Politecnico di Milano
MSc Course - MEMS and Microsensors - 2023/24

E02
MEMS Accelerometer DC vs AC Readout

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Problem description and questions

You work in the *R&D* laboratory af a medical company, and you have to design an electronic board in order to test a new capacitive accelerometer. The sensor application is vibration monitoring on a new wearable device for Parkinson disease. Vibrations (which are AC accelerations) are expected in a specific frequency range, from 40 Hz to 400 Hz. The MEMS sensor has a mass of 8 nkg, a mechanical stiffness of 5 N/m and a quality factor $Q = 2$. The capacitive sensing is performed through 5 cells of differential parallel-plate capacitors, each one with 200 µm-long stators. The gap between rotor and stators is 2.5 µm and the process height is 15 µm. The device readout is performed through a charge amplifier configuration, as represented in figure 1. The capacitance $C_P = 10\,\text{pF}$ accounts for capacitive couplings between the rotor (including its pad and interconnections) and the grounded substrate.

1. Considering the softening given by parallel plates biased at $V_{dd} = \pm 1.8\,\text{V}$, calculate the value of the feedback capacitor C_F in order to obtain a sensitivity of $6\,\text{mV}/\hat{\text{g}}$.
2. Consider now the bias currents of the operational amplifier ($i_{bias} = 0.05\,\text{pA}$), and neglect the parasitic. Does this leakage affect the behavior of the stage? Modify the topology of the circuit in order to solve this issue and evaluate the residual offset in gravity units.

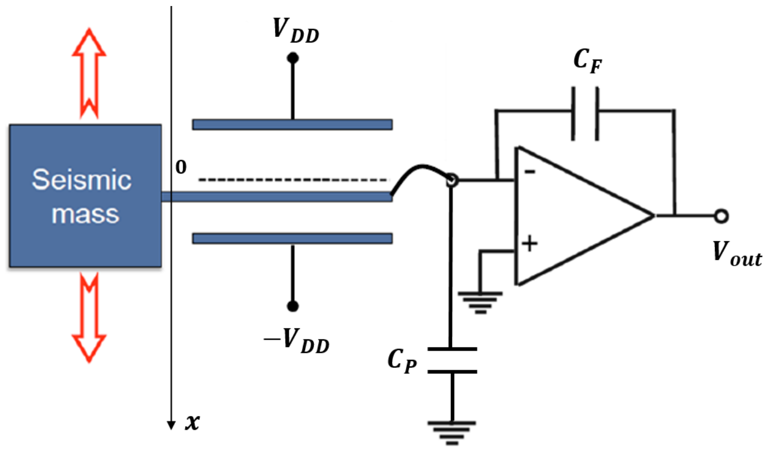

Figure 1: Schematic representation of the system.

3. Compare the device noise in terms of NEAD $\left[\frac{\hat{g}}{\sqrt{\mathrm{Hz}}}\right]$ and the front-end electronic noise in terms of $\left[\frac{\hat{g}}{\sqrt{\mathrm{Hz}}}\right]$. For the latter case evaluate three main contributions: the operational amplifier voltage noise $S_{v,n} = \left(10\,\frac{\mathrm{nV}}{\sqrt{\mathrm{Hz}}}\right)^2$, the operational amplifier current noise $S_{i,n} = \left(0.3\,\frac{\mathrm{fA}}{\sqrt{\mathrm{Hz}}}\right)^2$ and the resistance thermal noise, providing reasonable approximations for the frequency range of interest.

4. Is this kind of readout suitable for a measurement of the absolute inclination of the patient (standing or resting down)? If not, how can you modify the circuit to cope with this additional feature?

Question 1

From theoretical lectures, we know that the sensitivity of the system can be written as:

$$S = \frac{\Delta V}{\Delta a_{ext}} = \frac{\Delta x}{\Delta a_{ext}} \cdot \frac{\Delta C}{\Delta x} \cdot \frac{\Delta V}{\Delta C} =$$
$$= 2\frac{V_{DD}}{C_F} \cdot \frac{C_0}{g} \cdot \frac{1}{\omega_0^2}$$

When considering the expression above, just take care that:

- the resonance frequency has to be considered in operation, i.e. subject to the electrostatic softening effect. The total stiffness of the device, indeed, will be formed by the mechanical contribution, given in the data, and the electrostatic one, due to the presence of electrostatic forces arising from the voltage difference between rotor and stators. We know the formula of the electrostatic stiffness for a parallel plates configuration:

$$k_{elec} = -2V_{DD}^2 \frac{C_0}{g^2}$$

To find its value, we have to compute first the value of the rest capacitance of the accelerometer. This is easily obtained as:

$$C_0 = \frac{\epsilon_0 A}{g} = \frac{\epsilon_0 L_{PP} h N_{PP}}{g} = 53.1 fF$$

So, it results that $k_{elec} = -0.05\,\text{N/m}$. In this specific case the effect of electrostatic forces is low and the derived electrostatic stiffness results negligible (two orders of magnitude lower than the mechanical stiffness). Therefore, for the following calculations the total stiffness can be considered the same as the mechanical stiffness.

- the charge amplifier gain appearing in the expression of the sensitivity arises from an analysis of the overall current flowing through the feedback branch of the amplifier configuration of Fig. 1 (see also the theoretical lectures):

$$i_c = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

In our circuit topology, we have a DC voltage across the variable capacitance: since $dV/dt = 0$, only the second contribution to the current is relevant. Thus, we can calculate the transfer function from the capacitance variation ΔC to the voltage at the output of the charge amplifier stage using the circuital model of figure 2.

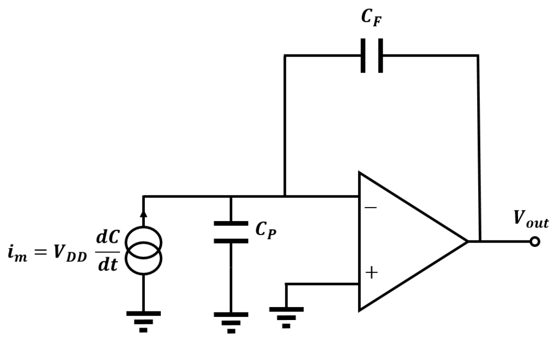

Figure 2: Circuital model for the calculation of the transfer function from capacitance variation to output voltage.

Using the properties of the Laplace transform, we can write the current as:

$$i(s) = sC(s)V_{DD}$$

This current, thanks to the negative feedback, flows through the virtual ground towards the feedback capacitance. The output voltage can be in turn written as:

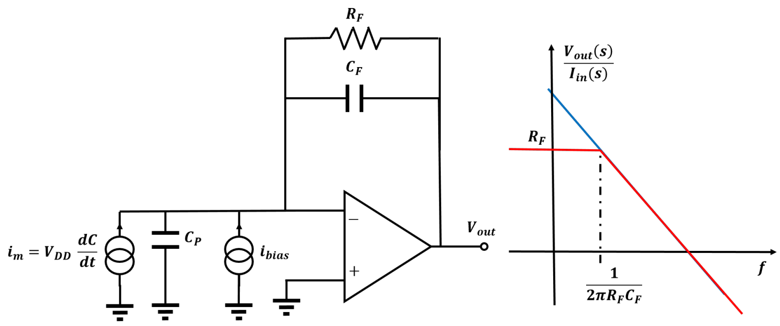

Figure 3: Circuit topology with feedback resistor, and Bode diagram of the transfer function between current and output voltage

$$V_{out} = -i(s)\frac{1}{sC_F} = -sC(s)V_{DD}\frac{1}{sC_F} \rightarrow \frac{\Delta V_{out}}{\Delta C}(s) = \frac{V_{DD}}{C_F}$$

So that one can draw the same conclusion that we obtained, through a different approach, in the lectures.

It is possible to find the value of C_F in order to match the required sensitivity. Just to get acquainted to numbers, we first calculate the *mechanical sensitivity*, i.e. the sensitivity of the sensor only, not including the contribution of the electronics:

$$S_{mech} = 2\frac{C_0}{g} \cdot \frac{1}{\omega_0^2} = 0.068\,\frac{\mathrm{fF}}{\mathrm{m/s^2}}$$

We can rewrite this sensitivity, as commonly done for accelerometers, in terms of gravity units $\hat{g}$. Exploiting the relation $1\hat{g} = 9.8\,\mathrm{m/s^2}$:

$$S_{mech,\hat{g}} = S_{mech} \cdot 9.8\,\mathrm{m/s^2} = 0.67\,\frac{\mathrm{fF}}{\hat{\mathrm{g}}}$$

