Politecnico di Milano
MSc Course - MEMS and Microsensors - 2024/2025

E05
Resonator Design

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Problem

You work in a company developing innovative smart-watches, and you are asked to design a MEMS resonator, based on the so-called Tang structure. The target resonance frequency, f_0, is 32.768 kHz. The equivalent resistance of the electrical model of the resonator, noted as R_m, should be lower than 10 MΩ. The electromechanical structure, see Fig. 1, has a moving mass with an equivalent area of 150 μm x 85 μm and a spring length of 91 μm. Additionally, the device has 8 differential parallel-plate cells used solely for electrostatic tuning. The fabrication technology has some rules and characteristic dimensions, listed in Table 1[1]. You are asked to:

1. Neglecting any softening, calculate the worst-case variability of the resonance frequency of the device, given the process tolerances ($3\sigma_x$ and $3\sigma_h$).
2. Knowing (i) that the final resonance frequency of the device has anyway to match the target frequency, f_0, and (ii) that it is possible to exploit electrostatic tuning

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[1] Due to process spreads, dimensions of real devices may differ from the design: etch spreads indicate that the structural layer can be wider or narrower than by design; thickness spreads refer to an epitaxial height thicker or thinner than the nominal target. This variability is described by a Gaussian function: the most probable case is for no process spread; numbers reported in Table 1 refer to the 3σ value of the distribution.

	Symbol	Value
Young's Modulus	E	160 GPa
Density	ρ	2320 kg/m^3
Process thickness	h	15 μm
Nominal spring width	w_{sp}	1.5 μm
Nominal process gap	g_{min}	1 μm
Etch spread	$\pm 3\sigma_x$	$\pm$0.05 μm
Thickness spread	$\pm 3\sigma_h$	$\pm$1 μm
N. of comb fingers per side	N_{CF}	38
Quality factor at room T	Q	670

Table 1: Technological constraints and rules.

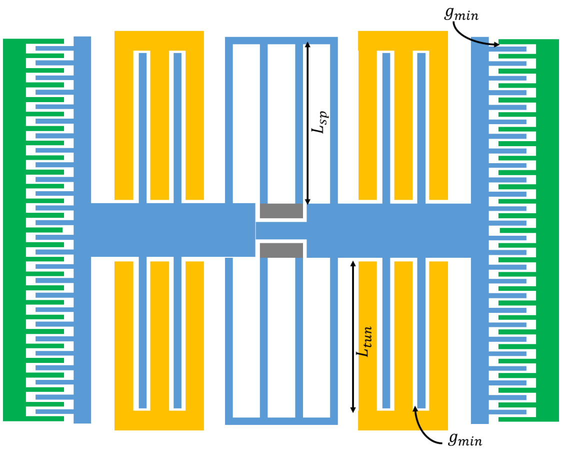

Figure 1: Structure of the MEMS resonator. Blue: moving mass; yellow: tuning electrodes; grey: anchored mass; green: drive and sense electrodes.

to solve over- or under-etch issues, find a clever target natural frequency, f_r, of the device and choose the proof mass value.

3. Calculate the maximum voltage that should be applied to the tuning electrodes to bring all the devices down to the target resonance frequency, f_0.
4. For the target parameters, find the minimum DC rotor voltage, V_{DC}, in order to comply with the requirement on R_{eq}, and extract the complete electrical equivalent model of the resonator.

INTRODUCTION

The Tang topology is a common structure for MEMS resonators and can be made very small with respect to typical dimensions of MEMS inertial sensors. It employs a comb-finger structure for both actuation and sensing to avoid non-linearity of parallel plates and reach large vibration amplitudes.

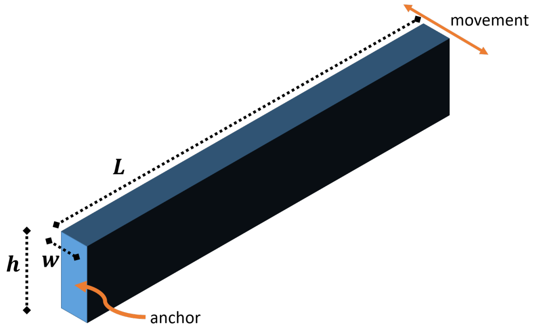

Figure 2: Generic, non folded, guided-end spring.

