Politecnico di Milano MSc Course - MEMS and Microsensors - 2024/2025

E13 Gyroscope Sense Design

Riccardo Nastri riccardo.nastri@polimi.it

14/11/2024

PROBLEM

Working for the European Space Agency, you are asked to design the sense electronics of a mode-split capacitive MEMS gyroscope for space applications. The device parameters are given in Table 1 (C_P is the parasitic capacitance between stators and ground). The circuit topology is shown in Fig. 1.

- 1. Calculate the sensitivity in terms of sense capacitance per unit angular rate, CA output voltage per unit angular rate, and INA output voltage per unit angular rate. Additionally, calculate the FSR.
- 2. Evaluate the intrinsic resolution of the sensor, due to thermo-mechanical noise only.
- 3. Size the feedback resistance of the CA-based front-end, in order to obtain a well-balanced system in terms of noise performance.
- 4. Calculate the voltage noise PSD of the INA required not to worsen the resolution of the sensor, and the needed bias current of the input op-amps of the INA, in order to fulfill this requirement.

Structure		
Process thickness	h	$22 \ \mu \mathrm{m}$
Gap	g	$1.9~\mu\mathrm{m}$
External mass (half structure)	m_e	2 nKg
Internal mass (half structure)	m_i	8 nKg
Sense axis		
Sense resonance frequency	f_{rs}	19800 Hz
Sense damping (half structure)	b_s	$2 \mu \mathrm{N/(m/s)}$
Parallel-plate length	L_{PP}	$308~\mu\mathrm{m}$
Parallel-plate cells (half structure)	N_{PP}	8
Rotor-to-stator DC voltage	V_{DC}	10 V
Drive axis		
Drive displacement amplitude	x_{da}	$6~\mu\mathrm{m}$
Drive resonance frequency	f_{rd}	19000 Hz
Drive damping (half structure)	b_s	$100 \mathrm{~nN/(m/s)}$
Electronics		
Amplifier supply voltage	V_{DD}	0-3.3 V
CA feedback capacitance	C_F	2 pF
Parasitic capacitance	C_P	10 pF
INA resistance	R_{GAIN}	4.94 kΩ
INA gain	G_{INA}	$1+49.4~\mathrm{k}\Omega~/~R_{GAIN}$
CA op-amp voltage noise PSD	$s_{n,OAv,V}$	$3.3~\mathrm{nV}/\sqrt{\mathrm{Hz}}$
CA op-amp current PSD	$s_{n,OAi,I}$	$1~\mathrm{fA}/\sqrt{\mathrm{Hz}}$
INA op-amp MOS overdrive voltage	V_{OV}	200 mV
INA op-amp MOS γ coefficient	γ	2/3

Table 1: Parameters of the gyroscope.

Introduction

A convenient way to deal with noise, in any sensor, is to write the resulting effect of the various noise sources at the output node of the system (or even at an intermediate node), and then to input-refer these quantities in terms of equivalent noise density through the sensitivity (or the partial sensitivity up to that intermediate point).

Note that, sometimes, noise is not characterized by a white density. In systems where modulation/demodulation is applied, and the sensor bandwidth is much lower than the working frequency (e.g. in gyroscopes operating around 20 kHz, with a bandwidth limited to few hundred Hz) we can ignore this issue. Indeed, one can calculate noise just around the operating frequency, assuming that, after demodulation, noise components far from it will be filtered out by the LPF (the concept is similar to what we already discussed for accelerometers when we have the rotor modulation).

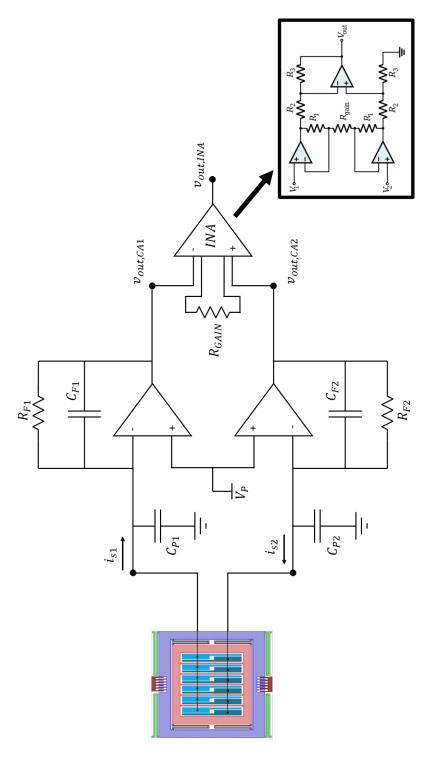


Figure 1: Sense readout circuit.

