Politecnico di Milano
MSc Course - MEMS and Microsensors - 2023/24

E03
Design of a torsional MEMS accelerometer

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Problem description and questions

You are asked to design a consumer-grade accelerometer for out-of-plane accelerations inside a 6-axis IMU. In order to use the same electronic circuit discussed for the in-plane devices, the OP accelerometer needs to target the same performance: $dC_{diff}/da_{ext} = 4.1\,\mathrm{fF}/\hat{\mathrm{g}}$, $f_0 = 4456\,\mathrm{Hz}$ (in operation), $FSR = 16\,\hat{\mathrm{g}}$, $V_{DD} = 3\,\mathrm{V}$. Other parameters are reported in Table 1. Also refer to Figure 1 for a better understanding of the geometry.

	Symbol	Value
Shear modulus	G	65 GPa
process tickness	h	24 µm
holes transduction coefficient	α	0.87
start point of PP (from the rotational axis)	x_0	10 µm
end point of PP (from the rotational axis)	x_f	150 µm
mass 1 width	r_1	150 µm
mass 2 width	r_2	300 µm
gap of PP	g	1.3 µm
maximum device length	L	950 µm
hole pitch / total pitch	h_p / t_p	3 µm / 10 µm

Table 1: Problem parameters.

1. Find the maximum tilt angle for the accelerometer in operation and the linearity error.

2. Given the target mechanical sensitivity and using the the hole transduction coefficient in the Table, calculate the required length of the parallel plates[1].

[1] the hole transduction coefficient indicates how much a holed vertical parallel plate transduces a capacitance variation with respect to an ideal full plate, see the upcoming laboratory

3. Evaluate the parallel-plate contribution in terms of electrostatic stiffness, and choose the springs geometry (hint: start from the silicon density, $\rho = 2320\ kg/m^3$, to find an effective mass density through the holes pitch).

4. Which are the parameters affected by a thickness variation of the process for both the accelerometers (IP and OP)?

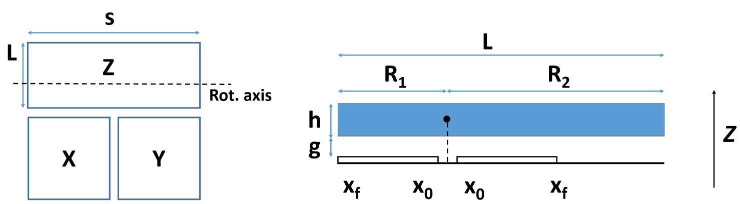

Figure 1: sketch of (i) the top view layout of a 3-axis accelerometer and of (ii) the lateral cross-section of the z-axis device.

Introduction

Why is it almost mandatory to use torsional accelerometers to reveal out-of-plane accelerations?
1) the OP translational stiffness becomes orders of magnitude larger with respect to the IP translational stiffness because of the cubic dependence on process height, a parameter you cannot act on by design.
2) it is not possible to get a differential sensing with translational motion and electrodes only underneath the proof mass.
The torsional accelerometer avoids these issues: while one half approaches the electrode, the other one moves away, see Figure 2.

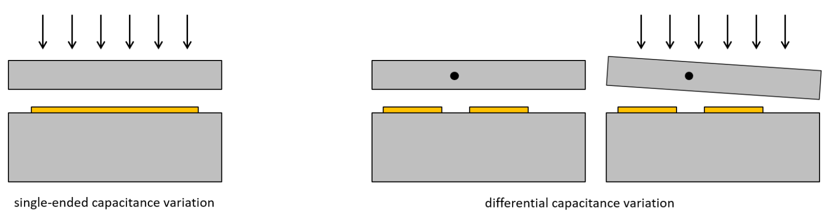

Figure 2: sketch of a vertical displacement and of a torsional displacement.

Thus, we are moving from the linear system described by mass-force-stiffness-displacement to a torsional system described by inertia-moment-torsional stiffness-angle.
Let us introduce some parameters (you can refer to class $n.6$ for further details):

- Shear modulus: it is defined as the ratio of shear stress to the shear strain. The shear modulus in polysilicon is about $G = 65GPa$.

- Torque: is defined as the cross product of the lever-arm distance vector and the force vector. Then, the dimension of torque is $N \cdot m$.