Why a resonance frequency of 32.768 kHz? This frequency value is commonly used because it is a power of 2 ($32768 = 2^{15}$). One can easily obtain a precise 1-Hz clock using binary frequency dividers, e.g. flip-flop chains used as binary counters. For this reason, this frequency value is the industry standard for real-time-clock applications.

QUESTION 1

Let us analyze the effect of the variation of the thickness of the PolySi layer on the resonance frequency. For a generic non-folded, guided-end spring, as the one shown in Fig. 2, the spring constant can be estimated as

$$k = E\frac{w^3 h}{L^3},$$

where E is Young's modulus, w is the width of the spring, h is its height, and L is the length. Note that this expression is valid for displacements occurring along the w-axis of the spring.

The mass of a suspended structure also depends on the thickness of the PolySi layer: $m = Ah\rho$, where A is the area of the structure, h is its thickness, ρ is its density. The resonance frequency of such a structure can be easily evaluated as:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{Ew^3 h}{Ah\rho L^3}} = \frac{1}{2\pi}\sqrt{\frac{Ew^3}{A\rho L^3}},$$

which is independent of the thickness of the device. This is a general rule for all the structures that have an in-plane mode. Note that the dependence is different if the device is a torsional structure: in this case, the spread on the process height becomes a critical parameter for the determination of its resonance frequency.

On the contrary, the effect of the over- or under-etch, i.e., of the variability of the in-plane dimensions, is very important for both in-plane and out-of-plane structures. Indeed, the stiffness of a spring has a cubic dependence on its width, hence a small change of this dimension has a huge effect on the spring constant, hence on the resonance frequency (while the mass is usually affected in a negligible way). In order to numerically evaluate the effects of this over-/under-etch, it is common practice to differentiate the expression of the resonance frequency with respect to the spring constant:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow \quad \frac{\partial f_r}{\partial k} = \frac{1}{4\pi\sqrt{km}} \cdot \frac{\sqrt{k}}{\sqrt{k}} = \frac{f_r}{2k} \quad \rightarrow \quad \frac{\partial f_r}{f_r} = \frac{1}{2}\frac{\partial k}{k}.$$

We can then differentiate the expression of the stiffness with respect to its width

$$k = E\frac{w^3 h}{L^3} \quad \rightarrow \quad \frac{\partial k}{\partial w} = \frac{3Ew^2 h}{L^3} = \frac{3}{w}\frac{Ew^3 h}{L^3} = \frac{3}{w}k \quad \rightarrow \quad \frac{\partial k}{k} = 3\frac{\partial w}{w}.$$

By combining the last two expressions, the resonance frequency variation due to a width variation is easily calculated:

$$\frac{\partial f_r}{f_r} = \frac{3}{2}\frac{\partial w}{w}.$$

In other terms,

$$\underbrace{f_r}_{\text{actual resonance}} = \underbrace{f_{r0}}_{\text{nominal resonance}} + \underbrace{\partial f_r}_{\text{frequency variation}} = f_{r0} + \frac{3}{2}\frac{f_{r0}}{w}\partial w.$$

The maximum resonance variation is obtained for the maximum over-etch, Δw_{max}, i.e., 2 times[2] the 3σ value of the spread probability density function:

$$\Delta f_{r,max} = \frac{3}{2}\frac{\Delta w_{max}}{w_{sp}} f_r = \frac{3}{2}\frac{2 \cdot 3\sigma_x}{w_{sp}} f_0 = \frac{3}{2}\frac{2 \cdot 0.05\,\mu\text{m}}{1.5\,\mu\text{m}} f_r = 3276.8\,\text{Hz}.$$

This is the unilateral maximum variation of the resonance frequency of the device due to under-/over-etch of the process. This means that such a device can randomly resonate from $f_0 - \Delta f_{r,max} = 29491$ Hz to $f_0 + \Delta f_{r,max} = 36045$ Hz.

QUESTION 2

Tang resonators are used for applications requiring precise oscillation frequencies. It is therefore mandatory to obtain the target resonance frequency value independently of process spreads.

As we learned in other classes, one can exploit electrostatic spring softening (frequency tuning) to change the resonance of an electromechanical structure.