So, the sizing of the feedback capacitance is readily obtained:

$$S_{\hat{g}} = S_{mech,\hat{g}} \cdot \frac{V_{DD}}{C_F} \rightarrow C_F = \frac{S_{mech,\hat{g}}}{S_{\hat{g}}} \cdot V_{DD} = 200\,\mathrm{fF}$$

Question 2

With the discussed circuit topology and neglecting the parasitic, a bias current would flow into the feedback capacitor resulting in an ramp-like output voltage in time (integration of the current). This, in turn, would unavoidably clip the output to saturation in about 7 s. Indeed, starting from the fundamental expression of the capacitor:

$$i = C\frac{dV}{dt} \rightarrow V_{out} = \frac{1}{C_F}\int i_{bias}dt = \frac{i_{bias}}{C_F} \cdot t$$

In such a situation, after a short interval from power-on, no accelerations can be readout as the output saturates. The solution consists in limiting the DC gain of the stage through a low-frequency pole introduced by a feedback resistor, as depicted in figure 3. The sizing of this resistor depends on the application. In this case, the sensor has to correctly measure frequencies from 40 Hz to 400 Hz, so we can fix the pole one decade before the lower limit, at $f_{pole} = 4\,\text{Hz}$. The required feedback resistance value becomes:

$$f_{pole} = \frac{1}{2\pi R_F C_F} \rightarrow R_F = \frac{1}{2\pi C_F f_{pole}} = 200\,\text{G}\Omega$$

Keep in mind that, as long as signals in the band of interest are concerned, we are still working beyond the pole of the Bode plot in fig. 3, where the signal is still integrated by the capacitive charge amplifier! Adopting this circuit correction, however, the bias current of the operational amplifier flows in DC through the resistor, simply introducing a DC offset of the output given by:

$$\Delta V_{out} = i_{bias} \cdot R_F = 10\,\text{mV} \rightarrow OS_{acc} = \frac{10\,\text{mV}}{6mV/\hat{g}} = 1.6\hat{g}$$

This DC offset is not problematic: indeed, when signals of interest are in the AC band (as in this example, 40 Hz to 400 Hz), an offset can be easily suppressed, e.g. by introducing a high-pass filter after the front-end output.

Question 3

The noise source from the sensor point of view is the Brownian contribution related to the random agitation of gas molecules. During the theoretical lectures we demonstrated that the effect of this fluctuation force turns into a white noise force power spectral density:

$$S_{Fn} = 4k_B T b \left[\frac{\text{N}^2}{\text{Hz}}\right]$$

The NEAD parameter, or Noise Equivalent Acceleration Density, represents the noise in terms of equivalent acceleration. Exploiting the transfer function between acceleration and force (F/a = m), it is possible to calculate the NEAD:

$$\text{NEAD} = \sqrt{S_{An}} = \sqrt{S_{Fn} \cdot \left(\frac{1}{m}\right)^2} = \sqrt{\frac{4k_B T \omega_0}{Qm}} = 160.9 \cdot 10^{-6}\ \frac{\text{m/s}^2}{\sqrt{\text{Hz}}}$$

To obtain the NEAD in terms of $\left[\frac{\hat{\text{g}}}{\sqrt{\text{Hz}}}\right]$:

$$\text{NEAD}_g = \frac{\text{NEAD}}{9.8} = 16.41 \frac{\mu\hat{\text{g}}}{\sqrt{\text{Hz}}}$$

In order to calculate the electronic noise we can exploit the small signal model of the system. As shown in figure 4, in this case the MEMS capacitance is in parallel with the

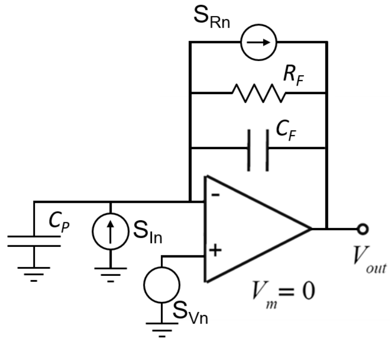

Figure 4: Small signal model for the calculation of the electronic noise contributions

parasitic capacitance, and this parallel is largely dominated by the parasitic capacitance $C_{MEMS} + C_P \approx C_P$.
Referring to figure 4, it is possible to start evaluating the three noise contributions:

- **Op-Amp Voltage Noise** $\rightarrow$ The voltage noise generator insists on the positive input of the amplifier, and it is thus brought to the output as a non-inverting gain:

$$S_{vn,out} = S_{vn} \cdot \left(1 + \frac{R_F // \frac{1}{sC_F}}{\frac{1}{sC_P}}\right)^2 = S_{vn} \cdot \left(1 + \frac{sC_P R_F}{1 + sC_F R_F}\right)^2 \left[\frac{\mathrm{V}^2}{\mathrm{Hz}}\right]$$

