Politecnico di Milano
MSc Course - MEMS and Microsensors - 2024/2025

E09
Gyroscope Drive Design

Riccardo Nastri
riccardo.nastri@polimi.it

31/10/2024

Problem

Working as a consultant for an analog company, you have to conceive the electronics to sustain the drive oscillation of MEMS gyroscopes for autonomous driving. The system shall guarantee a maximum sensitivity variation of $\pm 1.5\%$ within automotive temperatures ($-45°$C to $125°$C). The drive resonator is single-ended actuated and sensed.
The drive loop, shown in Fig. 1, is formed by a CA front-end, a differentiator and a hard-limiter with a dedicated supply. The complete drive oscillator with both primary loop and AGC is shown in Fig. 2. The differential INA gain is given as $G_{INA} = 1 + \frac{49.4\,\mathrm{k\Omega}}{R_{INA}}$. The variable-gain is implemented by acting on the supply voltage of the hard-limiter. Other electromechanical parameters are listed in Table 1.

1. Size the parameters of the primary loop only (Fig. 1), in order to obtain the target displacement amplitude, x_{a0}, at the reference temperature (313 K).
2. Considering the primary loop only (Fig. 1), calculate the maximum percentage variation of the drive amplitude, and evaluate the required compensation factor.
3. Size the parameters of the secondary (AGC) loop of the drive-mode oscillator (Fig. 2), considering the requirement on the maximum sensitivity variation.
4. Size the low-pass filter of the control loop.

Mechanical		
	Symbol	Value
Drive resonance frequency	f_{rd}	20 000 Hz
Drive Q-factor @ 300 K	Q_{d0}	8000
Internal mass	m_i	1.5 nKg
External mass	m_e	2.5 nKg
Process thickness	h	24 μm
Gap	g	1.8 μm
Number of drive comb fingers	N_{CF}	20
Target drive displacement amplitude	x_{a0}	5 μm
Electronics		
Rotor bias voltage	V_{DC}	5 V
Amplifier voltage output swing	V_O	$\pm$4 V
Minimum capacitance	C_{min}	0.2 pF
Secondary (AGC) loop		
Rectifier gain	G_{REC}	1
LPF gain	G_{LPF}	1
LPF resistance	R_{LPF}	3 MΩ

Table 1: Parameters of the gyroscope (half structure).

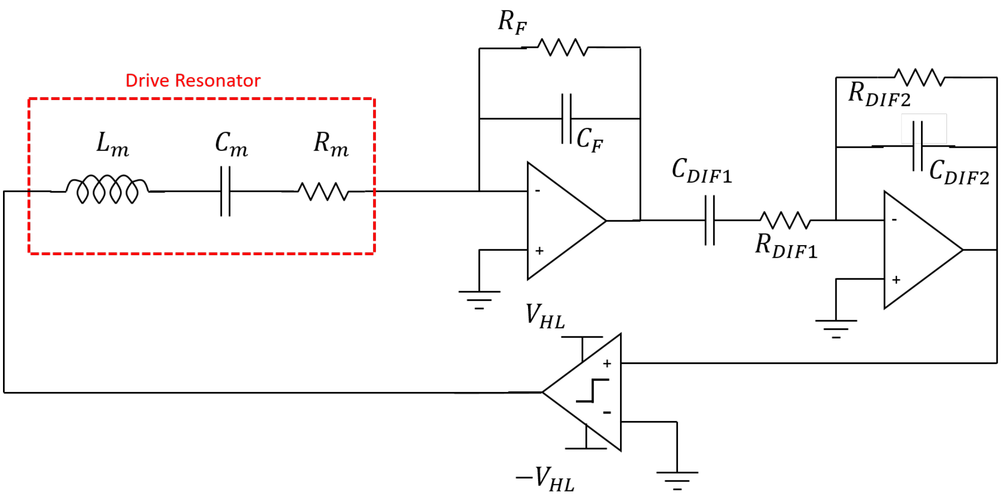

Figure 1: Primary loop of the drive oscillator.