- Torsional stiffness: it is involved in the relationship between angle of twist and the applied torque. For our purpose, the simplified formula of a single torsional bar is enough

$$k = G\frac{hw^3}{3l}\left[Pa\frac{\cancel{m} \cdot m^3}{\cancel{m}}\right] \rightarrow \left[\frac{N}{\cancel{m^2}}m^{\cancel{3}}\right] \rightarrow [N \cdot m]$$

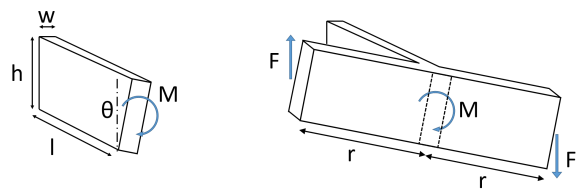

Figure 3: sketch of a torsional spring subject to a torque.

- Moment of inertia: is the second moment of mass with respect to distance from an axis. Then, the dimension of the inertia is $kg \cdot m^2$.

$$I = \int_m r^2 dm.$$

For a better understanding, we calculate the inertia for the given OP accelerometer. For the sake of simplicity, we can divide this calculation in two parts: one for the left half of the accelerometer and the second one for the right half (see Figure 1).

$$I = I_1 + I_2 = \int_{m_1} r^2 dm + \int_{m_2} r^2 dm$$

The mass is not considered as point-like: for more accurate results, we can consider it uniformly distributed along the device.

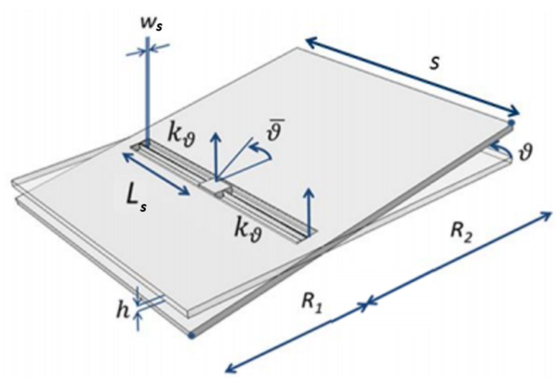

Figure 4: sketch of a torsional accelerometer, helpful for moment of inertia calculation.

Observing figure 4, it is clear that in this situation is possible to express the infinitesimal mass element as follows:

$$dm = sh\rho \cdot dr$$

Elaborating the definition of moment of inertia:

$$I = \int_0^{r_1} r^2 sh\rho \cdot dr + \int_0^{r_2} r^2 sh\rho \cdot dr = \frac{r_1^3}{3} sh\rho + \frac{r_1^3}{3} sh\rho = \frac{r_1^2 m_1 + r_2^2 m_2}{3}.$$

In the specific situation of our accelerometer,$r_2 = 2 \cdot r_1$ and consequently m_2 is twice m_1. It follows that:

$$I = 3 r_1^2 m_1.$$

To calculate the value of an external torque momentum, M_{ext}, it is convenient to solve the motion equation in the non-inertial reference, which for a torsional system is:

$$I\ddot{\theta} + b\dot{\theta} + k\theta = M_{ext}$$

Note that in a torsional system the unit of measurement of the damping is $[kg/s \cdot m^2]$.
Being $\Theta(s)$ the Laplace transformation of the relative angle and $M_{ext}(s)$ the Laplace transformation of the external torque momentum, one can find the transfer function $T_{\Theta M}(s)$ as

$$T_{\Theta M}(s) = \frac{\Theta(s)}{M_{ext}(s)} = \frac{1/I}{\left(s^2 + \frac{b}{I}s + \frac{k}{I}\right)}$$

Writing this equation in terms of '$j\omega$' and considering that $\omega_0 = \sqrt{k/I}$ and $Q = \omega_0 I/b$, we obtain

$$T_{\Theta M}(j\omega) = \frac{\Theta(j\omega)}{M_{ext}(j\omega)} = \frac{1/I}{\left(\omega_0^2 - \omega^2 + j\frac{\omega_0\omega}{Q}\right)}$$

Evaluating the transfer function modulus in quasi-stationary conditions typical of accelerometers, one can find the relation between angle and torque momentum below the resonance frequency ω_0

$$|T_{\Theta M}(j\omega)|_{\omega \ll \omega_0} = \frac{1/I}{\omega_0^2} = \frac{1}{k}$$

this means that when a quasi-stationary torque is applied to the seismic mass the angle is governed just by the torsional stiffness.

