Politecnico di Milano
MSc Course - MEMS and Microsensors - 2023/2024

E07
Gyroscope Electromechanical Design

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Problem

You work in a startup developing innovative gyroscopes for rehabilitation after vestibular disorders. You are asked to design a tuning-fork MEMS gyroscope that emulates the semicircular canals. The sensor parameters, for half the structure, are given in Table 1. The drive mode is actuated in a push-pull configuration through the set of comb electrodes $C_{da,1}$ and $C_{da,2}$, with square waves (see Fig. 1). Drive detection stators ($C_{dd,1}$ and $C_{dd,2}$) are kept to the oscillator front-end virtual ground; the rotor bias is $V_{DC} = 10$ V. The gyroscope is sketched in Fig. 2.

1. Determine the AGC reference V_{ref} to target a drive amplitude of 7 μm.
2. Evaluate the in-phase and anti-phase drive resonant modes, explaining which frames and springs they involve. Determine the sense stiffness to operate in matched mode.
3. Calculate the expression of the electrostatic drive force in the situation described by the figures, and, paying attention to the drive configuration, choose the number of actuation comb-fingers to have a nominal 0.5-V peak-to-peak square-wave actuation voltage.
4. Evaluate the electromechanical sensitivity of the gyroscope in fF/dps.

General		
ε_0	$8.85 \cdot 10^{-12}$ F/m	Dielectric constant in vacuum
E	150 GPa	PolySi Young's modulus
h	24 μm	Process thickness
m_e	2 nkg	External frame mass
m_i	2.22 nkg	Inner frame mass
g	2 μm	Comb and parallel plate gap
Drive		
b_d	$0.5 \cdot 10^{-7}$ N/(m/s)	Drive damping coefficient
$N_{CF,dd}$	30	Number of drive-detection comb fingers
L_{fd}	180 μm	Drive fold length
w_{fd}	3 μm	Drive fold width
n_{sd}	4	Number of drive springs
n_{fd}	2	Number folds for each drive spring
L_{ftf}	155 μm	Tuning fork fold length
w_{ftf}	3.1 μm	Tuning fork fold width
n_{stf}	2	Number of tuning fork springs
n_{ftf}	2	Number of folds for each tuning fork spring
Sense		
b_s	$1 \cdot 10^{-7}$ N/(m/s)	Sense damping coefficient
n_{PP}	4	Number of differential parallel-plate electrodes
L_{PP}	250 μm	Parallel plate length
Electronics		
C_F	500 fF	Feedback capacitance

Table 1: Gyroscope parameters. Data are for half device.

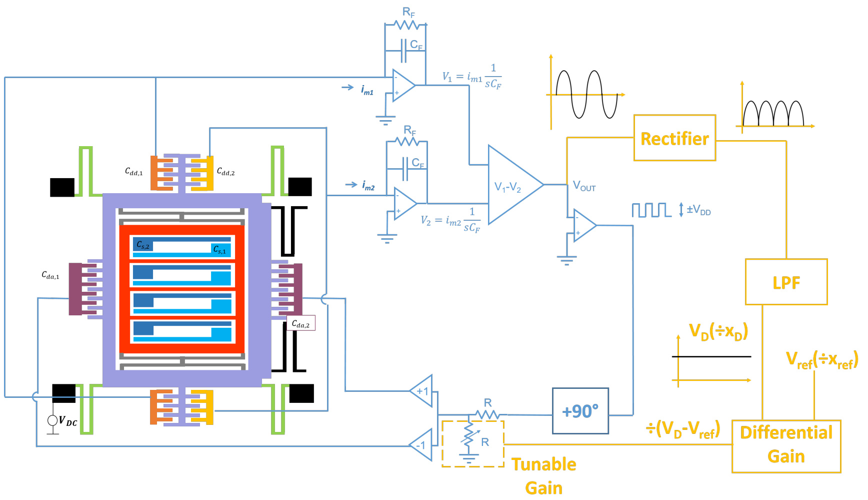

Figure 1: Drive oscillator.