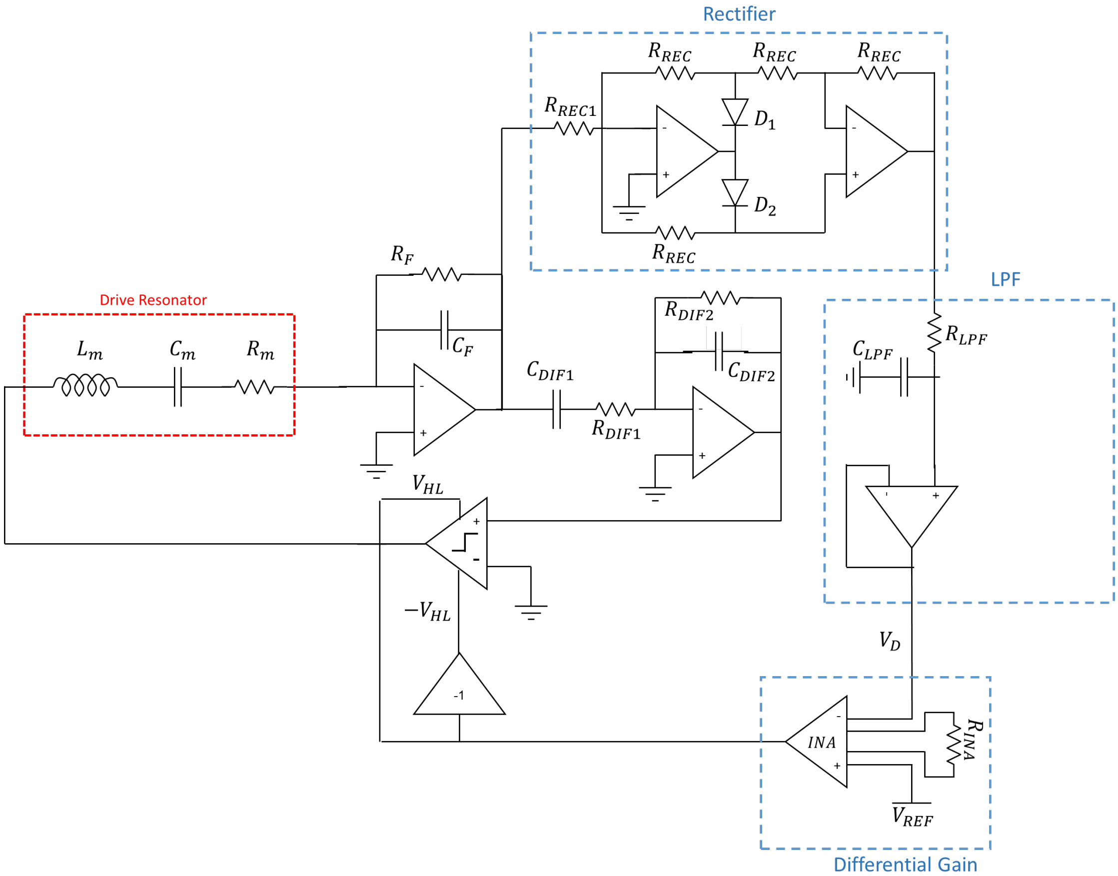

Figure 2: Drive oscillator with AGC.

Introductory comments

A MEMS gyroscope can be modeled as a resonator along the drive axis and an accelerometer along the sense axis, to detect sinusoidal motion induced by the Coriolis force.
The primary loop is needed to start and sustain the oscillation of the resonator at its resonant frequency. In this example, we will use a topology formed by a TCA-based front-end, that senses drive motion and provides an output voltage proportional to the drive displacement, followed by a differentiator, needed to obtain an overall 360° phase shift at resonance; regarding the drive actuation circuitry, instead of having a hard-limiter followed by the de-gain stage, the voltage amplitude reduction is obtained by supplying the comparator (hard-limiter) itself with a dedicated supply voltage, V_{HL}.