QUESTION 1

When a gyroscope is operated in mode-split, the resonance frequencies of the drive and sense modes are different: there is a certain, intended (by design) frequency mismatch, f_{Δ} between them. In our case,

$$f_{\Delta} = f_{rs} - f_{rd} = 800 \text{ Hz},$$

 $\omega_{\Delta} = 2\pi f_{\Delta} = 5026 \text{ rad/Hz}.$

Mode-split gyroscopes have several advantages with respect to mode-matched ones. First, we do not need to guarantee that the two resonance frequencies are exactly the same as in mode-matched gyroscopes.

The mechanical sensitivity of a mode-split gyroscope can be expressed as

$$S_{MS} = \frac{x_{da}}{\omega_{\Delta}} = \frac{x_{da}}{\omega_{rs} - \omega_{rd}}.$$

We can observe that this value does not depend on the quality factor of the sense axis Q_s . Hence, as shown in Fig. 2, Q-factor variations have no effects on the sensitivity. Mode-matched gyroscope, on the other hand, must face this issue, as their sensitivity, that can be expressed as

$$S_{MM} = \frac{x_{da}}{\frac{\omega_{rs}}{2Q_s}} = \frac{2x_{da}Q_s}{\omega_{rs}},$$

depends on Q_s .

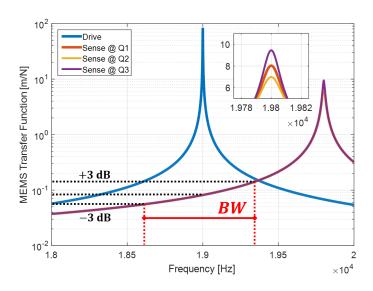


Figure 2: Mechanical transfer functions of drive- and sense-axis of a mode-split gyroscope.

Another pro relates with the bandwidth-vs-noise density trade-off. Mode-matched gy-roscope do show this kind of trade-off: indeed, both the bandwidth (which is limited

by the width of the mechanical transfer function peak) and noise density depend on the sense axis damping coefficient, b_s . This means that, acting in order to improve noise density would reduce the sensor bandwidth, and vice-versa. As we can see from Fig. 2, the bandwidth of a mode-split gyroscope is no more dependent on the width of the resonant peak; indeed, it is much higher, and can be easily limited with an electronic filtering placed at the end of the sense readout chain. With mode-split operation, the bandwidth-vs-noise density trade-off is broken, and it is possible, e.g., to lower the pressure inside the package as much as possible, thus increasing the noise performance of the device, without affecting its maximum sensing bandwidth. For these reasons, most state-of-the-art consumer MEMS gyroscope are mode-split operated.

The sense displacement amplitude sensitivity of our mode-split gyroscope, defined as the sense displacement amplitude variation per unit angular rate, is

$$S_{y_a} = rac{\partial y_a}{\partial \Omega} = rac{x_{da}}{\omega_\Delta} = rac{x_{da}}{2\pi f_\Delta} = 1.19 \ \mathrm{nm/(rad/s)}.$$

Note the measurement unit and remember that angular rate is expressed in rad/s in SI units. The sensitivity can be more conveniently expressed in terms of m/dps, as:

$$S_{y_a} = \frac{1.19 \text{ nm}}{\frac{180^o}{\pi rad}} = \frac{1.19 \text{ nm}}{57.3 \text{ dps}} = 20.8 \text{ pm/dps}.$$