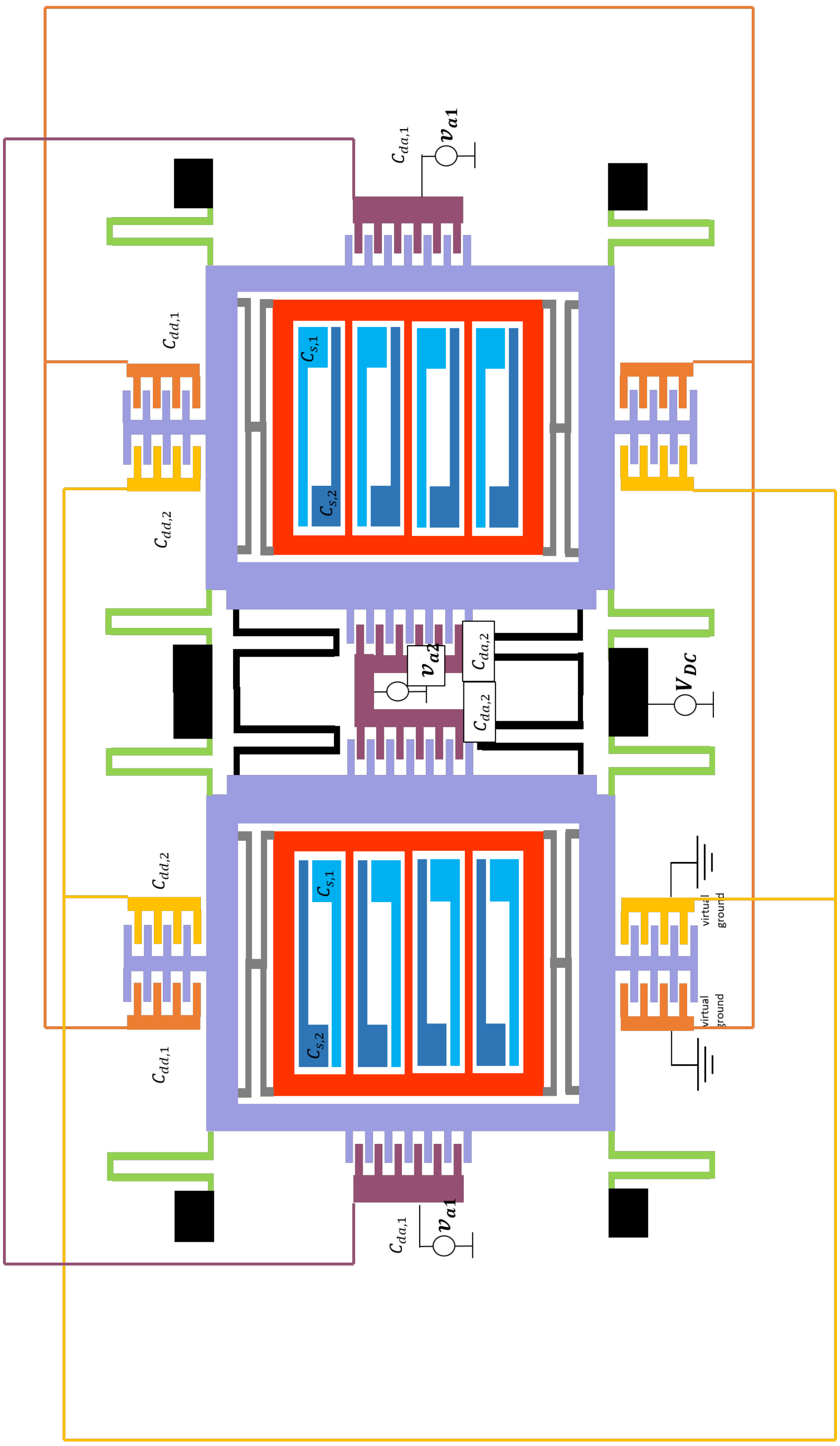

Figure 2: Sketch of the whole gyroscope.