Question 1

We want to calculate the actuation voltage amplitude v_a, to be applied to the actuation electrode in order to get the desired displacement amplitude x_{a0} at the reference temperature. We know that, if a MEMS resonator is actuated at resonance, assuming a small-signal hypothesis on the actuation voltage, the displacement amplitude is

$$x_a = \frac{Q_d}{k_d}\eta_d v_a.$$

In our case, Q_d is the quality factor of the drive resonator, k_d is the drive mode stiffness, η_d is the drive-actuation transduction coefficient, and v_{da} is the drive-actuation sinusoidal voltage amplitude. We know that

$$\eta_{da} = V_{DC}\frac{\partial C_{da}}{\partial x},$$

where V_{DC} is the DC voltage between the rotor and the stator electrode, and $\partial C_{da}/\partial x$ is the drive-actuation capacitance variation per unit displacement. With data of Table 1,

$$\frac{\partial C_{da}}{\partial x} = \frac{2\epsilon_0 h N_{CF}}{g} = 9.44\,\mathrm{fF}/\mu\mathrm{m}, \qquad \eta_{da} = V_{DC}\frac{\partial C_{da}}{\partial x} = 47.2\cdot 10^{-9}\,\mathrm{N/V}.$$

As the resonator is symmetric, $\eta_d = \eta_{da} = \eta_{dd} = 47.2\cdot 10^{-9}$ A/(m/s). We can then calculate the drive mode stiffness of the gyroscope:

$$k_d = (2\pi f_{rd})^2\, m_d = (2\pi f_{rd})^2\,(m_e + m_i) = 63.2\,\mathrm{N/m}.$$

And we can finally evaluate the amplitude of drive-actuation sine-wave voltage required to obtain the target displacement amplitude:

$$v_{da} = \frac{x_a \eta_{da}}{Q_d k_d} = 836\,\mathrm{mV}.$$

Remember that, with the circuit topology we are using, the voltage signal applied to the drive electrode is the hard-limiter output, which is a square-wave. Taking into account the $4/\pi$ factor:

$$v_{da,sq} = \frac{v_{da}}{4/\pi} = 0.66\,\mathrm{V}.$$

This means that the hard-limiter should be biased (V_{HL}) between -0.66 V and $+0.66$ V at the reference temperature.
Note that $v_{da} = 0.836$ V is much lower than the rotor-stator DC voltage (5 V) and the small-signal hypothesis is thus valid.

We can evaluate the motional current amplitude flowing through the drive-detection port of the drive resonator,

$$i_{ma} = x_a \eta_d\,(2\pi f_{rd}) = 29.6\,\mathrm{nA},$$

and the drive-detection capacitance variation amplitude,

$$C_{dda} = x_a \frac{\partial C_{dd}}{\partial x} = 47.2\,\text{fF}.$$

With a TCA-based front-end, the output voltage amplitude can be calculated as

$$V_{TCA,out,a} = G_{TCA} C_{dda} = |T_{TCA}(f_{rd})|\, C_{dda} = \frac{V_{DC}}{C_F} C_{dda},$$

or, equivalently,

$$V_{TCA,out,a} = \frac{1}{(2\pi f_{rd})\, C_F} i_{ma}.$$

A good design approach to choose the TCA gain would be to maximize it. This maximum gain shall ideally be the highest value that prevents any saturation of the TCA output. Since the amplifier voltage output swing (V_O) is ± 4 V, given the previously calculated i_{ma} and C_{dda}, the desired feedback capacitance, $\hat{C}_F$, would be

$$\hat{C}_F = \frac{V_{DC}}{V_O} C_{dda} = 59\,\text{fF},$$

which, unfortunately, is lower than the minimum one (200 fF). We will thus choose the minimum one, $C_F = 200\,\text{fF}$, which implies an output voltage amplitude:

$$V_{TCA,out,a} = \frac{1}{(2\pi f_{rd})\, C_F} C_{dda} = 1.18\,\text{V}.$$

For proper TCA behavior, the feedback pole frequency should be, at least, two decades before the oscillation frequency,

$$\frac{1}{2\pi R_F C_F} < \frac{1}{100} f_{rd},$$

that corresponds to a (minimum) feedback resistor of 4 GΩ. With this choice, we guarantee a small error (lower than 1 deg) on the ideal phase shift of the TCA (90°).

Regarding the differentiator stage, we choose its gain G_{DIF} following the same approach, i.e. setting an output swing of 4 V. Additionally, fixing its poles two decades beyond resonance, we get a set of three equations with four unknowns:

$$G_{DIF} = 2\pi f_{rd} C_{DIF1} R_{DIF2} = \frac{4V}{1.18V} = 3.38.$$

$$f_{p,DIF} = \frac{1}{2\pi C_{DIF1} R_{DIF1}} = 2MHz, \qquad f_{p,DIF} = \frac{1}{2\pi C_{DIF2} R_{DIF2}} = 2MHz$$

In principle, any quartets of the passive components that satisfy the above equations is a valid solution. A wise choice is to note that C_{DIF2} appears only at the denominator of the third equation. As the pole to match has a relatively large frequency, we select the minimum allowed capacitance value for this component.