We can now express the sensitivity in terms of single-ended sense capacitance variation per unit angular rate:

$$S_{C_s} = \frac{\partial C_s}{\partial \Omega} = \frac{\partial C_s}{\partial y} \frac{\partial y_s}{\partial \Omega} = \frac{\partial C_s}{\partial y} S_{y_a}.$$

Here, $\partial C_s/\partial y$ is the single-ended capacitance variation of the sense port per unit displacement along the y-axis,

$$\frac{\partial C_s}{\partial y} = \frac{C_{s0}}{g}, \qquad C_{s0} = \frac{\epsilon_0 h 2 L_{PP} N_{PP}}{g} = 506 \, \mathrm{fF}, \qquad \frac{\partial C_s}{\partial y} = \frac{C_{s0}}{g} = 266 \, \mathrm{fF}/\mu\mathrm{m}.$$

(the factor 2 above accounts for the two halves). The sense capacitance variation sensitivity, S_{C_*} , can be thus evaluated as

$$S_{C_s} = \frac{\partial C_s}{\partial \Omega} = \frac{C_{s0}}{g} S_{y_a} = 266 \, \mathrm{fF}/\mu \mathrm{m} \cdot 20.8 \, \mathrm{pm/dps} = 5.54 \, \mathrm{aF/dps}.$$

As mentioned, $\partial C_s/\partial \Omega$ is the single ended capacitance variation per unit angular rate. For a given angular rate, Ω , due to antiphase motion, the capacitance of one sense port increases of a factor $S_{C_s}\Omega$, while the other capacitance decreases by the same factor, i.e., $-S_{C_s}\Omega$. We can define the differential capacitance variation per unit angular rate, as

$$S_{C_{s,diff}} = rac{\partial C_{s,diff}}{\partial \Omega} = 2rac{\partial C_{s}}{\partial \Omega} = 2S_{C_{s}} = 11.1 \ \mathrm{aF/dps}.$$

We know that, if the sense-port is biased with a DC voltage applied between the rotor and the stator, V_{DC} , the sense-port capacitance variation induces a current flowing through the electrode that flows in the feedback path of the front-end used to detect the sense displacement. In our case, the sense current is integrated in the CA feedback capacitance.

$$V_{CA}(t) = G_{CA}C_s(t)$$
,

where

$$G_{CA} = \left| T_{CA} \left(j \omega_{rd} \right) \right| = \left| \frac{V_{CA} \left(j \omega_{rd} \right)}{C_s \left(j \omega_{rd} \right)} \right| = \frac{V_{DC}}{C_F} = 5 \cdot 10^{12} \; \mathrm{V/F}$$

is the gain of the CA, i.e., the transfer function from capacitance variation to output voltage of a CA evaluated at resonance, ω_{rd} .

Hence, the CA sensitivity of the gyro, defined as the single-ended voltage amplitude variation of the CA output per unit angular rate, can be expressed as

$$S_{V_{CA}} = \frac{\partial V_{CA}}{\partial \Omega} = G_{CA}S_{C_s} = 13.3 \text{ mV/Hz} = 27.7 \,\mu\text{V/dps}.$$

The differential CA sensitivity is thus

$$S_{V_{CA,diff}} = 2S_{V_{CA}} = 55.5 \,\mu\text{V/dps}.$$

The two outputs of the two CAs are connected with the inputs of an INA, whose gain is 11. The total sensitivity of the gyroscope, defined as the voltage amplitude variation of the INA output per unit angular rate, can be thus expressed as

$$S_{V_{OUT}} = S_{V_{CA}} 2G_{INA} = S_{V_{CA,diff}} G_{INA} = 610 \,\mu\text{V/dps}.$$

As the supply voltage of the INA is 0-3.3 V, the full scale range corresponds to:

$$FSR = \frac{\pm 1.65 V}{S_{V_{OUT}}} = \pm 2690 dps$$

QUESTION 2

We define the *intrinsic* sensor resolution as the one we get assuming noiseless readout electronics, i.e., considering the sensing element noise only. In case of MEMS, the intrinsic resolution is due to unavoidable thermomechanical noise associated with the structure. For gyroscopes, this resolution is referred to as noise equivalent rate density (NERD).

