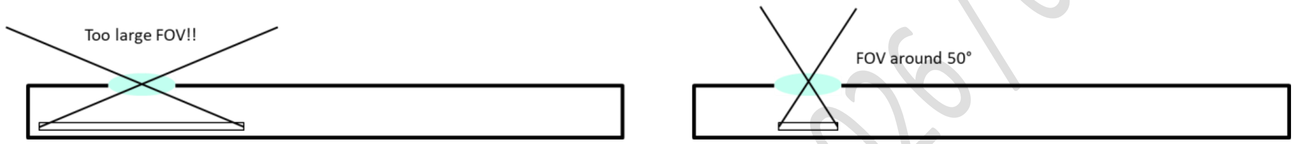
Last Name __DE MILANESE_ **Given Name** _OLIMPIA___ **ID Number** _20260218_

Question n. 1

Discuss the major differences in designing imaging sensors for smartphones or for semi-professional DSC cameras. To hep your discussion, consider differences in the optics, sensor, electronics and overall performances.

An imaging system usually tries to imitate what human vision does. As such, typical target parameters for a realistic "average" picture shall be a field of view in the range of 50°, a maximum dynamic range around 70-80 dB, the representation of color features, and few ten Mpixel resolution.

Mobile imaging has typically more constraints in terms of available area than a larger digital still camera. From the sketch below, one can easily recognize that the available distance between the lens and the sensor is usually few mm.



This yields two major challenges:

- one cannot design a large sensor area, otherwise the field of view would become enormous (though sometimes we desire wide-angle pictures, it is certainly not the typical case);
- the refractive power of the lens shall be very large, in order to get a short focal length. This implies the use of large curvature radii in the lenses, which are more prone to aberrations phenomena.

The following additional differences thus arise between mobile imaging and other digital still cameras:

- as the sensor size is smaller, for the same number of pixels also the pixel size will be smaller. Therefore, for the same imaging scenario the signal-to-noise ratio will be worse;
- additionally, given the lower pixel size, also the dynamic range will be worst in mobile imaging;
- even if the pixel size is smaller, resolution in mobile imaging will be worsened by the larger aberrations and larger diffraction (smaller aperture for the lens). It is thus often limited by the optics, rather than by the pixel size;
- it is difficult to integrate in mobile imaging systems several groups of lenses, to compensate for spherical aberrations with aspheric lenses and for chromatic aberrations with achromatic doublets. This further limits the resolution performance in mobile imaging;
- it is also difficult to implement variable focal length in mobile imaging, given the challenges in assembling the lenses in a small space.

Multichance students can skip point (iv) of question n. 2 and point (iii) of question n. 3.

The pixel-level electronics can be, overall, similar for the two systems, nowadays mostly relying on 4T topologies with pinned photodiodes. Sometimes, given the small pixel size required in mobile imaging, sharing some of the electronics among different pixels (with a 4T topology) is a strategy not to reduce too much the fill factor.

To partly relieve the issues arising from the hardware, mobile imaging leverages the use of multiple cameras, software combination of multiple images, and other algorithms, e.g. to boost the dynamic range through multiple captures, or to adjust on average the focal length.

For what concerns color capture, there is no conceptual difference, and both types of system rely on RGB filters, mostly based on the Bayer pattern. Some advanced semi-professional digital still cameras can adopt technologies based on layered junctions, to avoid color filters and boost the SNR: these are instead not suitable for mobile imaging due to their larger minimum pixel size.

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Question n. 2

An Asian mobile phone vendor requires the specifications in the table aside for the next generation of parallel-plate accelerometers.

Your available process features a 40-µm thick structural layer with 1.5-µm minimum gaps. The release distance is 10 µm. The MEMS design area in the module shall be limited to (400 µm)^2.

(i) identify the target resonance frequency (in Hz) and overall sensitivity (in mV/g);

(ii) sketch the accelerometer structure, and reasonably estimate the mass, mechanical stiffness, and number of parallel plates you can fit in the area;

(iii) with the parameters from the points above optimize your feedback capacitance to match the target sensitivity;

(iv) optimize the quality factor and amplifier noise to match the output noise density specification.