Once this choice id done, the calculation of the other values is straightforward:

$$C_{DIF2} = 200fF, \qquad R_{DIF2} = \frac{1}{2\pi \cdot 2MHz \cdot 200fF} = 398k\Omega$$

$$C_{DIF1} = \frac{G_{DIF}}{2\pi f_{rd} R_{DIF2}} = 67.8pF, \qquad R_{DIF1} = \frac{1}{2\pi \cdot 2MHz \cdot 67.8pF} = 1.1k\Omega$$

All the found values are suitable for integration within a CMOS IC technology.

Question 2

The sensitivity of a MEMS gyroscope is proportional to the displacement amplitude of the drive-axis oscillation, x_{da}:

$$S \propto x_{da}.$$

In absence of an amplitude-control loop, i.e., in presence of a fixed drive-actuation voltage amplitude, v_{da}, the drive displacement amplitude depends on the quality factor,

$$x_{da} = \frac{Q}{k}\eta_{da} v_{da}.$$

In presence of temperature variations, the quality factor changes and, in turn, the drive-displacement amplitude changes. As seen in a previous class, the quality factor variations vs temperature variations can be roughly approximated with the following linearization,

$$\frac{\Delta Q}{Q} = -\frac{1}{2}\frac{\Delta T}{T}.$$

Since

$$\frac{\Delta T}{T} = \frac{170\,\mathrm{K}}{300\,\mathrm{K}} \simeq 56\%,$$

the quality factor variation is about -28%. The sensitivity variation within the whole temperature operating range is estimated as:

$$\frac{\Delta S}{S} = \frac{\Delta x_{da}}{x_{da}} = \frac{\Delta Q}{Q} = -\frac{1}{2}\frac{\Delta T}{T} = -28\%.$$

A 28% sensitivity variation is too high for a high-performance product. For this reason, an AGC loop must be implemented in the drive oscillator of a gyroscope. As the the maximum allowable variation is 1.5%, an open-loop-actuated drive resonator would not fulfill the requirement. We need to introduce a compensation (control) loop, to reduce the displacement amplitude variations by a factor (at least) $28\%/\pm 3\% \simeq 10$. This is, in practice, the minimum value required by the AGC loop gain.

Question 3

To control the drive displacement amplitude, an AGC (automatic gain control) loop is introduced in most vibratory MEMS gyroscopes. The control circuit is designed as a negative feedback loop. Its raw schematic is reported in Fig. 3. We can consider the voltage difference $V_D - V_{REF}$ as the error signal of our negative feedback loop:

$$\varepsilon = V_{REF} - V_D = V_{REF}\frac{1}{1+G_{loop}}.$$

If the loop gain is sufficiently high, the error is $\simeq 0$, hence $V_D \simeq V_{REF}$. In other words V_D behaves as the *virtual ground* of our negative feedback, as shown in Fig. 3.

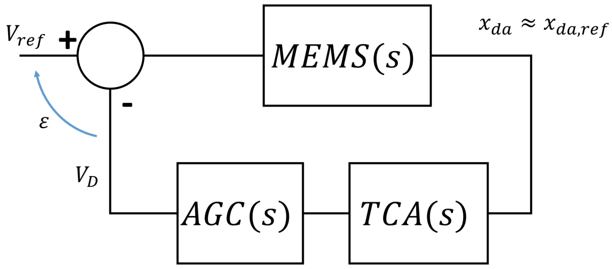

Figure 3: AGC as a negative feedback system: note how the MEMS represents the forward path from input voltage (force) to output displacement.

If the gain K_{VCA} from the drive displacement amplitude, x_{da}, to the DC value V_D, including the front-end and AGC stages, is known and constant,

$$V_D = K_{AGC}x_{da}, \qquad x_{da} = \frac{V_D}{K_{AGC}}, \qquad x_{da,ref} = \frac{V_{REF}}{K_{AGC}}$$

so that choosing V_{REF} is equal to choosing a certain reference drive displacement amplitude, $x_{da,ref}$, and the loop would force $x_{da} \simeq x_{da,ref}$, as shown in Fig. 3.