Thermo-mechanical noise PSD of the sense resonator can be easily modeled in terms of force noise PSD applied to the proof mass:

$$S_{n,MEMS,F} = 4k_bTb_s$$

expressed in N²/Hz; this noise source is white, i.e., it is constant with frequency.

$$s_{n,MEMS,F} = \sqrt{S_{n,MEMS,F}} = \sqrt{4k_bTb_s} = 181 \, \mathrm{fN}/\sqrt{\mathrm{Hz}}.$$

Remembering what we learned in the Introduction, if we evaluate the $\partial F/\partial\Omega$ parameter, i.e., the Coriolis force variation per unit angular rate, we can easily infer the intrinsic resolution of our gyroscope, by simply dividing the force noise PSD by $\partial F/\partial\Omega$. As the Coriolis force amplitude can be expressed as

$$F = 2m_s v_{da} \Omega = 2m_s \omega_{rd} x_{da} \Omega$$

linearly proportional to the angular rate,

$$\frac{\partial F}{\partial \Omega} = \frac{F}{\Omega} = 4\pi m_s f_{rd} x_{da} = 11.5 \; \mathrm{nN/(rad/s)} = 200 \; \mathrm{pN/dps},$$

where

$$m_s = m_i = 8 \text{ nkg}.$$

The intrinsic resolution (NERD), due to MEMS noise only, is thus

$$s_{n,MEMS,\Omega} = rac{s_{nF}}{rac{\partial F}{\partial \Omega}} = rac{181 \, \mathrm{fN}/\sqrt{\mathrm{Hz}}}{200 \, \mathrm{pN/dps}} = 909 \, \mu \mathrm{dps}/\sqrt{\mathrm{Hz}}.$$

Note the units of measurement: if we correctly evaluate the sensitivity at node X in terms of X/dps, we can divide the noise PSD, expressed in X/\sqrt{Hz} , by the sensitivity, thus obtaining the input-referred resolution directly expressed in dps/\sqrt{Hz} .

We can write the complete expression of the NERD, by expliciting previous equations:

$$s_{n,MEMS,\Omega} = \frac{\sqrt{k_b T b_s}}{2\pi m_s f_{rd} x_{da}}.$$

Be careful about the units of measurement. Remember that, if no conversion is considered, the sensitivity is expressed in N/(rad/s), hence, the obtained NERD would be expressed in $(rad/s)/\sqrt{Hz}$! If one wanted to evaluate the resolution in terms of dps/\sqrt{Hz} , a conversion factor should be included in the equation, and one might write:

$$s_{n,MEMS,\Omega} = rac{\sqrt{k_b T b_s}}{2\pi m_s f_{rd} x_{da}} rac{180 ext{ dps}}{\pi ext{ rad/s}} = 909 \ \mu ext{dps}/\sqrt{ ext{Hz}},$$

which is correctly expressed in dps/\sqrt{Hz} .

This is the ultimate resolution limit of our mode-split gyroscope, considering a single half-mass and a single damping coefficient. For a tuning-fork structure with two masses, NERD improves thus by the $\sqrt{2}$ down to 640 $\mu dps/\sqrt{Hz}$.

QUESTION 3

A well-balanced sensor from a noise performance point-of-view is a sensor where noise introduced by the readout electronics is equal to the intrinsic noise introduced by the sensor. It is somewhat silly to design a readout electronics that introduces much higher noise with respect to the intrinsic one: we would be indeed worsening the system resolution

(an exception is represented by situations where, due to power consumption constraints, electronic noise cannot be sized as low as desired). On the other hand, it is silly, as well, to design a too low-noise readout electronic circuit, whose noise is absolutely negligible with respect to the intrinsic one: we would be wasting design time, power consumption, area..., just to increase the resolution by a factor $\sqrt{2}$ with respect to the well-balanced case where intrinsic and electronic noise are equal.

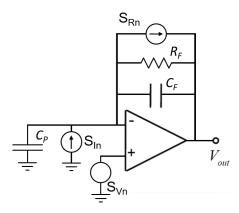


Figure 3: Electrical model for noise calculations on the CA.