Parameter	Symbol	Condition	Typ	Units
Acceleration Range	g_{FS2g}	Selectable via serial digital interface	±2	g
	g_{FS4g}		±4	g
	g_{FS8g}		±8	g
	g_{FS16g}		±16	g
Supply Voltage Internal Domains	V_{DD}		0–2.4	V
Total Supply Current in Normal Mode	I_{DD}	T_A=25°C, ODR_{max}, V_{DD} = V_{DDIO} = 2.4V	130	µA
Total Supply Current in Suspend Mode	I_{DDsum}	T_A=25°C V_{DD} = V_{DDIO} = 2.4V	2.1	µA
Operating Temperature	T_A			°C
Output Noise Density	n_{rms}	g_{FS16g}, T_A=25°C Nominal V_{DD} supplies Normal mode	120	µg/√Hz
Zero-g Offset	Off	g_{FS2g}, T_A=25°C, nominal V_{DD} supplies, over life-time	±50	mg
Zero-g Offset Temperature Drift	TCO	g_{FS2g}, Nominal V_{DD} supplies	±1.0	mg/K
Bandwidth	bw_8	2nd order filter, bandwidth programmable	8	Hz
	bw_{16}		16	Hz
	bw_{31}		31	Hz
	bw_{63}		63	Hz
	bw_{125}		125	Hz
	bw_{250}		250	Hz
Nonlinearity	NL	best fit straight line, g_{FS16g}	±0.5	%FS

Physical Constants

k_b = 1.38 10^{-23} J/K;
ε_0 = 8.85 10^{-12} F/m;
ρ = 2350 kg/m^3;
T = 300 K;

(i)

We begin by observing that we have a specification for nonlinearity given at 16-g, which we need to fulfill. Given the minimum gap, it is easy to find the displacement for the FSR acceleration as:

$$\epsilon_{\%} = \left(\frac{x}{g}\right)^2 100 = 0.5\% \ \rightarrow x = 106\ nm$$

$$x = \frac{FSR}{\omega_0^2} \rightarrow f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{FSR}{x}} = 6.1\ kHz$$

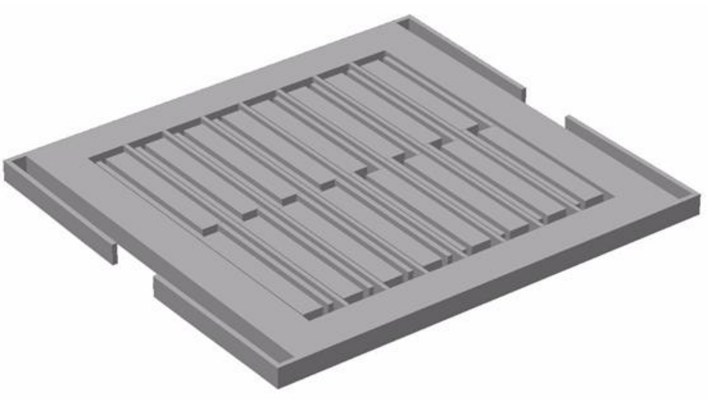
Assuming that electrostatic softening is negligible (we will verify this later), we can assume that this is also our mechanical natural resonance frequency.

The overall sensitivity is also easily found as a ratio between the supply (notice the single supply voltage, to be divided by 2) and target full-scale:

$$SF = \frac{\pm V_{DD}}{\pm\ FSR} = \frac{\pm 1.2V}{\pm\ 16\ g} = 75\frac{mV}{g} = 7.5\frac{mV}{m/s^2}$$

Multichance students can skip point (iv) of question n. 2 and point (iii) of question n. 3.

(ii)



Consider the conventional accelerometer structure aside. Leaving some space for the springs, we assume the overall outer size of the frame to be (400 µm x 380 µm). We also assume that ½ of the mass is cut by holes to accommodate parallel plates, thus we find an estimated mass value of:

$$m = \frac{1}{2}(380 \cdot 400 \cdot 40)10^{-18} m^3 \rho = 7\, nkg$$

If the electrostatic stiffness is negligible, the mechanical stiffness can be estimated as:

$$k_{mech} = \omega_0^2 \cdot m = 5.3 \frac{N}{m}$$

Within the 380-µm dimension, we leave about 25 µm per side for the frame. Given the release distance of 10 µm, we need to design stators for the parallel plates which are at least >20 µm wide (twice the release distance). We choose 30 µm, taking some margin. With 1.5 µm gaps, each differential parallel-plate cell takes twice the plate size and three times the gap size (towards each side of the rotor, and between the stators themselves):

$$L_{hole} = 30\mu m + 30\mu m + 1.5\mu m + 1.5\mu m + 1.5\mu m = 64.5\, \mu m$$

Within the available space, we can thus fit the following number of cells:

$$N_{PP} = \frac{380\, \mu m - 2 \cdot 25\mu m}{L_{hole}} = 5.11 \rightarrow N_{PP} = 5$$

For the length of the parallel plate, we assume again to subtract 25 µm per side from the full mass dimension (400 µm). We can now check the electrostatic stiffness:

$$k_{elec} = 2 \frac{\epsilon_0 L_{PP} h N_{PP}}{g} \frac{V_{DD}^2}{g^2} = 2 \frac{8.85\ 10^{-12} \frac{F}{m}\ 350\mu m\ 40\mu m\ 5}{(1.5\ \mu m)^3} (1.2\ V)^2 = 0.53 \frac{N}{m}$$

We can thus see that effectively the electrostatic stiffness is negligible.