This transfer has a zero and a pole. In which point of the transfer function is the system working point located? We know that for working frequencies larger than $40Hz$, the approximation $\omega R_F C_F \gg 1$ is valid. Additionally, we know that at the output a high-pass filter can be placed, which will not only suppress offset but also low-frequency noise. Therefore, the expression can be simplified:

$$S_{vn,out} = S_{vn} \cdot \left(1 + \frac{\cancel{sR_F} C_P}{\cancel{sR_F} C_F}\right)^2 = S_{vn} \cdot \left(1 + \frac{C_P}{C_F}\right)^2 \left[\frac{\mathrm{V}^2}{\mathrm{Hz}}\right]$$

From this result we can note that (i) noise increases with large parasitics C_P: it is thus important to keep parasitics as low as possible. Additionally, (ii) noise increases with a decrease of C_F, but note that the signal-to-noise ratio and the input referred noise (in terms of acceleration) do not change, as the sensitivity as well increases by decreasing C_F. This is easily demonstrated: to obtain the noise value in terms of $\left[\frac{\hat{\mathrm{g}}}{\sqrt{\mathrm{Hz}}}\right]$ we only need to divide the obtained value by the sensitivity calculated above in terms of [V/ĝ]:

$$\sqrt{S_{\hat{g}vn}} = \frac{\sqrt{S_{vn,out}}}{S_g} = 85.07 \, \frac{\mu\hat{\mathrm{g}}}{\sqrt{\mathrm{Hz}}}$$

- **Op-Amp Current Noise** → The current noise directly flows into the parallel of the feedback resistance and capacitance, as in this situation the parasitic capacitance has both the plates connected to ground.

$$S_{in,out} = S_{in} \cdot \left(\frac{R_F}{1 + sC_F R_F} \right)^2 \left[\frac{\mathrm{V}^2}{\mathrm{Hz}} \right]$$

  The result is different with respect to voltage noise in that here noise is not constant along the frequency range of interest. After the pole, indeed, noise lowers as the frequency increases. For the sake of simplicity, we evaluate this contribution in the (geometric) mean point of our frequency range (i.e. for $f = 126\,\mathrm{Hz}$). The approximation $\omega R_F C_F \gg 1$ is clearly valid, so the expression can be simplified:

$$S_{in,out} = S_{in} \cdot \left(\frac{\cancel{R_F}}{sC_F \cancel{R_F}} \right)^2 = S_{in} \cdot \left(\frac{1}{sC_F} \right)^2 \left[\frac{\mathrm{V}^2}{\mathrm{Hz}} \right]$$

  We finally evaluate this noise contribution in terms of $\left[\frac{\hat{\mathrm{g}}}{\sqrt{\mathrm{Hz}}} \right]$:

$$\sqrt{S_{\hat{g}in}} = \frac{\sqrt{S_{in,out}}}{S_g} = 315\, \frac{\mu\hat{\mathrm{g}}}{\sqrt{\mathrm{Hz}}}$$

  In this frequency range, the amplifier current noise is larger than the voltage noise contribution.

- **Feedback Resistance Thermal Noise** → This contribution can be treated exactly as for the former case. The resistor current noise flows into the parallel of the feedback capacitance and resistance, generating a voltage noise at the output.

$$S_{rn,out} = \frac{4k_B T}{R_F} \cdot \left(\frac{R_F}{1 + sC_F R_F} \right)^2 \left[\frac{\mathrm{V}^2}{\mathrm{Hz}} \right]$$

  As before, for the frequency point of interest:

$$S_{rn,out} = \frac{4k_B T}{R_F} \cdot \left(\frac{\cancel{R_F}}{sC_F \cancel{R_F}} \right)^2 = \frac{4k_B T}{R_F} \cdot \left(\frac{1}{sC_F} \right)^2 \left[\frac{\mathrm{V}^2}{\mathrm{Hz}} \right]$$

$$\sqrt{S_{\hat{g}rn}} = \frac{\sqrt{S_{rn,out}}}{S_g} = \sqrt{\frac{4k_B T}{R_F}} \cdot \left(\frac{1}{\omega C_F} \right) \cdot \frac{1}{S_g} = 302\, \frac{\mu\hat{\mathrm{g}}}{\sqrt{\mathrm{Hz}}}$$

  This contribution is comparable to the current noise of the operational amplifier.