It is possible to identify three main noise sources introduced by the front-end electronics: (i) the op-amp voltage noise, (ii) the op-amp current noise, (iii) the feedback resistance thermal noise. As shown in Fig. 3, it is convenient to model both the op-amp current noise and the feedback resistance thermal noise as current noise sources at the front-end input, while op-amp voltage noise is modeled as a voltage noise source applied at the positive input of the op-amp.

Let us first evaluate the input-referred noise of the CA front-end, as sense capacitance noise. In this way, we will be familiar with typical numbers related to capacitive sensors: remember that, in the MEMS world, the interface between the mechanical domain and the electronic domain is usually indicated as the sense capacitance variation; hence, if one wanted to compare two electronic front-ends in terms of resolution, they may compare them in terms of input-equivalent capacitance noise density, which is expressed in F/\sqrt{Hz} . Remember additionally that our signal is a sinusoidal tone at f_{rd} . Hence, if needed, we will evaluate the transfer functions at f_{rd} , neglecting any frequency dependence.

We will initially consider only one front-end. Regarding the op-amp voltage noise, it is convenient to calculate the voltage noise at the output of the CA, and then bring it back to the capacitance:

$$S_{n,OAv,C_s} = S_{n,OAv,V} \frac{\left| T_{V_{out}/V_{+}} \left(f_{rd} \right) \right|^{2}}{\left| T_{V_{out}/C_s} \left(f_{rd} \right) \right|^{2}},$$

$$s_{n,OAv,C_s} = s_{n,OAv,V} \frac{\left| T_{V_{out}/V_{+}}\left(f_{rd}\right) \right|}{\left| T_{V_{out}/C_s}\left(f_{rd}\right) \right|}.$$

As

$$\left|T_{V_{out}/V_{+}}\left(f_{rd}\right)\right| = 1 + \frac{C_{P}}{C_{F}}, \qquad \left|T_{V_{out}/C_{s}}\left(f_{rd}\right)\right| = \frac{V_{DC}}{C_{F}},$$

we get:

$$s_{n,OAv,C_s} = s_{n,OAv,V} rac{\left(1 + rac{C_P}{C_F}
ight)}{rac{V_{DC}}{C_F}} \simeq s_{n,OAv,V} rac{C_P}{V_{DC}} = 4 \ {
m zF}/\sqrt{{
m Hz}},$$

Note that, as long as $C_P \gg C_F$, s_{n,OAv,C_s} does not depend on the feedback capacitance value. This is a common property of CA based front-ends.

The current noise of the op-amp can be expressed as an equivalent sense capacitance variation noise PSD, as well:

$$s_{n,OAi,C_s} = s_{n,OAi,I} \frac{\left| T_{V_{out}/I} \left(f_{rd} \right) \right|}{\left| T_{V_{out}/C_s} \left(f_{rd} \right) \right|} = s_{n,OAi,I} \frac{\frac{1}{2\pi f_{rd}C_F}}{\frac{V_{DC}}{C_F}}$$

$$s_{n,OAi,C_s} = s_{n,OAi,I} \frac{1}{2\pi f_{rd} V_{DC}} = 0.837 \ {
m zF}/\sqrt{{
m Hz}}.$$

Note that op-amp current noise contribution is much lower than the voltage noise one. Note that also s_{n,OAi,C_s} does not depend on C_F . The same approach can be used for the thermal noise of the feedback resistance:

$$s_{n,RF,I} = \sqrt{\frac{4k_bT}{R_F}}.$$

Hence

$$s_{n,RF,C_{s}} = s_{n,RF,I} \frac{\left| T_{V_{out}/I} \left(f_{rd} \right) \right|}{\left| T_{V_{out}/C_{s}} \left(f_{rd} \right) \right|} = s_{n,RF,I} \frac{\frac{1}{2\pi f_{rd}C_{F}}}{\frac{V_{DC}}{C_{F}}} = \sqrt{\frac{4k_{b}T}{R_{F}}} \frac{1}{2\pi f_{rd}V_{DC}}.$$