(iii)

The expression of the scale factor (or sensitivity) of a parallel-plate accelerometer is:

$$SF_{m/s^2} = 2 \frac{C_0}{g} \frac{V_{DD}}{C_F} \frac{1}{\omega_0^2}$$

The only missing parameter is the feedback capacitance, which we thus optimize as (warning: use the SF considering V/(m/s^2) and not V/g):

$$C_F = 2 \frac{C_0}{g} \frac{V_{DD}}{SF_{m/s^2}} \frac{1}{\omega_0^2} = 60\, fF$$

If the value of such a capacitance cannot be integrated in the used electronic technology, one shall put an amplification in the following INA stage to fill the voltage dynamics.

(iv)

We assume half of the noise power to be given by the MEMS, and half by the electronics (in particular, we consider only the amplifier voltage noise).

Thus, the NEAD shall cope with:

$$NEAD = \frac{\sqrt{4k_B T b}}{m} = \sqrt{\frac{4k_B T \omega_0}{Qm}} \frac{1}{9.81 \frac{m}{s^2}/g} = \frac{120 \frac{\mu g}{\sqrt{Hz}}}{\sqrt{2}} \rightarrow Q = \frac{\frac{4k_B T \omega_0}{m}}{(9.81)^2} \frac{1}{\left(84 \frac{\mu g}{\sqrt{Hz}}\right)^2} = 0.13$$

This is a rather low value for the Q factor, but with such a large resonance (6 kHz) and much smaller bandwidth (250 Hz), we can accept it.

The electronic noise can be finally written as:

$$\sqrt{S_{a_{Vn}}} = \frac{\sqrt{2S_{Vn}}\left(1 + \frac{C_P}{C_F}\right)}{SF} = 84 \frac{\mu g}{\sqrt{Hz}} \rightarrow \sqrt{S_{Vn}} = 26 \frac{nV}{\sqrt{Hz}}$$

Where the value is found assuming a 10-pF parasitic capacitance.

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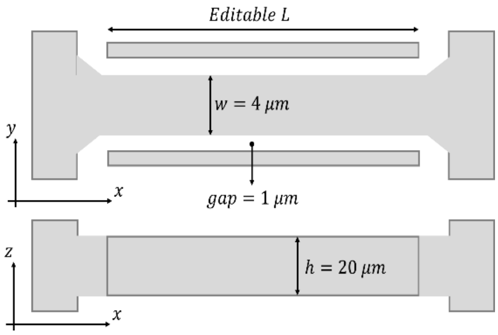
Question n. 3

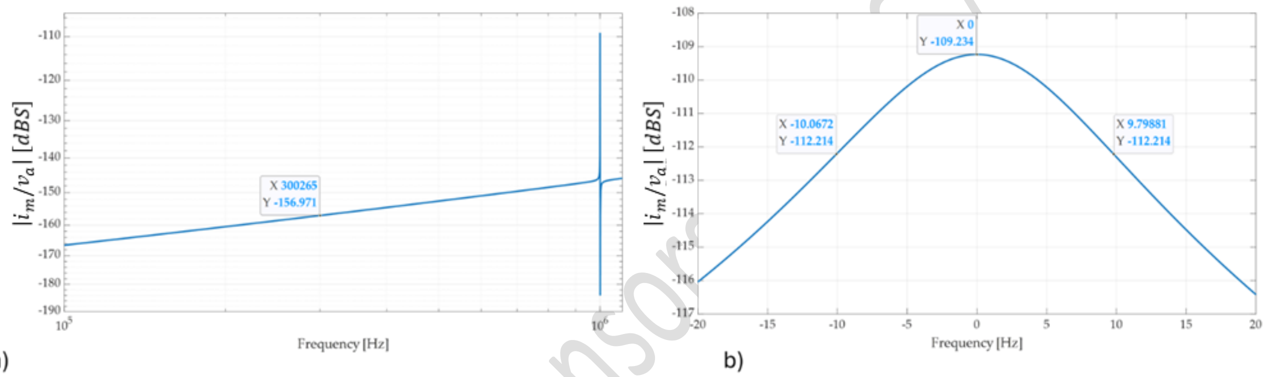


A MEMS oscillator at 1 MHz is required as a clock to synchronize different electronic components within an IMU. You are asked to design the MEMS resonator and the sustaining electronics.