Without excessive theoretical demonstrations, it is intuitive to find out how the loop gain value acts on the variability of the gyroscope sensitivity to temperature changes. Indeed, this variability is inherent to the MEMS transfer function, so it falls in the forward path between applied voltage and displacement. Conversely, in a system based on a negative feedback, we know that the gain is ideally set only through the feedback branch (K_{VCA}), and that the effects of the variability of elements in the forward path to the closed-loop gain is reduced by approximately the loop gain. We can thus conclude that the relative displacement amplitude variation is now:

$$\frac{\Delta x_{da}}{x_{da0}} = \frac{\Delta Q}{Q_0}\frac{1}{1+G_{loop0}} = \left(\frac{1}{2}\frac{\Delta T}{T_0}\right)\frac{1}{1+G_{loop0}}.$$

Hence, variations on the gyroscope sensitivity, which is linear with the drive displacement amplitude, are reduced by a factor $1+G_{loop0}$ as well, when the AGC loop is introduced.

As the required compensation factor found in the previous point is 10, we will design the AGC loop with a target loop gain a litlle larger than this, e.g. of 20, so to account for the variability of the G_{loop} itself with temperature. The drive displacement behavior as a function of temperature with the control loop is reported in Fig. 4.

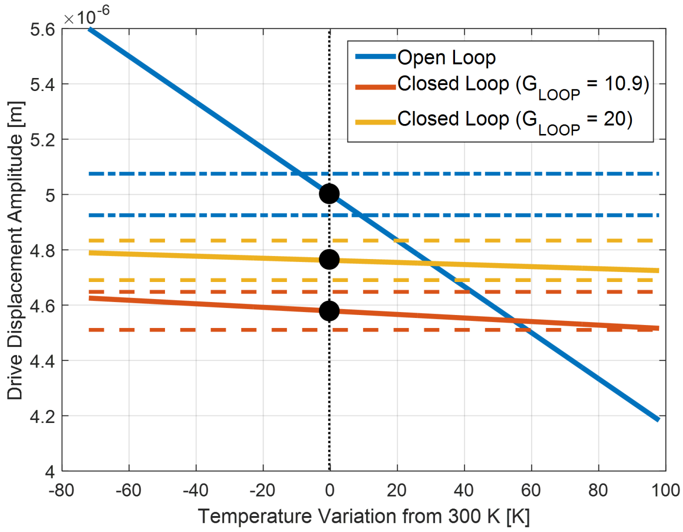

Figure 4: Drive displacement amplitude as a function of temperature variations, for different situations.

Note that the previously defined x_{da0} represents the drive displacement amplitude at the reference temperature. With a loop gain equal to 20, $x_{da0} = 4.76$ μm, i.e., $\simeq 5\%$ lower than the target value, 5 μm. However, its variability is limited as expected.

We are now ready to evaluate the loop gain. Referring to Fig. 5, the loop gain we are going to calculate is the one marked as $G_{loop,AGC}$.
To calculate its value, we can (i) open the loop (possibly at the output of a low-impedance voltage source or at the input of a high-impedance amplifier), (ii) inject an *amplitude variation* as our test signal (i.e. apply an amplitude modulation to the ideally harmonic signal generated by the drive loop), and (iii) evaluate the quantity that *returns* to where the loop was cut.
Keep in mind that all calculations should be done *on the amplitude variations of the sinusoidal or DC signals*, which is what the AGC acts on. Additionally, we begin by calculating it for quasi-stationary variations of the input (i.e. for slow changes of temperature and Q factor). The loop is composed by the following stages:

1. A full-wave rectifier (FWR), which, given a sine-wave at its input, outputs the fully-rectified waveform:

$$V_{FWR,in}(t) = (V_{CA} + v_a)\sin(\omega_0 t) \qquad \rightarrow \qquad V_{FWR,out}(t) = (V_{CA} + v_a)\left|\sin(\omega_0 t)\right|,$$