To infer noise equivalent rate densities of the considered sources, one should take into account the differential readout. Assume a certain noise PSD at the output of both CAs (due to electronics only); remember that the two noise contributions are not correlated, as they are produced by uncorrelated sources. The differential readout doubles the signal (in amplitude) and doubles the noise PSD (in power). Hence, when considering the SNR, a differential readout improves the resolution by a factor $\sqrt{2}$. As a general rule, if

$$c = a - b,$$
 $S_{n,c} = S_{n,a} + S_{n,b}.$

In our case,

$$V_{diff} = (+V_{s.e.}) - (-V_{s.e.}) = 2V_{s.e.},$$

and

$$S_{n,diff} = 2S_{n,s.e.},$$
 $s_{n,diff} = \sqrt{2}s_{n,s.e.}.$

When dealing with a sensor, the input-referred resolution is

$$s_{n,i} = \frac{s_{n,diff}}{S_{diff}},$$

where S_{diff} is the differential sensitivity, which is twice the single-ended one. Hence:

$$s_{n,i} = \frac{s_{n,diff}}{S_{diff}} = \frac{1}{\sqrt{2}} \frac{s_{n,s.e.}}{S_{s.e.}},$$

which is $\sqrt{2}$ better with respect to the one calculated when considering one front-end alone! As the sense capacitance variation per unit angular rate is known, it is possible to input-refer the previously calculated capacitance noises, as

$$s_{n,OAv,\Omega} = \frac{1}{\sqrt{2}} \frac{s_{n,OAv,C_s}}{S_{C_s}} = \frac{1}{\sqrt{2}} \frac{s_{n,OAv,C_s}}{\frac{\partial C_s}{\partial \Omega}} = 510 \,\mu\text{dps}/\sqrt{\text{Hz}},$$

$$s_{n,OAi,\Omega} = \frac{1}{\sqrt{2}} \frac{s_{n,OAi,C_s}}{S_{C_s}} = \frac{1}{\sqrt{2}} \frac{s_{n,OAi,C_s}}{\frac{\partial C_s}{\partial \Omega}} = 106 \,\mu\text{dps}/\sqrt{\text{Hz}},$$

$$\frac{\sqrt{4k_bT}}{\sqrt{4k_bT}} = 1$$

$$s_{n,RF,\Omega} = \frac{1}{\sqrt{2}} \frac{s_{n,RF,C_s}}{S_{C_s}} = \frac{1}{\sqrt{2}} \frac{\sqrt{\frac{4k_bT}{R_F}} \frac{1}{2\pi f_{rd}V_{DC}}}{\frac{\partial C_s}{\partial \Omega}}.$$

We can calculate

$$s_{n,OA,\Omega} = \sqrt{s_{n,OAv,\Omega}^2 + s_{n,OAi,\Omega}^2} = 521 \,\mu\mathrm{dps}/\sqrt{\mathrm{Hz}}.$$

Op-amp noise is lower than the intrinsic one. We can then calculate R_F for which

$$s_{n,OA,\Omega}^2 + s_{n,RF,\Omega}^2 = s_{n,MEMS,\Omega}^2$$
.

We can then write

$$\frac{1}{\sqrt{2}} \frac{s_{n,RF,C_s}}{S_{C_s}} = \frac{1}{\sqrt{2}} \frac{\sqrt{\frac{4k_bT}{R_F}} \frac{1}{2\pi f_{rd}V_{DC}}}{\frac{\partial C_s}{\partial \Omega}} = \sqrt{s_{n,MEMS,\Omega}^2 - s_{n,OA,\Omega}^2} = 371 \,\mu\text{dps}/\sqrt{\text{Hz}},$$

$$R_F = \frac{1}{2} \frac{4k_bT}{\left(2\pi f_{rd}V_{DC}\frac{\partial C_s}{\partial \Omega}\right)^2 \left(s_{n,MEMS,\Omega}^2 - s_{n,OA,\Omega}^2\right)} = 1.38 \,\text{G}\Omega.$$