From the calculations it is clearly visible that the most important noise contributions for the system in this frequency range are current noise of the operational amplifier and feedback resistance thermal noise. This contributions overcome the intrinsic noise of the device, making the system limited by the electronic noise. Overall, we can write:

$$\sqrt{S_{\hat{g}n,tot}} = \sqrt{S_{\hat{g}vn} + S_{\hat{g}in} + S_{\hat{g}rn} + NEAD_g} = 444\,\frac{\mathrm{\mu\hat{g}}}{\sqrt{\mathrm{Hz}}}$$

Note that this result suggests lowering the device quality factor Q from its value (2) down to 0.5. This change, indeed, will worsen a little bit thermomechanical noise, which however will remain negligible compared to electronic noise. On the other side, this action will avoid ringing that may arise under shocks when using an under-damped Q.

Question 4

The accelerometer, combined with the designed electronics, is not suitable to measure DC accelerations like gravity. Though suitable for vibration monitoring (vibrations are AC accelerations), it cannot also measure whether the patient is standing or lying down. Indeed, the low-frequency pole introduced in order to avoid saturation of the charge amplifier stops the integration of signals with frequency lower than 4 Hz.
A solution that allows to readout also DC accelerations consists in a high-frequency modulation of the suspended mass, with each of the sensing stators kept to the virtual ground of a charge amplifier (see figure 5).

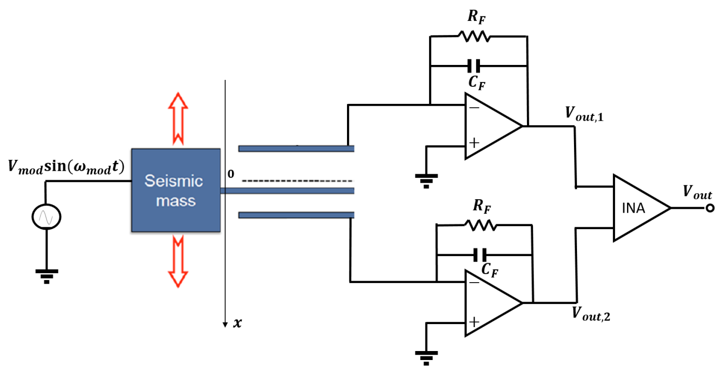

Figure 5: High-frequency capacitive readout circuit topology.

In the expression of the current flowing in each capacitor:

$$i_c = \frac{dQ}{dt} = \frac{d(CV)}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

the $C\frac{dV}{dt}$ term is no longer null. In the following, its relative weight with respect to the term $V\frac{dC}{dt}$ will be evaluated. We begin by assuming a generic sinusoidal capacitance variation at a frequency ω_a. This generic situation (any signal can be seen as the sum of sine components) will be later simplified when DC accelerations only are acting on the device:

$$C = C_0 + C_a \cdot \cos(\omega_a t)$$

Once again, note that ω_a here represents the frequency of the acceleration (and thus of the capacitance variation), while ω_{mod} is the frequency of the applied voltage sinewave at the rotor. We can thus write the current:

$$i_c = C\frac{dV}{dt} + V\frac{dC}{dt} = (C_0 + C_a \cos(\omega_a t))\omega_{mod} V_{mod} \cos(\omega_{mod} t) - C_a \omega_a V_{mod} \sin(\omega_a t) \sin(\omega_{mod} t)$$

We thus have two contributions, both modulated around ω_{mod}. Their amplitude ratio is roughly:

$$\frac{C\frac{dV}{dt}}{V\frac{dC}{dt}} \approx \frac{\omega_{mod}}{\omega_a}$$

So, considering that accelerometers typically measure signals up to a maximum acceleration frequency of few 100 Hz, using a $f_{mod} = 100\,\text{kHz}$ the $C\frac{dV}{dt}$ term is 1000 times larger than the $V\frac{dC}{dt}$ term, and will dominate in the sum.
Additionally, in the specific case of DC accelerations, $\omega_a = 0$ and only $C\frac{dV}{dt}$ remains, with no approximation. We can calculate the expression of the signals $V_{out,1}$ and $V_{out,2}$:

$$V_{out,1} = -\int \frac{C_0 + \Delta C}{C_F} V_{mod} \cdot \omega_{mod} \cos(\omega_{mod} t)\ \mathrm{d}t = -\frac{C_0 + \Delta C}{C_F} \cdot V_{mod} \sin(\omega_{mod} t)$$

$$V_{out,2} = -\int \frac{C_0 - \Delta C}{C_F} V_{mod} \cdot \omega_{mod} \cos(\omega_{mod} t)\ \mathrm{d}t = -\frac{C_0 - \Delta C}{C_F} \cdot V_{mod} \sin(\omega_{mod} t)$$

