Politecnico di Milano
MSc Course - MEMS and Microsensors - 2023/2024

E01
Design of a MEMS accelerometer

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Problem description and questions

You are a young MEMS designer and your supervisor asks you to redesign a consumer-grade accelerometer for a next-generation mobile phone, and to give some forecasts on the electro-mechanical performance of the sensor. The bounded geometrical parameters are reported in Table 1. Additionally, the accelerometer must ensure a full scale range $FSR = \pm 16\ \hat{g}$, and the parallel plates can be polarized at $V_{DD} = \pm 3\,\mathrm{V}$.

1. Given that the maximum acceptable linearity error, ϵ_{lin}, is equal to 1%, calculate the maximum capacitance variation.
2. Calculate the electromechanical sensitivity [fF/ĝ] and the resonance frequency during the operation of the accelerometer.
3. Evaluate the contribution of the parallel plates in terms of electrostatic stiffness and calculate the mechanical stiffness (bonus question: choose the geometry of the springs).
4. Calculate the needed quality factor, Q, to guarantee a NEAD $= 25\,\frac{\mu\hat{g}}{\sqrt{\mathrm{Hz}}}$.

	Symbol	Value
Young's modulus	E	150 GPa
process tickness	h	24 µm
seismic mass	m	4.5 nkg
maximum spring length	l_{max}	200 µm
minimum spring width	w_{min}	1.7 µm
# diff PP cells	N_{PP}	10
length of PP	l_{PP}	300 µm
gap of PP	g_{PP}	2 µm

Table 1: fixed parameters.

Introduction

A MEMS accelerometer is a microelectromechanical system used to reveal linear accelerations. MEMS devices are modeled with a spring-mass-damper system, in order to describe their static and dynamic behavior. The equation of motion of such a system can be written as follows:

$$m\ddot{x} + b\dot{x} + kx = F_{ext}.$$

Being $X(s)$ the Laplace transform of the relative position and $F_{ext}(s)$ the Laplace transform of the external force, the transfer function $T_{XF}(s)$ can be calculated as

$$T_{XF}(s) = \frac{X(s)}{F_{ext}(s)} = \frac{1/m}{\left(s^2 + \frac{b}{m}s + \frac{k}{m}\right)}.$$

Substituting $s = j\omega$ and defining the resonance frequency $\omega_0 = \sqrt{k/m}$ and quality factor $Q = \omega_0 m/b$, the transfer function is rewritten in the Fourier domain as

$$T_{XF}(j\omega) = \frac{X(j\omega)}{F_{ext}(j\omega)} = \frac{1/m}{\left(\omega_0^2 - \omega^2 + j\frac{\omega_0\omega}{Q}\right)}.$$

The relation between displacement and force can be simplified in a useful way considering three different frequency ranges:

- if $\omega << \omega_0$:

  $$|T_{XF}(j\omega)|_{\omega\ll\omega_0} = \frac{1/m}{\omega_0^2} = \frac{1}{k}$$

  which means that when a quasi-stationary force is applied to the seismic mass, the displacement is governed only by the stiffness constant and is in phase with the applied force;

- if $\omega = \omega_0$:

  $$|T_{XF}(j\omega)|_{\omega=\omega_0} = \frac{1/m}{\sqrt{\left(\frac{\omega_0^2}{Q}\right)^2}} = \frac{Q/m}{\omega_0^2} = \frac{Q}{k}$$

  which means that applying a force at the device resonance frequency, the displacement increases by a factor Q with respect to the quasi-stationary case. Additionally, if one looks at the phase a 90^o phase lag is found;

- if $\omega >> \omega_0$:

  $$|T_{XF}(j\omega)|_{\omega\gg\omega_0} = \frac{1/m}{\omega^2}$$

  which means that, beyond the resonance frequency, the displacement is proportional to the inverse of frequency. Accelerometers typically work in the first frequency range, thus with an input acceleration bandwidth smaller than ω_0. The reason is simply that accelerations usually occur at low frequency for most of the applications, and there is no easy way to modulate the inertial force $F_{acc} = m \cdot a$.