Question 1

In a gyroscope, it is necessary to introduce an automatic-gain-control (AGC) loop in the drive oscillator. In this way, the displacement amplitude of the drive-axis motion is controlled and stabilized against part-to-part or temperature-induced changes of the quality factor.
In our example (see Fig. 1), the differential voltage signal taken at the output of the front-end, V_{out}, is rectified with a full-wave rectifier, and it is then averaged with a low-pass filter (LPF). The LPF output voltage, V_D, is compared with a reference voltage, V_{ref}. The error signal is processed, and the loop is then closed. If the loop is properly designed, i.e., if the loop gain is high, V_D is equal to the reference voltage, V_{ref}. Assuming a constant and well-known gain from x_{da} to V_D, *controlling V_{ref} is equivalent to controlling the drive displacement amplitude x_{da}*.

Our aim is to calculate the proper V_{ref} that forces x_{da} to be equal to 7 μm. To do this, we need to evaluate the gain from x_{da} to V_D. The half-structure drive-detection electromechanical transduction factor, η_{dd}, can be evaluated as

$$\eta_{dd} = V_{DC}\frac{\partial C_{dd}}{\partial x}, \qquad \frac{\partial C_{dd}}{\partial x} = \frac{2\epsilon_0 N_{CF,dd} h}{g} = 6.37\,\mathrm{fF}/\mu\mathrm{m}, \qquad \eta_{dd} = 63.7\cdot 10^{-9}\,\mathrm{N/V}.$$

The motional current amplitude flowing through each drive-detection port is

$$i_{ma} = 2\eta_{dd} x_{da}\omega_{rd},$$

where the factor 2 takes into account the second half of the device, i.e., takes into account that the current flowing through the unique drive-detection-1 electrode is twice the one calculated for half the structure.
Motional currents flowing through the two drive detection ports, $i_{m1}(t) = i_{ma}\sin(\omega_{rd}t)$ and $i_{m2}(t) = -i_{ma}\sin(\omega_{rd}t)$, are integrated in the feedback capacitance of the TCA-based front-end. The output voltage variation amplitudes of the TCA are thus

$$V_{1,a} = -V_{2,a} = i_{ma}\left|\frac{1}{j\omega_{rd}C_F}\right| = i_{ma}\frac{1}{\omega_{rd}C_F} = \frac{2\eta_{dd}x_{da}}{C_F} = \frac{V_{DC}}{C_F}\left(2\frac{\partial C_{dd}}{\partial x}\right)x_{da} = 1.78\,\mathrm{V}.$$

The differential output voltage is thus

$$V_{out,a} = V_{1,a} - V_{2,a} = 3.57\,\mathrm{V}.$$

The sinusoidal voltage signal V_{out} is then rectified and averaged. The rectification with a full-wave rectifier has a gain equal to $2/\pi$. The LPF output is then,

$$V_D = V_{out,a}\frac{2}{\pi} = \frac{2}{\pi}2\frac{V_{DC}}{C_F}\left(2\frac{\partial C_{dd}}{\partial x}\right)x_{da} = 2.27\,\mathrm{V}.$$

This means, that, in order to have a 7-μm drive displacement amplitude, the AGC reference voltage should be set equal to 2.27 V.

Question 2

A dual-mass decoupled gyroscope presents two fundamental resonances: the anti-phase drive mode and the sense mode. In general, other spurious modes will be present but, with the designer care, they will lie at much larger frequencies. An exception is found along the drive axis, where we unavoidably find an additional undesired mode which is the in-phase drive mode. In a gyroscope, the drive oscillator keeps the anti-phase mode in stable oscillation at resonance, while the undesired in-phase mode shall not be excited.

In-phase drive mode

The in-phase drive mode is characterized by the simultaneous motion of the two halves of the structure along the same direction.