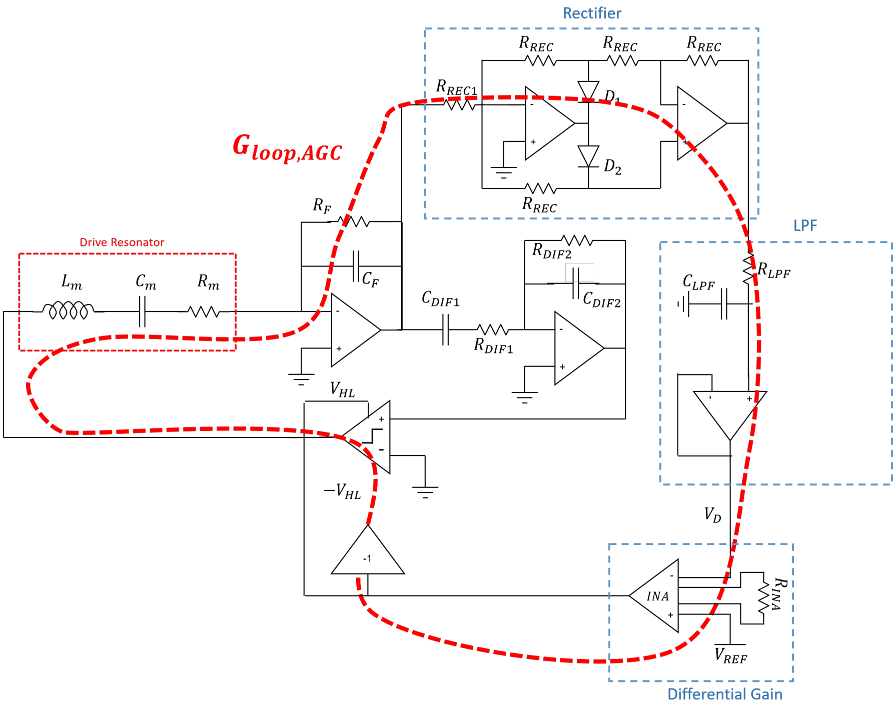

Figure 5: Complete drive-mode oscillator. The signal path of the amplitude loop is highlighted.

V_{CA} being the nominal amplitude of the oscillating signal at the CA output, and v_a being the small amplitude variation we are applying to evaluate the loop gain. As there is no change in amplitude, the gain of the FWR is simply 1.

2. A low-pass filter (LPF) that extracts the DC component (i.e. the mean value) of its input signal. You can easily verify that the mean value of a rectified sine-wave is $2/\pi$:

$$V_{LPF,out}(t) = \text{mean}\left[(V_{CA} + v_a)\left|\sin(\omega_0 t)\right|\right] = \frac{2}{\pi}(V_{CA} + v_a)$$

The gain of the LPF on the amplitude variation v_a is therefore $2/\pi$.

Here the LPF is implemented with a single-pole RC low-pass filter, but we are free to choose other topologies.

3. A differential gain stage, implemented with an instrumentation amplifier (INA). The INA output is buffered (± 1) to the positive and negative supply of the hard-limiter. With this topology, given a certain INA output voltage, $V_{out,INA}$, the drive actuation signal is

$$V_{da}(t) = V_{INA,out}\text{sqw}(\omega_0 t),$$

that describes a square-wave signal at ω_0, that toggles between $+V_{INA,out}$ and $-V_{INA,out}$. The combination of the INA and hard limiter yields thus a gain of

$G_{INA} \cdot 4/\pi$ on the signal amplitude (and thus on the test amplitude variation we applied).

4. the MEMS at the resonance frequency just corresponds to a gain of $1/R_{eq}$.
5. Finally, the TCA gain at resonance just corresponds to $/(\omega_{rd} C_F)$.