Question 1

To evaluate the maximum angle undergone by the accelerometer, θ_{FSR}, one can write the angle in terms of applied torque moment on the seismic mass through the transfer function $|T|$, remembering that the accelerometer works at $\omega < \omega_0$:

$$\theta = M \cdot |T(\omega < \omega_0)| \rightarrow \theta_{FSR} = \frac{M_{FSR}}{k} \rightarrow \theta_{FSR} = \frac{I}{k}\frac{M_{FSR}}{I} \rightarrow \theta_{FSR} = \frac{1}{\omega_0^2}\frac{M_{FSR}}{I}$$

where the inertia of the system is $I = 3r_1^2 m_1$, as reported above, and the torque moment at the full scale range can be calculated, assuming the application point of the inertial force in the mid-point of each half-structure (i.e. the forces are applied in $r_1/2$ and $r_2/2$ for the two half-masses):

$$M_{FSR} = M_1 + M_2 = \vec{l_1} \times \vec{F_1} + \vec{l_2} \times \vec{F_2} = -\frac{r_1}{2}m_1 \cdot FSR \cdot \hat{g} + \frac{r_2}{2}m_2 \cdot FSR \cdot \hat{g} = \frac{-r_1 m_1 + 4r_1 m_1}{2} \cdot FSR \cdot \hat{g} \rightarrow$$

$$M_{FSR} = \frac{3}{2}r_1 m_1 \cdot FSR\hat{g} \Rightarrow \theta_{FSR} = \frac{1}{\omega_0^2}\frac{FSR \cdot \hat{g}}{2r_1}$$

where $FSR \cdot \hat{g}$ indicates the full scale range acceleration: the value expressed in gravity units is multiplied by the gravity constant $9.8m/s^2$. Given the parameters and the fixed lateral geometry, the accelerometer undergoes a maximum angle $\theta_{FSR} = 6.66 \cdot 10^{-4} rad$. Note that the angle is determined by the external acceleration, by the resonance frequency and by the geometry.
To evaluate the linearity error, ϵ_{lin}, introduced by the parallel plates configuration, we need to write the real capacitance variation and the linearized capacitance variation at the full scale range, indeed:

$$\epsilon_{lin,FSR} = \frac{\Delta C_{real,FSR} - \Delta C_{lin,FSR}}{\Delta C_{real,FSR}} \cdot 100$$

where $\Delta C_{real,FSR}$ can be computed as

$$\Delta C_{real,FSR} = C_2 - C_1 = \epsilon_0 \int_{x_0}^{x_f} \frac{l_{PP}}{g - tan(\theta)x} dx - \epsilon_0 \int_{x_0}^{x_f} \frac{l_{PP}}{g + tan(\theta)x} dx$$

For the sake of simplicity, we can approximate the capacitance variation obtained with this integral through an equivalent parallel plate that translates vertically by a displacement z equal to the mean value across the electrode width. Naming x_m the medium point of the parallel plate electrode:

$$x_m = \frac{x_f + x_0}{2} \quad ; \qquad z = tan(\theta)x_m \sim \theta x_m \quad .$$

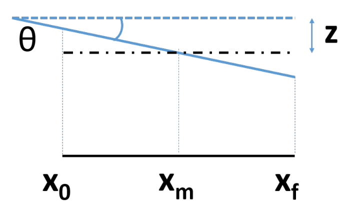

Figure 5: displacement approximation for torque capacitance variation

At this point, the expression of the real and linearized capacitance variations are identical to the IP case:

$$\Delta C_{real} = \epsilon_0 \frac{A_{PP}}{g - z} - \epsilon_0 \frac{A_{PP}}{g + z} = \epsilon_0 \frac{l_{PP} \cdot (x_f - x_0)}{g} \left(\frac{1}{1 - \frac{z}{g}} - \frac{1}{1 + \frac{z}{g}} \right) = C_0 \frac{2\frac{z}{g}}{1 - \left(\frac{z}{g}\right)^2}$$

$$\Delta C_{lin} = 2C_0 \frac{z}{g}.$$

In turn, the maximum linearity error can be as well calculated as for in-plane devices:

$$\left(\frac{z_{FSR}}{g}\right)^2 = \frac{\epsilon_{lin}}{100} \rightarrow \epsilon_{lin} = (\frac{x_m \cdot \theta_{FSR}}{g})^2 \cdot 100.$$

With the given geometry and parameters, the linearity error results $\epsilon_{lin} = 0.17\%$.