- Springs: the displacements of the two halves are equal. The tuning fork springs (the black folded springs in the middle of Fig. 2) therefore do not bend. Hence, they do not contribute to the determination of the resonance frequency of this mode. When estimating the stiffness of the in-phase mode, $k_{d,ip}$, the sole springs that should be taken into account are the *drive springs* (the green folded springs in Fig. 2). Considering half of the structure:

$$k_{d,ip} = \frac{n_{sd}}{n_{fd}} Eh \left(\frac{w_{fd}}{l_{fd}} \right)^3 = 33.33\,\mathrm{N/m},$$

  where $n_{sd} = 4$ is the springs number, $n_{fd} = 2$ is the folds number for each spring.

- Mass: the external, light violet, frames are named *drive frames*: they move along the drive direction and they are electrically interfaced with the drive stators. Of course, this part of the device shall be taken into account in the evaluation of an in-phase mode. In addition the *Coriolis frames* (the red inner ones), are as well dragged during drive motion. For this reason, we can consider the total drive mass of one half of the device, m_d, as the sum of the external mass and the inner mass:

$$m_d = m_e + m_i = 4.22\,\mathrm{nkg}.$$

We thus evaluate the in-phase resonance frequency, $f_{rd,ip}$, as

$$f_{rd,ip} = \frac{1}{2\pi}\sqrt{\frac{k_{d,ip}}{m_d}} = \frac{1}{2\pi}\sqrt{\frac{2k_{d,ip}}{2m_d}} = 14.14\,\mathrm{kHz}.$$

Anti-phase drive mode

When the anti-phase drive mode is excited, the two halves of the structure move in opposite directions.

- Springs: the tuning fork takes now part into the overall *drive stiffness*. During the anti-phase mode, the two ends of the tuning fork perform the same displacement while, due to simmetricity considerations, the central point of the tuning fork spring remains in a steady position. Then, we can consider this point as virtually anchored. The total drive stiffness, k_d, is the sum of the stiffness of the drive springs and of the tuning fork

$$k_d = k_{d,ap} = k_{ip} + k_{tf} = k_{ip} + \frac{n_{stf}}{n_{ftf}} Eh \left(\frac{w_{ftf}}{l_{ftf}}\right)^3 = 62.13\,\mathrm{N/m}.$$

  where $n_{sd} = n_{fd} = 2$ is the number of folds for each spring group.

- Mass: the mass is again the sum of the external and the internal frames

$$m_d = m_e + m_i = 4.22\,\mathrm{nkg}.$$

The anti-phase drive resonance frequency is thus

$$f_{rd} = \frac{1}{2\pi}\sqrt{\frac{k_d}{m_d}} = 19.31\,\mathrm{kHz}.$$

Note how the in-phase resonance frequency is lower than the anti-phase resonance frequency, their difference being due to the tuning fork. In this case the distance is around 5 kHz and, in general, should be kept large to avoid spurious in-phase excitation.

Sense mode

In mode-matched operation, the drive resonance frequency, ω_{rd}, and the sense resonance frequency, ω_{rs}, are equal. The sense-axis spring constant, k_s, can be thus evaluated as

$$k_s = m_s\left(2\pi f_{rs}\right)^2 = 32.68\,\mathrm{N/m}.$$

Note that, for this calculation, we used only the *sense mass*, $m_s = m_i$, equal to the one of the inner frame.

Question 3

Introduction to push-pull actuation

We have seen how a resonant sine wave of amplitude v_a can applied to the driving stator of a resonator, with the rotor kept to a DC voltage V_{DC}. The stator used to detect the drive displacement is kept to the virtual ground of the drive-detection front-end, as shown in Fig. 3. This type of 3-port resonator is called a *single-ended* resonator. In presence of a square-wave drive actuation, one can equivalently describe the actuation

