 + sR_FC_F}.$$

Remembering that

$$i = \frac{\partial C}{\partial t} V_{DC}, \qquad I(s) = sV_{DC}C(s)$$

we can re-evaluate the transfer function of the TCA, now defined as the ratio between the output voltage, V_{out}, and the capacitance variation C, as

$$T_{TCA}(s) = \frac{V_{out}(s)}{C(s)} = -sV_{DC}\frac{R_F}{1 + sR_FC_F},$$

which introduces a high-pass filtering action, whose pole frequency is

$$f_p = \frac{1}{2\pi R_F C_F}.$$

If the pole frequency is designed to be equal to (or lower than) the minimum signal frequency, 20 Hz, the gain of the TCA (in the audio range) is flat and equal to

$$\frac{\partial V_{out}}{\partial C} = \left|T_{TCA}(\omega)\right|_{\omega \gg \omega_p} = \frac{V_{DC}}{C_F}.$$

The input-referred noise of the front-end, assuming that the dominant noise source is the voltage noise of the op-amp, can be expressed as

$$s_{n,ELN,C} = s_{n,OA,v}\frac{1 + \frac{C_P}{C_F}}{\frac{V_{DC}}{C_F}} \simeq s_{n,OA,v}\frac{C_P}{V_{DC}}.$$

In the previous expression, we assumed $C_P \gg C_F$. Note that, as shown in Fig. 6, the parasitic capacitance is given by the sum of two contributions: the interconnections capacitance (wire bonding + chip metal wires), modeled as C_I, and the DC (rest) capacitance of the membrane, C_0, that contributes, as well, to the calculation of the overall capacitance from the virtual ground to ground:

$$C_P = C_I + C_0 = 7.57\,\text{pF}.$$

The DC bias voltage, V_{DC}, that forces electronics noise to be equal to MEMS thermo-mechanical noise is thus:

$$V_{DC} = \frac{s_{n,OA,v} C_P}{s_{n,ELN,C}} = \frac{s_{n,OA,v} C_P}{s_{n,MEMS,C}} = 3.70\,\text{V}.$$

Question 3

The feedback capacitance of the TCA determines the gain of the front-end,

$$\frac{\partial V_{out}}{\partial C} = \left|T_{TCA}(\omega)\right|_{\omega \gg \omega_p} = \frac{V_{DC}}{C_F}.$$

It can be sized in such a way that, given the maximum capacitance variation (that corresponds to the maximum displacement, i.e. to the maximum sound pressure), the output voltage variation is equal to V_{max}:

$$V_{max} = \frac{V_{DC}}{C_F} C_{max},$$

where $C_{max} = 259$ fF from the previous formulas at the FSR of 28.3 Pa. Hence,

$$C_F = \frac{V_{DC} C_{max}}{V_{max}} = 383\,\text{fF}.$$

A short comment about non-linearity. The linearity error, ϵ_{lin}, of a single-ended parallel-plate-based capacitance measurement can be evaluated as

$$\epsilon_{lin} = \frac{\Delta C_{real} - \Delta C_{lin}}{\Delta C_{real}} = \frac{\left(\frac{\varepsilon_0 A}{g-y} - \frac{\varepsilon_0 A}{g}\right) - \frac{\varepsilon_0 A}{g^2} y}{\frac{\varepsilon_0 A}{g-y} - \frac{\varepsilon_0 A}{g}} = \cdots = \frac{y}{g}.$$

This means that, with the previously calculated maximum displacement, about 100 nm, the linearity error is about 10%! Remember that, if the capacitance measurement is differential, the linearity error can be expressed as

$$\epsilon_{lin,diff} = \frac{\Delta C_{diff,real} - \Delta C_{diff,lin}}{\Delta C_{diff,real}} = \cdots = \left(\frac{y}{g}\right)^2.$$

Hence, if the readout was differential, with the same maximum displacement, the linearity error would be much lower, in the range of 1%. This is why a differential readout would

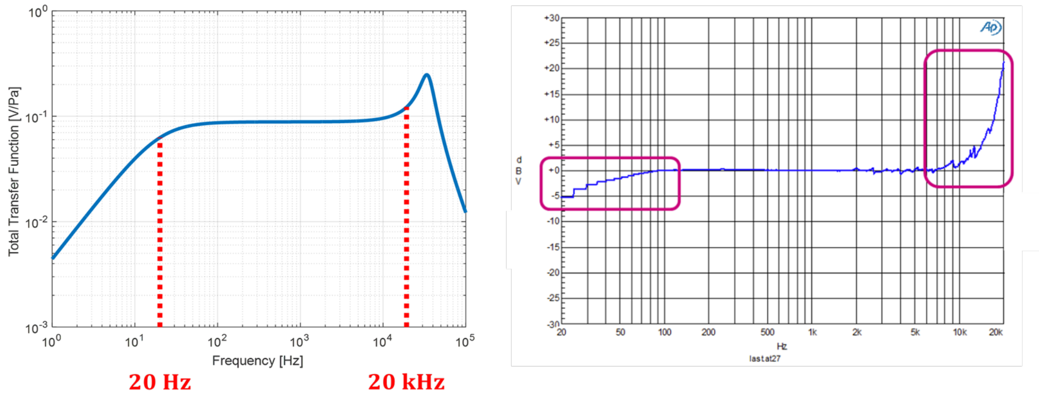


Figure 7: Magnitude of the mechanical transfer function of the sensor, from sound pressure to displacement (left), compared to the results on a real microphone.

be beneficial. Given a feedback capacitance of 383 fF, the required feedback resistance, in order to have a pole frequency at 20 Hz is $R_{max} = \frac{1}{2\pi C_F f_p} = 20\,\mathrm{G}\Omega$.
The overall transfer function of the sensor, defined as the ratio between the output voltage and the input pressure, is

$$T_{TOT}\left(j\omega\right) = \frac{V_{out}\left(j\omega\right)}{P\left(j\omega\right)} = -\pi r^2 \frac{1}{k + j\omega b - \omega^2 m} \frac{\partial C}{\partial y} j\omega V_{DC} \frac{R_F}{1 + j\omega R_F C_F},$$

whose magnitude, reported in Fig. 7 resembles a pass-band filter where the bass-band frequency range is, correctly, the frequency range of interest! Note that this theoretical frequency response is very similar to an experimental one (see again Fig. 7). Fig. 8 reports a photograph of the membrane of a commercial MEMS microphone.

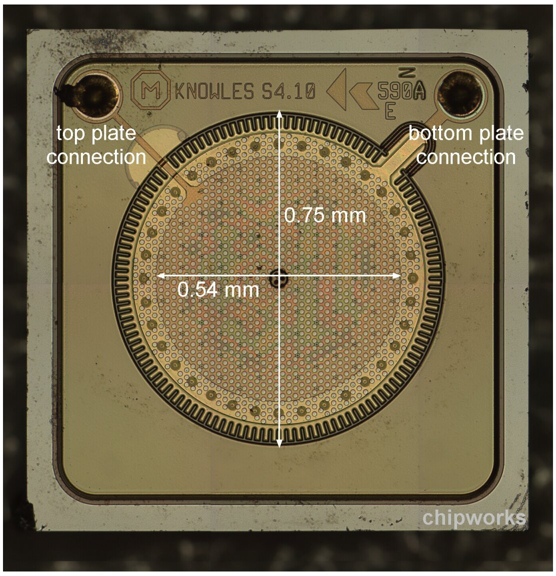


Figure 8: SEM photograph of the membrane of a MEMS microphone.