Question 1

The parallel plate configuration is common in accelerometers because of the higher sensitivity with respect to a comb finger configuration for the same area occupation. However, this configuration suffers from a geometrical non-linearity: this phenomenon is described by a percentage linearity error ϵ_{lin}. From the specifications, the acceptable value of this parameter turns out to be 1% and is defined as:

$$\epsilon_{lin} = \frac{\Delta C_{real,FSR} - \Delta C_{lin,FSR}}{\Delta C_{real,FSR}} \cdot 100$$

where $\Delta C_{real,FSR}$ is the real capacitance variation at the full scale range between the capacitance C_1 and C_2 (see Lecture n. 5):

$$\Delta C_{real} = C_2 - C_1 = 2C_0 \frac{x}{g} \frac{1}{1 - \left(\frac{x}{g}\right)^2}$$

where C_0 is the "rest" capacitance, i.e. with no displacement applied to the MEMS accelerometer.
On the other hand, using the small-displacement approximation, $x \ll g$, the linearized expression for ΔC_{lin} can be derived as:

$$\Delta C_{lin} = 2C_0 \frac{x}{g}.$$

Imposing $\epsilon_{lin} = 1\%$, one can fix the maximum acceptable full-scale displacement x_{FSR}, i.e. the displacement corresponding to the maximum acceleration that the accelerometer is designed for:

$$\epsilon_{lin} = \frac{C_0 \frac{2\frac{x}{g}}{1-\left(\frac{x}{g}\right)^2} - 2C_0 \frac{x}{g}}{C_0 \frac{2\frac{x}{g}}{1-\left(\frac{x}{g}\right)^2}} \cdot 100$$

$$(100 - \epsilon_{lin}) \left| \frac{1}{1 - \left(\frac{x}{g}\right)^2} \right|_{FSR} = 100 \quad \rightarrow \left(\frac{x_{FSR}}{g}\right)^2 = \frac{\epsilon_{lin}}{100}$$

$$x_{FSR} = g\sqrt{\frac{\epsilon_{lin}}{100}}.$$

Hence, the maximum displacement is governed by the linearity of parallel plates: in this case $x_{FSR} = 200\,\mathrm{nm}$ and the corresponding value of capacitance variation results $\Delta C_{real,FSR} = 64.36\,\mathrm{fF}$.

Question 2

You have seen during the classes (Lecture n. 4) that the sensitivity[1] of a parallel plates MEMS accelerometer, from input acceleration to differential capacitance variation, can be expressed as:

$$\frac{dC}{da} = 2\frac{dx}{da} \cdot \frac{dC}{dx} = 2\frac{1}{\omega_0^2} \cdot \frac{C_0}{g}$$

Anyway, in our particular situation, we do not have any information about the accelerometer resonant frequency. There is an alternative (and much simpler) way to find the sensitivity when we have information about full scale range and linearity error. Indeed, we already know the capacitance variation for an external acceleration $a_{FRS} = 16\hat{g}$, so the sensitivity is given by the following ratio:

$$\left.\frac{dC}{da}\right|_{FSR} = \frac{\Delta C_{real,FSR}}{FSR} = 4.1\,\mathrm{fF}/\hat{\mathrm{g}} = 0.4\,\mathrm{fF}/(\mathrm{m/s^2})$$

In order to evaluate the working resonance frequency, one can write the displacement in terms of applied force on the seismic mass through the transfer function $|T|$, remembering that the accelerometer works in the low-frequency range $\omega \ll \omega_0$,

$$X = F \cdot |T(\omega < \omega_0)| \rightarrow x = \frac{F}{k} \rightarrow x = \frac{m \cdot a}{k} \rightarrow x = \frac{a}{{\omega_0}^2} \rightarrow \omega_0 = \sqrt{\frac{a_{FSR}}{x_{FSR}}}$$

The obtained resonance frequency value is referred to the accelerometer in operation (the term k above is the overall stiffness, as detailed in the following point of the exercise). The reader is invited to note that the only data used so far are the linearity error, the gap and the value of the full scale range. The resonance frequency results $f_0 = 4456\,\mathrm{Hz}$, since $f_0 = \omega_0/2\pi$.

[1] note that the sensitivity can be given at various levels... from acceleration to either displacement, or capacitance, or output voltage... be sure to check what the exercise is asking for. The sensitivity from acceleration to output voltage takes also the name of scale-factor

Question 3

The considered sensor presents a capacitive readout based on a parallel-plate configuration. In order to evaluate the effect of electrostatic softening introduced by the electrostatic force of the parallel plates, one can rewrite the characteristic equation of the spring-mass-damper system:

$$m\ddot{x} + b\dot{x} + kx = F_{ext} \quad \rightarrow m\ddot{x} + b\dot{x} + kx = ma_{ext} + F_{elec}$$

Then, one can calculate the expression for the electrostatic force, F_{elec} in the case of differential parallel plates configuration.