With this value of feedback resistance, the total noise equivalent rate density is

$$s_{n,TOT,\Omega} = \sqrt{s_{n,MEMS,\Omega}^2 + s_{n,ELN,\Omega}^2} = \sqrt{s_{n,MEMS,\Omega}^2 + \left(s_{n,OAv,\Omega}^2 + s_{n,OAi,\Omega}^2 + s_{n,RF,\Omega}^2\right)}$$
$$s_{n,TOT,\Omega} = 0.909 \text{ mdps}/\sqrt{\text{Hz}}.$$

The main contributions are given by the MEMS thermo-mechanical noise and the voltage noise of the op-amp, which are almost equal. Note that with this value of feedback resistance, the feedback pole is

$$f_{p,CA} = \frac{1}{2\pi R_F C_F} = 59 \text{ Hz},$$

which is well below the operation frequency, f_{rd} . The CA is thus correctly designed.

QUESTION 4

Up to now we considered only the noise introduced by the front-end. What about the noise introduced by the INA? We can calculate the differential voltage noise PSD at the INA input, $S_{n,TOT,V_{CA,diff}}$, due to all the previously considered noise sources (MEMS + front-end), and then compare this with the input-referred noise of the INA, $S_{n,INA,V_{CA,diff}}$. The differential voltage noise PSD at the output of the CA due to both MEMS and front-end noise, can be easily calculated by multiplying the previously calculated input-referred rate noise density by the CA voltage differential sensitivity, $S_{V_{CA,diff}}$:

$$s_{n,TOT,V_{CA,diff}} = s_{n,TOT,\Omega} \cdot S_{V_{CA,diff}} = 50.5 \text{ nV}/\sqrt{\text{Hz}}.$$

In order to have a negligible INA noise, its equivalent input noise, $S_{n,INA,V}$, should be lower than 50.5 nV/ $\sqrt{\text{Hz}}$. As a rule of thumb, we can dimension it to be 0.1 times in power (i.e., $\simeq 1/3$ in voltage):

$$s_{n,INA,V,max} = 16.8 \text{ nV}/\sqrt{\text{Hz}}$$

Input-referred INA noise is due to the two op-amps voltage noise sources, $S_{n,OAINAv,V}$, and the thermal noise of the gain resistance, $S_{n,RINA,V}$:

$$S_{n,INA,V} = 2S_{n,OAINAv,V} + S_{n,RINA,V},$$

where

$$S_{n,RINA,V} = 4k_BTR_{GAIN} = (9.04 \text{ nV}/\sqrt{\text{Hz}})^2$$
.

Hence, $S_{n,OAINAv,V}$ should be equal to

$$S_{n,OAINAv,V} = \sqrt{\frac{S_{n,INA,V,max} - S_{n,RINA,V}}{2}}$$

$$S_{n,OAINAv,V} = \sqrt{\frac{\left(16.8 \text{ nV}/\sqrt{\text{Hz}}\right)^2 - \left(9.04 \text{ nV}/\sqrt{\text{Hz}}\right)^2}{2}} = 10.0 \text{ nV}/\sqrt{\text{Hz}}.$$

The expression of the input voltage noise of an op-amp is

$$S_{n,OA,V} = 2\frac{4k_bT\gamma}{q_m},$$

where the factor 2 takes into account both input transistors, $\gamma = 2/3$, and g_m is the transconductance of the input differential pair:

$$g_m = \frac{2I_D}{V_{OV}}.$$

where I_D is the bias current of the input transistors, and V_{OV} is their overdrive voltage. The current needed to have an op-amp with an input voltage noise $S_{n,OAINAv,V}$ is thus:

$$I_D = \frac{4k_b T \gamma V_{OV}}{S_{n,OAINAv,V}} = 22.0 \,\mu\text{A}.$$

Fig. 4 reports all noise contribution, as a pie chart. As expected, MEMS thermomechanical noise and voltage op-amp noise contributions are equivalent and constitute the 84% of the overall resolution.

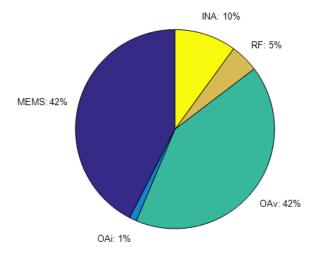


Figure 4: Pie chart showing all noise contributions.