Question 2

The mechanical sensitivity is defined as the capacitance variation per gravity unit of acceleration. Then we can find the needed dimensions of parallel plates to satisfy the target (we are considering the 'vertical displacement approximation')

$$\frac{dC_{diff}}{da_{ext}} = \frac{\Delta C_{FSR}}{a_{FSR}} \quad \rightarrow \quad \frac{dC_{diff}}{da_{ext}} \cdot a_{FSR} = \Delta C_{FSR} = 64.36fF \rightarrow$$

$$\Delta C_{FSR} = \alpha \cdot 2 \frac{\epsilon_0 \cdot l_{PP} \cdot (x_f - x_0)}{g^2} \cdot z_{FSR}$$

Where $z_{FSR} = \Theta_{FSR} \cdot x_m$. In order to reach the target sensitivity, the parallel plates length becomes $l_{PP} = 944\mu m$.

We can also write, for the sake of completeness, the mechanical sensitivity formula by combining the equations found so far:

$$S_{mech} = \frac{\Delta C}{\Delta a_{ext}} = \frac{\Delta C_{FSR}}{z_{FSR}} \cdot \frac{z_{FSR}}{\theta_{FSR}} \cdot \frac{\theta_{FSR}}{a_{FSR}} =$$
$$= 2\alpha\frac{C_0}{g} \cdot x_m \cdot \frac{1}{\omega_0^2 \cdot 2 \cdot r_1} = 2\alpha\frac{C_0}{g} \cdot \frac{1}{\omega_0^2} \cdot \frac{x_m}{2 \cdot r_1}$$

The difference with respect to the in-plane accelerometer accounts for the capacitive fringe effects (α), the geometrical distribution of the mass (r_1) and the positioning of the parallel plates (x_m).

Question 3

The considered sensor presents a capacitive readout based on parallel plates configuration. To evaluate the effect of electrostatic softening introduced by the electrostatic torque momentum of the parallel plates, one can write according to the small displacement approximation:

$$M_{elec} = 2F_{elec} \cdot x_m = 2\alpha\frac{C_0}{g^2}V_{DD}^2 \cdot z \cdot x_m = 2\alpha\frac{C_0}{g^2}V_{DD}^2 \cdot \theta \cdot x_m^2$$

For a stationary torque applied to the sensor, the general equation of motion and thus the definition of electrostatic torsional stiffness become:

$$I\cancel{\ddot{\theta}} + b\cancel{\dot{\theta}} + k\theta = M_{ext} + M_{elec} \rightarrow k\theta - M_{elec} = M_{ext} \rightarrow \theta\left(k_{mec} + k_{elec}\right) = M_{ext}$$

$$k_{elec} = -2\alpha\frac{C_0}{g^2}V_{DD}^2 \cdot x_m^2.$$

which is the same expression as for the in-plane electrostatic stiffness, now multiplied by the squared distance between the electrode mid-point and the rotational axis.
In our case $k_{elec} = -5.3 \cdot 10^{-8} N \cdot m$. The total stiffness can be calculated from the resonance frequency in operation and the moment of inertia:

$$f_0 = \frac{1}{2\pi}\sqrt{k_{tot}/I} = \frac{1}{2\pi}\sqrt{k_{tot}/(2 * r_1^2 * m_1)}$$

The value of the mass m_1 corresponds to a silicon parallelepiped, which however has an effective density lower than a full one, because of the holes:

$$m_1 = 150\mu m \cdot 950\mu m \cdot 24\mu m \cdot \rho \cdot \frac{A_{full}}{A_{pitch}} =$$
$$= 7.9nkg \cdot \frac{A_{full}}{A_{pitch}} = 7.9nkg \cdot \frac{(10\mu m)^2 - (3\mu m)^2}{(10\mu m)^2} = 7.9nkg \cdot 0.91 = 7.2nkg$$

where the ratio takes the name of effective polysilicon density (0.91 in this case). The total stiffness turns out to be $k_{tot} = 3.8 \cdot 10^{-7} N \cdot m$.
Considering that $k + k_{elec} = k_{tot}$, it turns out that our mechanical stiffness is $k = 4.33 \cdot 10^{-7} N \cdot m$, about an order of magnitude larger than the electrostatic stiffness.