$$F_{elec} = F_{elec,2} + F_{elec,1} = \frac{1}{2}\frac{\partial C_2}{\partial x}V_{DD}^2 + \frac{1}{2}\frac{\partial C_1}{\partial x}V_{DD}^2 = \frac{V_{DD}^2}{2}\left(\frac{\partial C_2}{\partial x} + \frac{\partial C_1}{\partial x}\right) =$$

$$= \frac{V_{DD}^2}{2}C_0\left[\frac{\frac{1}{g}}{\left(1-\frac{x}{g}\right)^2} - \frac{\frac{1}{g}}{\left(1+\frac{x}{g}\right)^2}\right]$$

Because of the small displacement approximation, $x \ll g$, the electrostatic force F_{elec} can be linearized and results in the following formula:

$$F_{elec} = \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{1}{1-2\frac{x}{g}+\cancel{\left(\frac{x}{g}\right)^2}} - \frac{1}{1+2\frac{x}{g}+\cancel{\left(\frac{x}{g}\right)^2}}\right] = \frac{V_{DD}^2}{2}\frac{C_0}{g}\frac{4\frac{x}{g}}{1-4\cancel{\left(\frac{x}{g}\right)^2}} = 2\frac{C_0}{g^2}V_{DD}^2 x$$

In this way, we can notice the linearized behavior of the electrostatic force with respect to the displacement performed by the seismic mass. Let us consider a stationary acceleration (i.e. at low frequency if compared to ω_0) applied to the sensor. In this case the general equation of motion becomes:

$$m\cancel{\ddot{x}} + b\cancel{\dot{x}} + kx = ma_{ext} + F_{elec} \rightarrow kx - F_{elec} = ma_{ext} \rightarrow x\left(k_{mech} + k_{elec}\right) = ma_{ext}$$

having defined

$$k_{elec} = -2\frac{C_0}{g^2}V_{DD}^2.$$

In our case $k_{elec} = -1.43\,\mathrm{N/m}$.
To obtain the value of the mechanical stiffness k_{mech}, we can use the definition of resonance frequency:

$$\omega_0 = \sqrt{\frac{k_{mech} + k_{elec}}{m}}$$

With these data, the required mechanical stiffness becomes $k_{mech} = 4.96\,\mathrm{N/m}$.

Now we can start with the design of the springs. MEMS sensors usually feature more than one spring. If the springs share the same 'starting-point' (different anchorages to the substrate are ideally the same point!) and the same 'end-point' (different ends connected to the same moving frame are ideally the same point!) then they are in a parallel configuration, sketched in Figure 1.

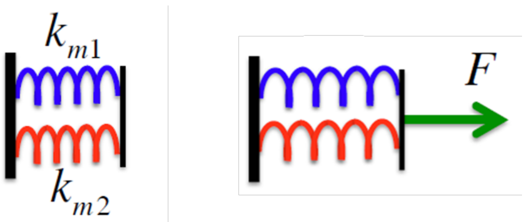

Figure 1: parallel configuration of springs.

In this case, the total force applied to the system, F_{tot}, can be expressed as the sum of the two elastic forces, F_1 and F_2, acting on each spring. Moreover the displacement of each spring is the same, thus the force balance can be written as:

$$F_{tot} = F_1 + F_2 = k_1 x + k_2 x = (k_1 + k_2)x = k_{tot} x$$

Hence we can generalize the total stiffness $k_{tot,par}$ of a system of n parallel springs as:

$$k_{tot,par} = \sum_{m=1}^{n} k_m = n \cdot k_m$$

where the latter expression holds if the n springs are identical.
Conversely, if the 'end-point' of a spring is the 'starting-point' of another one they are in a series configuration, as shown in Figure 2.

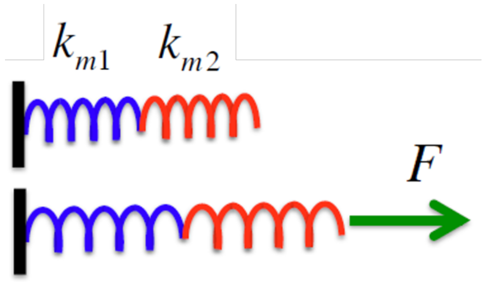

Figure 2: series configuration of springs.

In this case the total displacement of the system, x_{tot}, is obtained by the sum of the displacements of both springs, x_1 and x_2. Besides, due to the action-reaction principle, the force F applied to the system is equal to that applied to both springs. Thus the total displacement in a series configuration can be written as:

$$x_{tot} = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2} = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{F}{k_{tot}}$$

from which we derive a generalization of the total stiffness $k_{tot,ser}$ of a series configuration of n springs:

$$\frac{1}{k_{tot,ser}} = \sum_{m=1}^{n} \frac{1}{k_m} = \frac{n}{k_m}.$$

As a tip, one can remember that the overall stiffness of series/parallel of springs is computed like the overall capacitance of series/parallel of capacitors in an electrical network.