The AGC loop gain is thus the combination of the mentioned gains:

$$G_{loop,AGC} = \frac{2}{\pi} G_{INA} \frac{4}{\pi} \frac{1}{R_{md}} \frac{1}{(2\pi f_{rd}) C_F} = \frac{2}{\pi} G_{INA} \frac{4}{\pi} \frac{\eta_d^2}{b_d} \frac{1}{(2\pi f_{rd}) C_F}$$

The damping coefficient and in turn the equivalent resistance can be calculated as:

$$R_{md} = \frac{b_d}{\eta_d^2} = \frac{2\pi f_{rd} m}{Q_d \eta_d^2} = 28.2 M\Omega$$

Targeting a loop gain of 20, the INA gain, G_{INA}, should in the end be:

$$G_{INA} = \frac{G_{loop,AGC}}{\frac{2}{\pi} \frac{1}{(2\pi f_{rd}) C_F} \frac{1}{R_{md}} \frac{4}{\pi}} = 17.49,$$

that can be obtained with an INA gain resistance of $R_{INA} = \frac{49.4\,\text{k}\Omega}{G_{INA}-1} = 3.00\,\text{k}\Omega$.

Question 4

We have to size the low-pass filter of the AGC loop. As the gain of the LPF is unitary, the transfer function of the LPF can be expressed as

$$\frac{V_{LPF,out}(s)}{V_{LPF,in}(s)} = \frac{1}{1 + sR_{LPF} C_{LPF}}.$$

From a spectral point of view, the signal at the input of the FWR is a pure tone at ω_{rd}; on the other hand, the rectified signal will have a DC tone, whose amplitude is $2/\pi$ times the amplitude of the input sine-wave, and other harmonics, as shown in Fig. 6. The only harmonic that we want is at DC, proportional to the drive amplitude. The filter role is to cut out all other harmonics. As temperature variations have a very narrow bandwidth (such variations are very slow), we could place the pole at low frequency. How low? As in every negative feedback, the choice of the poles is related to stability. In other words, we evaluated the DC loop gain, G_{loop0}, but what about the loop gain at other frequencies? One can demonstrate that all the stages can be modeled as a frequency-independent gain, except for the MEMS resonator and the LPF. Indeed, when dealing with actuation-voltage amplitude variations, the resonator can be modeled (see Appendix) as a base-band equivalent system whose transfer function is

$$\frac{I_a(s)}{V_a(s)} = \frac{1}{R_m} \frac{1}{1 + s\frac{2Q(s)}{\omega_o}}.$$

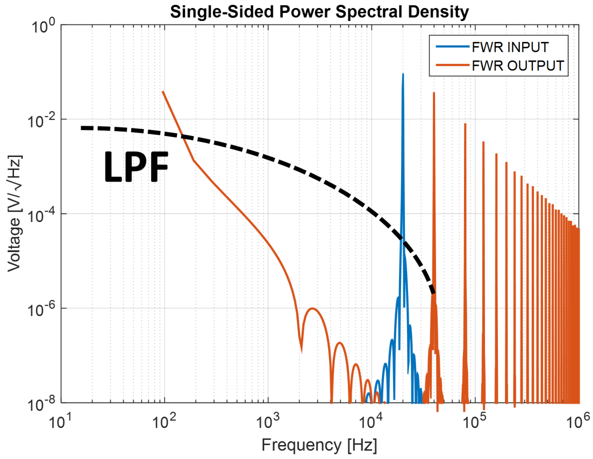

Figure 6: Action of the AGC loop low-pass filter.

Be careful! This is not the ratio between motional current and actuation voltage! This is the ratio between motional current *amplitude variations* and actuation voltage *amplitude vriations*, when actuated at resonance. The variable s here refers only to the frequency content of the Q factor changes! The frequency-dependent AGC loop gain can be thus written as

$$G_{loop,AGC} = G_{INA}\frac{2}{\pi}\frac{1}{(2\pi f_{rd})\,C_F}\frac{1}{R_{md}}\frac{4}{\pi}\frac{1}{1+s\dfrac{2Q}{\omega_{rd}}}\frac{1}{1+sR_{LPF}C_{LPF}}.$$

This is a two-poles system, where the first pole frequency, due to the MEMS, is given,

$$f_{p1} = \frac{1}{2\pi\dfrac{2Q_d}{\omega_{rd}}} = \frac{f_{rd}}{2Q_d} = 1.25\,\text{Hz},$$

while the second one, f_{p2} depends on the LPF design. As in every negative feedback loop, to ensure the stability of the loop, the second pole of the loop gain should be just higher than the gain-bandwidth product (GBWP) of the loop. In other words,

$$f_{p2} \geq G_{loop0}\cdot f_{p1} = 25\,\text{Hz}, \qquad C_{LPF} = \frac{1}{2\pi\cdot(G_{loop0}\cdot f_{p1})\cdot R_{LPF}} = 2.12\,\text{nF},$$

This is the minimum pole frequency of the LPF, for which a 45° phase margin is ensured. The situation is represented in Fig. 7. All the components have thus been sized.