To size the springs, first of all let us keep in mind the geometry presented in figure 4. In order to obtain a correct torsion of the accelerometer, it is necessary to use two springs in a parallel configuration resulting in a mechanical torsional stiffness:

$$k = 2G\frac{w_s^3 h}{3l_s}$$

How to select the spring length? We prefer the torsional element to be as long as possible, because this allows a wider spring for the same stiffness (so, less 'over-etch' problems). The available dimension is $s = 950\mu m$. We have to fit within this space two springs and their anchor point. Let us assume a single anchor area, $50\mu m$ long: the corresponding length for the spring becomes $l_s = 450\mu m$. Now we can find the spring width as

$$w_s = \sqrt[3]{\frac{3lk}{2Gh}} = 5.72\mu m$$

Question 4

A typical challenge associated to the design of MEMS is related to the variation of fabrication process parameters. The technological flow is formed by different cascaded steps, and each of them can introduce imperfections in the geometry/packaging definition. Some of such typical issues are (i) variation of the thickness (due to nonuniformities in the epitaxial growth), (ii) variation of the springs/gaps width (due to the DRIE etching nonuniformities) and (iii) variation of the pressure inside the module. It is important, during the design phase, to take into account these spreads and to find a way to mitigate their effects.
Specifically discussing the request of this exercise, which are the parameters affected by a thickness spread, and how do they change as a function of its value? Let us review this for both types of accelerometers.

IP

$$S_{IP} = 2\frac{C_0}{g_{IP}} \cdot \frac{V_{DD}}{C_F} \cdot \frac{1}{\omega_0^2} = 2\frac{C_0}{g_{IP}} \cdot \frac{V_{DD}}{C_F} \cdot \frac{m}{k + k_{elec}}$$

- Of course, the seismic mass is proportional to thickness, $m \propto h$.
- The electrostatic stiffness results itself proportional to thickness, $C_0 \propto h$.
- The IP stiffness is proportional to the thickness, $k \propto h$, then also the total stiffness (electrostatic+mechanical).
- The IP resonance frequency is thus independent of the thickness: this result arises from the proportionality of the resonance frequency to the total stiffness and its inverse proportionality to the mass. This result is very important because it means that also the performed displacement per applied acceleration is itself independent of h.
- The IP differential capacitance variation per unit acceleration (i.e. the mechanical sensitivity) is finally proportional to the thickness. **It means that for the same planar geometry, a larger scale-factor is obtained in thicker devices**. If repeatable devices in terms of sensitivity are required, we should ensure a uniform wafer thickness!

OP

$$S_{OP} = 2\alpha\frac{C_0}{g_{OP}} \cdot \frac{V_{DD}}{C_F} \cdot \frac{1}{\omega_0^2} \cdot \frac{x_m}{2 \cdot r_1} = 2\alpha\frac{C_0}{g_{OP}} \cdot \frac{V_{DD}}{C_F} \cdot \frac{I}{k + k_{elec}} \cdot \frac{x_m}{2 \cdot r_1}$$

- The seismic mass is linear with thickness and so the other parameter related to the seismic mass, i.e. the moment of inertia, $I \propto h$.
- The torsional stiffness is proportional to thickness, $k \propto h$.
- The electrostatic stiffness, in OOP accelerometer, is independent of process height, because now C_0 (vertical gap) is independent of h.
- The resonant frequency thus slightly varies with process height, because the electrostatic stiffness remains constant. Given that the electrostatic contribution is usually much lower than the mechanical one, we can say that the resonant frequency is anyway almost constant against thickness spreads.
- As a consequence, **for OOP accelerometers, the sensitivity remains roughly uniform if process height fluctuations occur**.