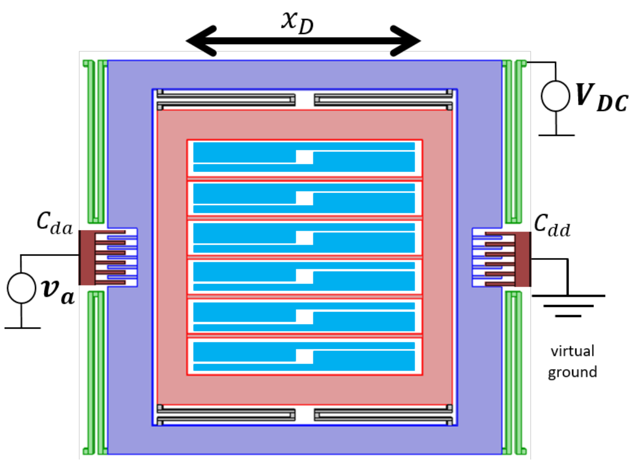

Figure 3: Sketch of a *single-ended* drive configuration in gyroscopes: we note a single electrical drive signal and a single electrical output current at the virtual ground.

signal through its first harmonic only, as other harmonics will be filtered out by the sharp resonator transfer function:

$$v_a(t) \simeq \frac{4}{\pi} v_{a,sq} \sin(\omega_{rd} t),$$

where $v_{a,sq}$ is the peak (not peak-to-peak) amplitude of the square-wave.
The net electrostatic force on the proof mass of a single-ended resonator (Fig. 3), if the number of comb fingers for the actuation and the detection electrodes is the same, can be written as:

$$F_{e,d} = \underbrace{\frac{1}{2}\frac{\partial C_{da}}{\partial x} v_a^2}_{\mathrm{dc}} + \underbrace{\frac{\partial C_{da}}{\partial x} V_{DC} v_a \sin(\omega_{rd} t)}_{\omega_{rd}} + \underbrace{\frac{1}{2}\frac{\partial C_{da}}{\partial x} v_a^2 \cos(2\omega_{rd} t)}_{2\omega_{rd}}.$$

The force applied to the resonator has thus three contributions: (i) a DC voltage, (ii) a contribution at ω_{rd}, (iii) a contribution at $2\omega_{rd}$. In order to have the $2\omega_{rd}$ component negligible, we need to guarantee that

$$v_a \ll V_{DC}.$$

If this condition is satisfied, neglecting the DC term which is compensated by the force at the other electrode, we get:

$$F_{e,d} = \underbrace{\left[\frac{\partial C_{da}}{\partial x} V_{DC}\right]}_{\eta_{da}} v_a \sin(\omega_{rd} t) = \eta_{da} v_a \sin(\omega_{rd} t),$$

In several situations, this condition is enough to guarantee the proper operation, e.g., the Tang resonator we studied in previous classes for time-keeping applications.

There might be situations, however, for which this condition is not enough. It may happen, for example, that the device has a spurious undesired mode at a frequency close to $2\omega_{rd}$, a typical situation for complex mechanical structures such as MEMS gyroscopes. Such a mode might be excited by the spurious drive signal at $2\omega_{rd}$, which must be avoided. Additionally, as large drive displacements are required, having a constraint on the amplitude of the applied AC voltage may be itself a limiting factor.

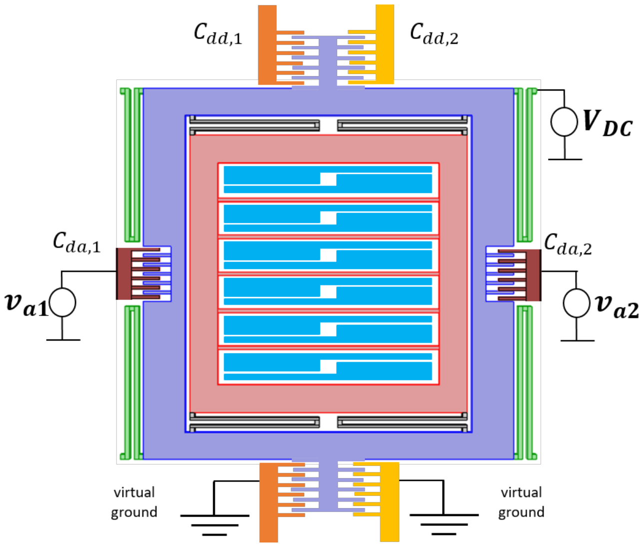

Figure 4: Sketch of a push-pull drive actuation configuration for MEMS gyroscopes: note two drive signals and two output differential currents.