We know that frequency tuning can shift the resonance only towards values lower than the nominal one. For this reason, a wise design shall target a higher (than 32.768 kHz) nominal (no over-/under-etch) frequency, f_r. In particular, one shall design the structure such that, for the maximum over-etch, the obtained frequency exactly matches 32.768 kHz (refer to Fig. 3).

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[2] We consider two times the over-/under-etch because a spring has two over-/under-etched edges.

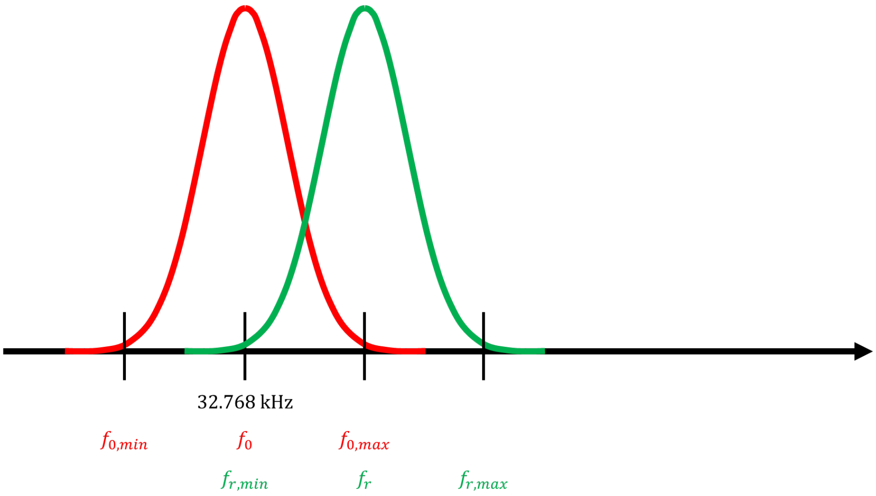

Figure 3: Schematic representation of the final (f_0) and design (f_r) target frequency for the device, evidencing the spread due to under-/over-etch.

Starting from the relationship obtained above, for a certain resonance frequency, f_r, the maximum frequency variation has to be $\Delta f_{r,max} = f_r - f_0$. Hence:

$$\frac{\Delta f_{r,max}}{f_r} = \frac{f_r - f_0}{f_r} = \frac{3}{2}\frac{\Delta w_{max}}{w_{sp}} = \frac{3}{2}\frac{2 \cdot 3\sigma_x}{w_{sp}},$$

where f_r is the equation unknown. Hence, $f_r = \dfrac{f_0}{1 - \dfrac{3}{2}\dfrac{2 \cdot 3\sigma_x}{w_{sp}}} = 36.409\,\text{kHz}$.

With a resonance frequency of 36.409 kHz, the maximum unilateral frequency variation due to under-/over-etch becomes:

$$\Delta f_{r,max} = \frac{3}{2}\frac{2 \cdot 3\sigma_x}{w_{sp}} \cdot f_r = 3641\ \text{Hz}.$$

In this way, assuming a linear dependence between spring width and resonance frequency, the minimum obtainable frequency for the resonator structure turns out to be exactly $f_{r,min} = f_r - \Delta f_r = f_0 = 32.768$ kHz, while the maximum one $f_{r,max} = f_r + \Delta f_{max} = 40.050$ kHz, as shown in Fig. 4.

We have now to size the mass of our device, so to target f_r. With the Tang resonator structure shown in Fig. 1, the spring on each side can be described by the parallel of two beams (from the anchor point to the rigid link), themselves in series with two further beams in parallel (from the rigid link to the suspended frame). Taking into account that we have two of such springs, the overall mechanical stiffness becomes