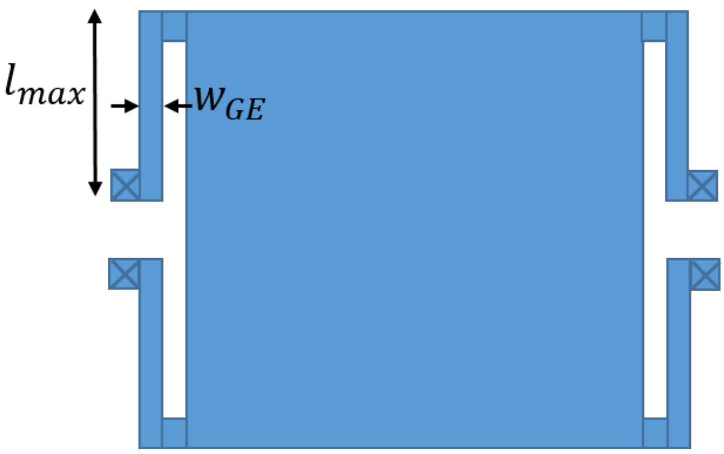

Figure 3: Four parallel guided-end springs.

To design the spring system of our accelerometer, one can start from the easiest approach:

1. to define a 1-D displacement, 4 guided-end springs are designed in parallel and placed at the four corners of the frame, $n_{spring} = 4$;
2. the simplest spring design has just a single fold per spring, and the stiffness of each spring is

$$k_{spring} = Eh\left(\frac{w}{l}\right)^3$$

In order to maximize the width of each spring, the maximum available length for the suspended springs is selected (see Table 1). In this way, we can obtain the maximum value $w_{max,GE}$, given by

$$k_{mech} \overset{\text{set}}{=} k_{GE} = n_{spring}k_{spring} = n_{spring}Eh\left(\frac{w_{max,GE}}{l_{max}}\right)^3$$

$$w_{max,GE} = l_{max}\sqrt[3]{\frac{k_{mech}}{n_{spring}Eh}}$$

obtaining $w_{max,GE} = 1.4\,\mu\text{m}$. This dimension is not allowed by the process rules of this technology. In order to widen the springs, one can use a folded spring topology.

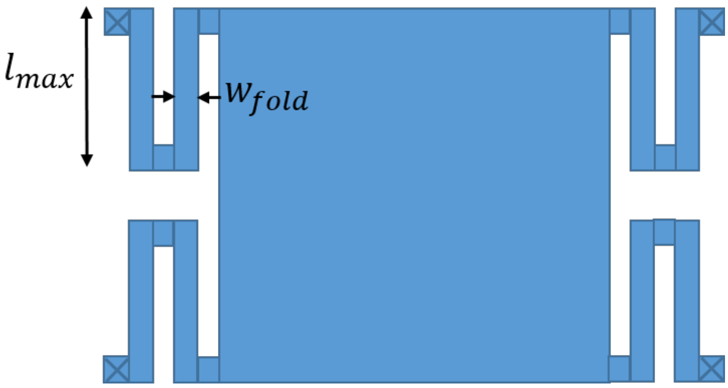

Figure 4: Four parallel folded springs (two folds per spring).

To 'fold a spring' means to put n_{fold} beams of the same length in series. Given the required elastic stiffness, we can write the relation between the width of springs in the 'folded' topology and the width of 'guided-end' springs as a function of the number of folds, as

$$k_{folded} \overset{\text{set}}{=} k_{GE}$$

$$k_{folded} = \frac{\cancel{n_{spring}}}{n_{fold}}\cancel{Eh}\left(\frac{w_{fold}}{\cancel{l_{max}}}\right)^3 \overset{\text{set}}{=} \cancel{n_{spring}}\,\cancel{Eh}\left(\frac{w_{GE}}{\cancel{l_{max}}}\right)^3 = k_{GE}$$

$$w_{fold} = w_{GE}\sqrt[3]{n_{fold}}$$

knowing that $w_{min} = 1.7\mu m$, we can obtain the minimum number of folds (an integer number) as:

$$w_{fold} = w_{GE}\sqrt[3]{n_{fold}} > w_{min} \rightarrow n_{fold} = \left(\frac{w_{min}}{w_{GE}}\right)^3 = 1.79$$

$$\Rightarrow n_{fold} = 2$$

then the corresponding value of the folded spring is $w_{fold} = 1.75\,\mu\text{m}$.