We thus conclude that a static acceleration signal produces a current, and in turn an output voltage for each stator, at a frequency f_{mod}. Note that f_{mod} should be much higher than the resonant frequency of the accelerometer, in order not to excite the device (the transfer function between force and displacement falls down at high frequencies). The chosen value of $100kHz$ copes with this requirement.
The voltages $V_{out,1}$ and $V_{out,2}$ of each readout channel are then subtracted, using an INA:

$$V_{out} = -2\Delta C \cdot \frac{V_{mod}}{C_F} \sin(\omega_{mod} t)$$

thus the output amplitude will be:

$$|V_{out}| = 2\Delta C \cdot \frac{V_{mod}}{C_F}$$

At the output of the chain, we have a sinusoidal signal whose amplitude is proportional to ΔC, from which we can recover information about the acceleration to be sensed. Consequently, we can easily obtain the sensitivity expression for this circuit configuration:

$$\frac{\Delta\,|V_{out}|}{\Delta a} = \frac{\Delta x}{\Delta a} \cdot \frac{\Delta C}{\Delta x} \cdot \frac{\Delta\,|V_{out}|}{\Delta C} = \frac{1}{\omega_0^2} \cdot \frac{C_0}{g} \cdot 2\frac{V_{mod}}{C_F}$$

It is also important to note that the modulation of the rotor voltage is useful to move the signal away from electronics flicker noise (neglected for the sake of simplicity in previous noise calculations) and from the DC offset (e.g. the offset given by opamp bias currents in question 2), as depicted in figure 6.

As discussed above, the capacitance variation is multiplied by a sinusoidal wave at f_{mod}. On the other hand, the flicker noise and the offset voltage are introduced by the operational amplifier, **after the modulation**, and so they remain at low frequencies. In other words, we are modulating the rotor voltage, that modulates the current but does not change the transfer function of the electronic noise to the output. Then, after demodulation and low-pass filtering, the signal is moved back to base-band and low-frequency noise is shifted at high frequency and then cut-off.

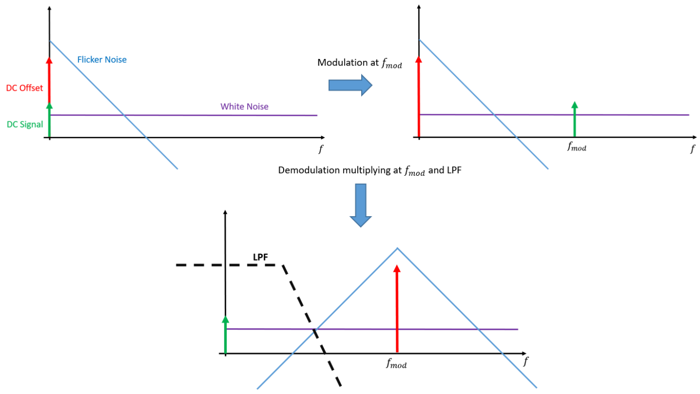

Figure 6: Effect of modulation and demodulation on signal, offset and noise.

So, the weight of noise contributions is completely changed (see figure 7): the resistor noise and the current noise of the amplifier are now totally negligible, and the contribution $S_{v,n}$ dominates: the overall noise floor is thus also reduced.

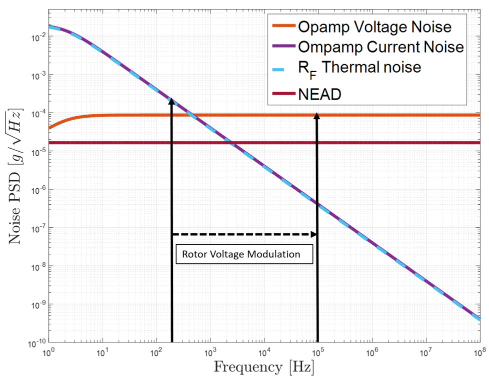

Figure 7: Noise contributions of the system. Note the differences between the two different operating frequencies.