A commonly-used configuration to get rid of this issue is the so-called *push-pull actuation*, depicted in Fig. 4. Such a configuration requires to double the number of stators for both drive actuation and drive detection. This leads to two advantages: (i) the intrinsic elimination of the $2\omega_{rd}$ component and (ii) the elimination of the small-signal hypothesis on the AC actuation voltage. Indeed, a push-pull actuation consists in the application of two AC (sine- or square-wave) signals for the drive actuation with a 180° phase delay:

$$v_{a1}(t) = v_a \sin(\omega_{rd} t), \qquad v_{a2}(t) = -v_a \sin(\omega_{rd} t),$$

With a push-pull drive actuation, the total electrostatic force can be expressed as:

$$F_{e,d} = F_{e,da,1} - F_{e,da,2} + F_{e,dd,1} - F_{e,dd,2}$$

$$F_{e,d} = \frac{1}{2}\frac{\partial C_{da,1}}{\partial x}(V_{DC} + v_a \sin(\omega_{rd} t))^2 - \frac{1}{2}\frac{\partial C_{da,2}}{\partial x}(V_{DC} - v_a \sin(\omega_{rd} t))^2 + \\ + \frac{1}{2}\frac{\partial C_{dd,1}}{\partial x}V_{DC}^2 - \frac{1}{2}\frac{\partial C_{dd,2}}{\partial x}V_{DC}^2.$$

Assuming that the two actuation electrodes are equal one another, and that the two detection electrodes are equal one another:

$$\frac{\partial C_{da,1}}{\partial x} = \frac{\partial C_{da,2}}{\partial x} \equiv \frac{\partial C_{da}}{\partial x}, \qquad \frac{\partial C_{dd,1}}{\partial x} = \frac{\partial C_{dd,2}}{\partial x} \equiv \frac{\partial C_{dd}}{\partial x}$$

the drive-detection terms get canceled, and the electrostatic force can be simplified as

$$F_{e,d} = \frac{1}{2}\frac{\partial C_{da}}{\partial x}\left[(V_{DC} + v_a \sin(\omega_{rd} t))^2 - (V_{DC} - v_a \sin(\omega_{rd} t))^2\right].$$

Reminding that $(a+b)^2 - (a-b)^2 = 4ab$, with no assumption the equation simplifies to:

$$F_{e,d} = \frac{1}{2}\frac{\partial C_{da}}{\partial x} 4 V_{DC} v_a \sin(\omega_{rd} t) = \underbrace{\left[2\frac{\partial C_{da}}{\partial x} V_{DC}\right]}_{\eta_{da}} v_a \sin(\omega_{rd} t) = 2\eta_{da} v_a \sin(\omega_{rd} t),$$

where η_{da} is the transduction coefficient of *one* drive-actuation electrode. With a comb-based electrode, the force can be re-written as:

$$F_{e,d} = 2\frac{2\epsilon_0 h N_{CF}}{g} V_{DC} \cdot v_a \sin(\omega_{rd} t).$$

As anticipated, the push-pull configuration eliminates high-frequency spurious tones and the validity of its harmonic content is not limited to a small signal approximation.
We can now go back to our initial question. A 0.5-V peak-to-peak square-wave driving has a first harmonic of amplitude:

$$v_a = \frac{4}{\pi}\frac{v_{a,sq}}{2} = 318\,\mathrm{mV}.$$

The drive-mode quality factor, Q_d, can be calculated as:

$$Q_d = \frac{\sqrt{k_d m_d}}{b_d} = 10\,240.$$

Since

$$x_{da} = \frac{Q_d}{k_d} 2\eta_{da} v_a,$$

where the factor 2 takes into account the two halves, the target drive-actuation transduction coefficient is

$$\eta_{da} = \frac{1}{2}\frac{x_{da}}{\frac{Q_d}{k_d} v_a} = 33.4 \times 10^{-9}\,\mathrm{VF/m}, \qquad \frac{\partial C_{da}}{\partial x} = \frac{\eta_{da}}{V_{DC}} = 3.35\,\mathrm{fF}/\mu\mathrm{m}.$$