Question 4

The motion of an accelerometer, and in general of each MEMS sensor, is affected by a random fluctuation force, F_n. In most cases this force arises from the interaction of the inertial mass with the residual gas particles in the MEMS cavity and the fluctuation force spectrum, $S_{F,n}$, in units of [N^2/Hz] is given by (see Lecture n. 5):

$$S_{F,n} = 4k_B T b$$

The fluctuation force spectrum is independent of the frequency, thus it is a white noise.
Now, we can obtain the expression for the noise equivalent acceleration density, NEAD, through the transfer function that relates acceleration to force, T_{AF} (squared, because we are dealing with power spectral densities):

$$|T_{AF}|^2 = \frac{1}{m^2}$$

$$S_{A,n} = \frac{S_{F,n}}{m^2}$$

$$\text{NEAD} = \sqrt{S_{A,n}} = \sqrt{\frac{S_{F,n}}{m^2}} = \sqrt{\frac{4k_B T b}{m^2}} = \sqrt{\frac{4k_B T \omega_0}{mQ}}$$

$$Q = \frac{4k_B T \omega_0}{m\ \text{NEAD}^2}$$

To match the required input-referred acceleration noise density, it turns out that our quality factor should be $Q = 1.74$. This is a pretty nice value for an accelerometer, as this kind of device does not benefit from large Q values which, on the contrary, would generate undesired ringing under shocks.

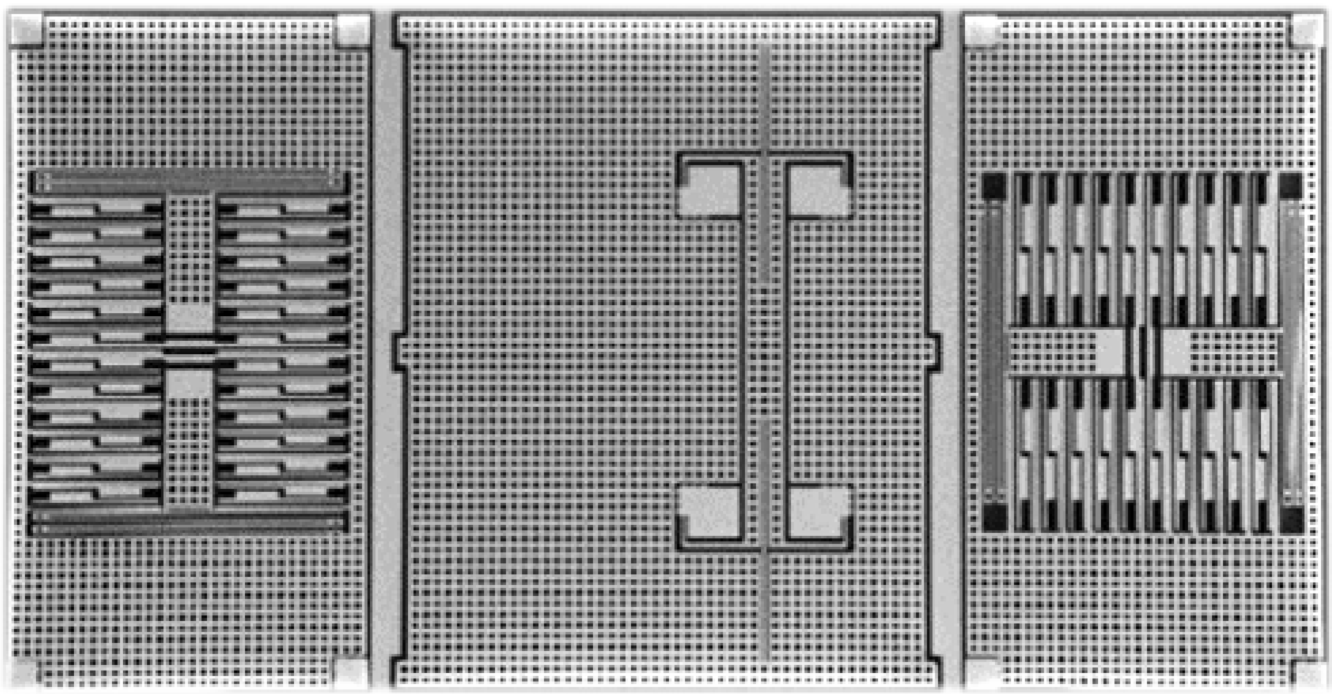

Figure 5: A 3-axis MEMS accelerometer from ST used on Huawei phones. The in-plane axes have parameters similar to those designed in this exercise.