$$k = k_{sb}\frac{1}{2}2 \cdot 2 = 2k_{sb},$$

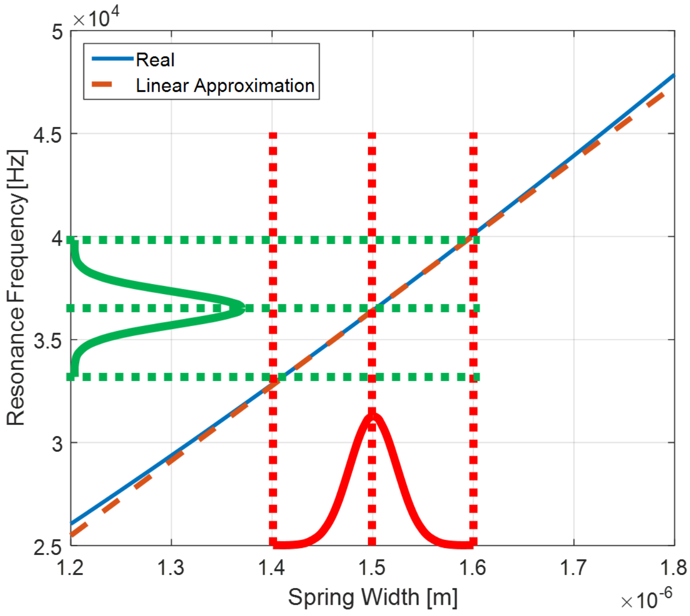

Figure 4: Real (blue curve) behavior of the resonance frequency for different over-/under-etch values and its linear fitting (red-dashed curve).

where k_{sb} is the spring constant of a single beam; the factor 1/2 accounts for the series connection, one factor 2 indicates the parallel between the beam pairs, and another factor 2 indicates the two springs in parallel. With the chosen values of width, $w_{sp} = w_{sp}$, and length, L_{sp}, the stiffness can be evaluated as:

$$k = 2k_{sb} = 2\frac{Ew_{sp}^3 h}{L_{sp}^3} = 21.5\,\mathrm{N/m}.$$

Once this nominal stiffness is targeted, the actual stiffness may vary between 26 N/m (for a spring width of 1.6 μm) and 17.4 N/m (for a spring width of 1.4 μm). Note that this latter case corresponds to the nominal resonance frequency (indeed, for the maximum etching the resonance value is the lowest, and matches 32768 Hz, as expected, without requiring electrostatic tuning).
Given the target resonance frequency, f_r, we calculate the required mass:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow \quad m = \frac{1}{4\pi^2}\frac{k}{f_r^2} = 0.41\,\mathrm{nKg}.$$

We should now make a comparison between the required mass, m, and the maximum mass we can effectively have from the geometry. With the provided data, the total area forming the mass of the device, $A_{eq} = 150$ μm x 85 μm, corresponds to a mass of

$$m_{full} = A_{eq} h \rho = 0.44\,\mathrm{nKg}.$$

This is larger than the value found above, indicating that we can use holes in the structure to reduce the effective mass value.

QUESTION 3

Given the resonance frequency, f_r, and its variability due to process spreads, the maximum resonant frequency, $f_{r,max}$, can be found, and it is possible to calculate the maximum voltage that has to be applied to the tuning stators in order to tune the resonance frequency down to f_0.

As shown in Fig. 1, there are 8 differential parallel plates tuning cells, 4 in the upper part of the structure and 4 in the lower part. The differential cells are all composed by two stators and the rotor.

In order to calculate the voltage to be applied to the tuning stators, we have to calculate the contribution to the stiffness given by the tuning process, i.e., the electrostatic stiffness. Starting from the maximum frequency, we can calculate the maximum mechanical stiffness of our device:

$$f_{r,max} = \frac{1}{2\pi}\sqrt{\frac{k_{max}}{m}} \quad \rightarrow \quad k_{max} = f_{r,max}^2 \cdot 4\pi^2 \cdot m = 26\,\mathrm{N/m}.$$

In order to tune the resonance frequency down to $f_0 = 32.768$ kHz even in this worst case, the needed electrostatic stiffness, k_{el}, can be evaluated from the following equation:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_{max}+k_{el,max}}{m}} \quad \rightarrow \quad k_{el,max} = 4\pi^2 \cdot f_0^2 \cdot m_{target} - k_{max} = -8.6\,\mathrm{N/m}.$$