The number of required comb fingers for each drive-actuation electrode on each half structure is thus:

$$N_{CF,da} = \frac{\frac{\partial C_{da}}{\partial x}}{2\varepsilon_0 \frac{h}{g}} = 16,$$

Question 4

We want now to find the mechanical sensitivity of the gyroscope, S, defined as the differential sense capacitance variation per unit angular rate. This can be split into two sub-terms as:

$$S = \frac{\partial C_{s,diff}}{\partial \Omega} = 2\frac{\partial C_s}{\partial \Omega} = 2\frac{\partial C_s}{\partial y}\frac{\partial y}{\partial \Omega},$$

where $\partial C_s/\partial y$ is the capacitance variation per unit displacement of one sense port, and $\partial y/\partial \Omega$ is the sense displacement amplitude variation per unit angular rate. The factor 2 takes into account the differential readout. As usual, the capacitance variation per unit displacement of one sense port, $\partial C_s/\partial y$, can be expressed as

$$\frac{\partial C_s}{\partial y} = \frac{C_s}{g} = \frac{\epsilon_0\,(2N_{PP})\,L_{PP}h}{g^2} = 106\,\mathrm{fF}/\mu\mathrm{m},$$

where the factor 2 here takes into account the two halves of the device.
The sense displacement amplitude induced by the Coriolis force, y_a, is:

$$y_a = \frac{Q_s}{k_s}F_{ca} = \frac{Q_s}{k_s}2m_s\omega_{rd}x_{da}\Omega = \frac{2Q_s}{\omega_{rs}}\omega_{rs}\frac{m_s}{k_s}\omega_{rd}x_{da}\Omega = \frac{2Q_s}{\omega_{rs}}\frac{\omega_{rd}}{\omega_{rs}}x_{da}\Omega.$$

In mode-match operation, the drive and sense resonance frequencies are equal, hence:

$$y_a = \frac{2Q_s}{\omega_{rs}}\frac{\cancel{\omega_{rd}}}{\cancel{\omega_{rs}}}x_{da}\Omega \simeq \frac{x_{da}}{\Delta\omega_{BW}}\Omega, \qquad \Delta\omega_{BW} = \frac{\omega_{rs}}{2Q_s}$$

where $\Delta\omega_{BW}$ is the sense-axis bandwidth. Hence,

$$\frac{\partial y}{\partial \Omega} = \frac{x_{da}}{\Delta\omega_{BW}}.$$

The sense-axis quality factor and bandwidth can be calculated as:

$$Q_s = \frac{\sqrt{k_s m_s}}{b_s} = \frac{(2\pi f_{rs})\,m_s}{b_s} = 2690 \qquad \Delta\omega_{BW} = \frac{\omega_{rs}}{2Q_s} = 22.52\,\mathrm{rad/s} = 3.5\,\mathrm{Hz}.$$

The corresponding variation of the Coriolis frame position per unit angular rate is

$$\frac{\partial y}{\partial \Omega} = \frac{x_{da}}{\Delta\omega_{BW}} = 310\,\mathrm{nm/(rad/s)} = 310\,\frac{\mathrm{nm}}{\frac{180\,\mathrm{dps}}{\pi\,\mathrm{rad/s}}\,\mathrm{rad/s}} = 5.4\,\mathrm{pm/dps}$$

The total capacitance variation per unit angular rate, which is the electromechanical sensitivity, is thus

$$S = 2\frac{\partial C_s}{\partial y}\frac{\partial y}{\partial \Omega} = 66\,\mathrm{fF/(rad/s)} = 1.15\,\mathrm{fF/dps}.$$