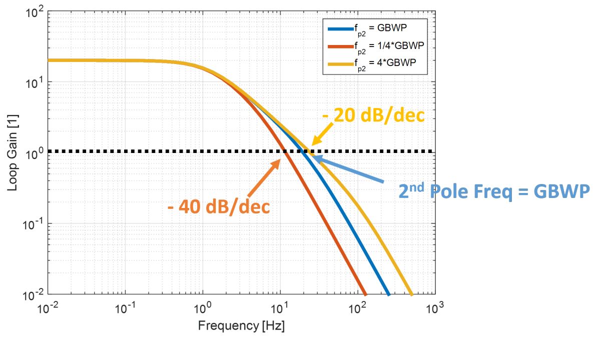

Figure 7: Loop gain of the AGC for different filter sizing.

Facultative appendix: base-band model of a mass-spring-damper system for actuation amplitude variation effects

A mass-spring-damper system is modeled in the Laplace domain as:

$$\frac{X(s)}{F(s)} = \frac{1}{k}\frac{1}{1 + s\dfrac{Q}{\omega_o} + \dfrac{s^2}{\omega_o^2}}.$$

Assume an external force, F, in the form $F(t) = F_a(t)\sin(\omega_o t)$, where F_a is the slowly varying amplitude of the force and ω_o is the structure resonance. As actuation occurs at resonance, displacement, velocity and acceleration are sinewaves as well:

$$x(t) = -x_a(t)\cos(\omega_o t).$$

$$\dot{x}(t) = -\dot{x}_a(t)\cos(\omega_o t) + x_a(t)\,\omega_o \sin(\omega_o t),$$

$$\ddot{x}(t) = -\ddot{x}_a(t)\cos(\omega_o t) + 2\dot{x}_a(t)\,\omega_o \sin(\omega_o t) + x_a(t)\,\omega_o^2 \cos(\omega_o t).$$

Hence,

$$m\left(-\ddot{x}_a(t)\cos(\omega_o t) + 2\dot{x}_a(t)\,\omega_o\sin(\omega_o t) + x_a(t)\,\omega_o^2\cos(\omega_o t)\right) +$$
$$+b\left(-\dot{x}_a(t)\cos(\omega_o t) + x_a(t)\,\omega_o\sin(\omega_o t)\right) + k\left(-x_a(t)\cos(\omega_o t)\right) = F_a(t)\sin(\omega_o t),$$

which can be re-written as

$$\left[2\dot{x}_a(t)\,m\omega_o + x_a(t)\,b\omega_o - F_a(t)\right]\sin(\omega_o t) = 0,$$

$$\left[-m\ddot{x}_a(t) + x_a(t)\,m\omega_o^2 - b\dot{x}_a(t) - kx_a(t)\right]\cos(\omega_o t) = 0$$

We can re-write the first equation as

$$2\dot{x}_a(t)\,m\omega_o + x_a(t)\,b\omega_o = F_a(t),$$

which can be rewritten in Laplace domain:

$$2sm\omega_o X_a(s) + b\omega_o X_a(s) = F_a(t), \qquad X_a(s)\left[2sm\omega_o + b\omega_o\right] = F_a(t)$$

$$\frac{X_a(s)}{F_a(s)} = \frac{1}{2sm\omega_o + b\omega_o} = \frac{\dfrac{Q}{k}}{1 + s\dfrac{2Q}{\omega_o}}.$$

This is the base-band model of a MEMS resonator when dealing with slow variations of the actuation signal amplitude. Following the same approach, one can demonstrate that the baseband-equivalent motional impedance of the resonator can be modeled as:

$$\frac{I_a(s)}{V_a(s)} = \frac{1}{R_m}\frac{1}{1 + s\dfrac{2Q}{\omega_o}}.$$

This frequency dependent behavior should be taken into account when dealing, e.g., with the AGC stability, i.e., *when the transfer function of the amplitude variations of the signal is of concern.*