The expression of the electrostatic stiffness for differential tuning electrodes is:

$$k_{el} = 2V_{tun}^2\frac{C_0 N_{tun}}{g_{tun}^2} = 2V_{tun}^2\frac{\epsilon_0 h L_{tun} N_{tun}}{g_{tun}^3},$$

where V_{tun} is the voltage difference between the rotor and the tuning electrodes, C_0 is the rest capacitance of a single tuning electrode, g_{tun} is the tuning-electrode gap, L_{tun} is the length of each tuning electrode, and N_{tun} is the number of tuning electrodes. As the length of the electrodes, L_{tun}, is not yet decided, we choose to make it as long as possible, so that to minimize the tuning voltage. Leaving some distance between the stators and the moving parts, e.g. 2.5 μm for both the upper and the lower part of the electrodes, its maximum length can be easily evaluated as

$$L_{tun} = L_{sp} - 2 \cdot 2.5\,\mu\mathrm{m} = 86\,\mu\mathrm{m}.$$

The gap between tuning electrodes and springs becomes in this case $g_{tun} = 0.9$ μm due to minimum etching on both sides. The number of differential cells is given ($N_{tun} = 8$). The maximum voltage to be applied to the tuning electrode can thus be found:

$$k_{el,max} = -2V_{tun,max}^2\frac{\epsilon_0 h L_{tun} N_{tun}}{g_{tun}^3} \quad \rightarrow \quad V_{tun,max} = \sqrt{\frac{-k_{el} \cdot g_{tun}^3}{2\epsilon_0 h L_{tun} N_{tun}}} = 5.86\,\mathrm{V}.$$

Note that, since $|k_{el,max}| < |k_{max}|$, the obtained tuning voltage is obviously lower than pull-in.

QUESTION 4

Let us infer the electrical model of a resonator in the Laplace domain. The output (or motional or sense) current, I, of a resonator can be evaluated as:

$$I\left(s\right)=\eta_s\dot{X}\left(s\right),$$

where $\eta_s=V_{DC}\frac{\partial C_s}{\partial x}$ is the sense-electrode electromechanical transduction coefficient, and $\dot{X}$ is the velocity of the proof mass. The velocity is the derivative of the displacement, $\dot{X}\left(s\right)=sX\left(s\right)$. Hence,

$$I\left(s\right)=\eta_s sX\left(s\right).$$

The displacement, X is related to the actuation force through the well-known second-order mechanical transfer function of a MEMS resonator:

$$X\left(s\right)=\frac{1}{k+bs+ms^2}F\left(s\right).$$

Assuming a small-signal actuation voltage, one can linearize the actuation voltage-vs-force relationship as

$$F=\eta_a V,$$

where $\eta_a=V_{DC}\frac{\partial C_a}{\partial x}$ is the actuation-electrode (sometimes referred to as drive) electromechanical transduction coefficient. One thus obtains

$$I\left(s\right)=\eta_s\frac{s}{k+bs+ms^2}\eta_a V\left(s\right).$$

We can now define the transfer function between actuation (driving) voltage and motional (sense) current as

$$Y\left(s\right)=\frac{I\left(s\right)}{V\left(s\right)}=\eta_s\frac{s}{k+bs+ms^2}\eta_a,$$

which has the dimensions of a physical admittance. Its inverse, $Z=1/Y$ has the expression of an impedance, and can be referred to as the motional impedance of a MEMS resonator, where motional refers to the fact that models the *moving, dynamic* section of the resonator. With identical drive- and sense- electromechanical transduction coefficients, $\eta_a=\eta_s=\eta$, the motional impedance can be expressed as:

$$Z\left(s\right)=\frac{k+bs+ms^2}{s}\frac{1}{\eta^2}=\frac{k}{s\eta^2}+\frac{b}{\eta^2}+\frac{m}{\eta^2}s=\frac{1}{sC_m}+R_m+L_m s,$$

where

$$R_m=\frac{b}{\eta^2},\qquad L_m=\frac{m}{\eta^2},\qquad C_m=\frac{\eta^2}{k}$$

are the motional resistance, inductance, and capacitance of the series electrical model of a MEMS resonator, as shown in Fig. 5.

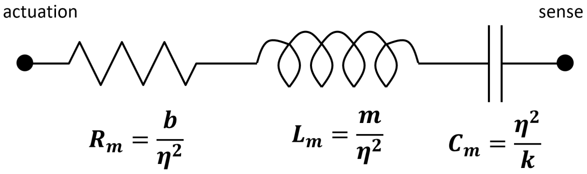

Figure 5: Series-RLC electrical model of a MEMS resonator.

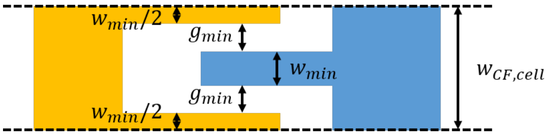

Figure 6: Single comb-finger structure.

As inferred from the electrical model of the resonator, the DC voltage applied to the moving mass, V_{DC}, is directly related to its motional resistance, R_m,

$$R_m = \frac{b}{\eta_a \eta_s} = \frac{1}{\eta_a \eta_s} \frac{2\pi f_r m}{Q}.$$

Note that, the higher is the quality factor, i.e., the lower is the damping coefficient, the lower is the equivalent resistance.

In order to evaluate the needed DC bias voltage, we first need to evaluate the electromechanical transduction coefficient of our resonator. The rotor-stator capacitance of a generic comb electrode can be expressed as

$$C = 2 \cdot N_{CF} \frac{\epsilon_0 h \left(x + L_{ov}\right)}{g},$$

where L_{ov} is the rotor-stator overlap length at rest (i.e., with $x = 0$), x is the rotor displacement, g is the gap between rotor and stators; the factor 2 takes into account both faces of the N_{CF} comb structures (see Fig. 6).

The capacitance variation per unit displacement of the comb-structure is thus:

$$\frac{\partial C}{\partial x} = \frac{2\epsilon_0 h N_{CF}}{g},$$

which is independent of the displacement. As mentioned, this is a huge benefit of comb-finger electrodes, when compared to parallel-plate ones.

For a symmetric resonator, as the one considered in this example, where the number of drive- and sense-comb structures are the same, η_a and η_s coefficients are equal, as $\partial C_a/\partial x = \partial C_s/\partial x$. The equivalent resistance can be thus expressed as

$$R_m = \frac{1}{\left(V_{DC}\frac{\partial C}{\partial x}\right)^2} \frac{2\pi f_r m}{Q}.$$

The equivalent resistance depends on the inverse of the square of V_{DC}: the higher is the DC voltage, the lower is R_{eq}. We have all the parameters needed to calculate its value:

$$R_m = \frac{b}{\eta^2} = \frac{1}{\left(V_{DC}\dfrac{\partial C}{\partial x}\right)^2}\frac{2\pi f_r m}{Q} = \frac{2\pi f_r m}{Q\left(V_{DC}\dfrac{2\epsilon_0 h N_{CF}}{g}\right)^2} < 10\mathrm{M}\Omega \quad \rightarrow \quad V_{DC} > 11.2\mathrm{V}.$$

Be careful! The voltage applied to the tuning electrodes has now to be increased! Remember that the spring softening effect depends on the voltage difference between the rotor and the tuning electrodes: the value for the tuning voltage we found above is not the absolute tuning voltage, but it is the voltage difference that has to be applied between rotor and tuning electrodes.
Once this value is calculated, it is easy to evaluate η and so the values of the components of the equivalent electrical model of the resonator, whose admittance is depicted in Fig. 7 for different V_{DC} values. Using the value found above, $V_{DC} = 11.2$ V, and the total stiffness relative to the final frequency f_0, i.e., $k_{tot} = 4\pi^2 f_0^2 m = 17.4$ N/m, one gets

$$R_m = \frac{b}{\eta^2} = 10\,\mathrm{M}\Omega, \qquad L_m = \frac{m}{\eta^2} = 32\,\mathrm{kH}, \qquad C_m = \frac{\eta^2}{k} = 0.72\,\mathrm{fF}$$

These values are typical for comb-based, small-size MEMS resonators.

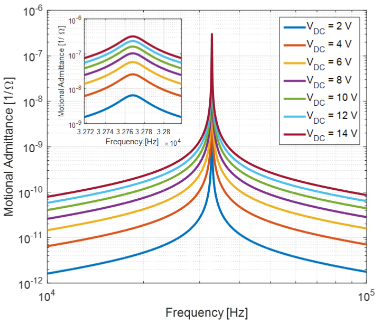

Figure 7: Magnitude of the resonator admittance for different V_{DC} values.