The foundry gives you a model for a clamped-clamped beam resonator, as shown aside, where the design and dimensions are reported (x and y are the in plane coordinates, z is the out of plane one). One parallel plate is used to actuate the beam, the other one to sense the beam motion. You are asked to:

- **(i)** identify the correct beam length L to obtain an oscillation frequency of 1 MHz (hint: consider the clamped-clamped beam as the *parallel of two guided-end beams*, neglecting electrostatic softening);

After fabrication you test the voltage-to-current transfer function, applying a rotor bias of 5 V. The measured admittance modulus is reported below (a), including a zoom centred around the resonance frequency (b):



a) b)

- **(ii)** draw the equivalent LRC model (help yourself with the provided numerical markers);

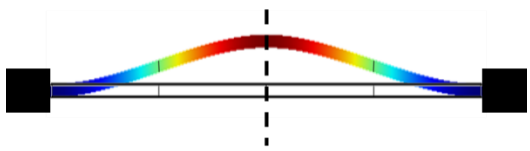
The integrated circuit technology can accommodate minimum capacitances of 20 fF.

- **(iii)** design and size the main parameters of an oscillator circuit, to sustain the resonator motion, using a trans-resistance amplifier front-end. No AGC is required. Assume a $\pm 1\ V$ supply voltage and target a nominal displacement of 0.25 μm at 300 K.

Physical Constants

k_b = 1.38 10^{-23} J/K
E = 160 GPa
T = 300 K (if not specified)
ρ = 2350 kg/m^3;
ε_0 = 8.85 10^{-12} F/m

(i)



As mentioned by the hint, and as sketched aside, the deformation of a clamped-clamped beam can be seen as the deformation of two guided-end beams connected in parallel (each of the two halves lies between the central point of the beam and an anchor to the substrate). Given the stiffness of a guided-end beam, the stiffness of the clamped-clamped beam can be thus easily written as a function of its dimensions:

$$k_{guided,\frac{L}{2}} = Eh\frac{w^3}{(L/2)^3} \rightarrow k_{clamped-clamped} = 2Eh\frac{w^3}{(L/2)^3} = 16Eh\frac{w^3}{L^3}$$

Multichance students can skip point (iv) of question n. 2 and point (iii) of question n. 3.

For what concerns the calculations of the effective mass (the mass that effectively takes part in the motion), we see that the ends of the beam are not moving, while the center is moving by the maximum amount. So, on average, half of the mass is moving. Therefore

$$m = \frac{1}{2}\,\rho L w h$$

Finally, the resonance frequency can be written as:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{16Eh\frac{w^3}{L^3}}{\frac{1}{2}\,\rho L w h}} = \frac{1}{2\pi}\sqrt{\frac{32E\frac{w^2}{L^3}}{\rho L}} = \frac{4w}{2\pi L^2}\sqrt{\frac{2E}{\rho}}$$

Setting it to 1 MHz, we find the only unknown, which is the beam length:

$$L = \sqrt{\frac{4w}{f_0 2\pi}\sqrt{\frac{2E}{\rho}}} = 172.4\ \mu m$$

(ii)

From the graphs we easily identify the quality factor from the peak width markers as:

$$\Delta f_{-3dB} = 10\ Hz \quad Q = \frac{f_0}{2\Delta f} = 50000$$

From the previous calculations, we can also write the mass and stiffness as:

$$m = \frac{1}{2}\,\rho L w h = 0.016\ nkg$$

$$k_{clamped-clamped} = 16Eh\frac{w^3}{L^3} = 639.7\,\frac{N}{m}$$

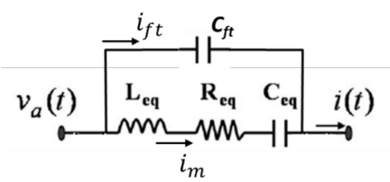
We additionally calculate the transduction factor for both drive and sense ports, being:

$$\eta = V_{rot}\frac{dC}{dx} = V_{rot}\frac{C_0}{g} = V_{rot}\frac{\epsilon_0 h L}{g^2} = V_{rot}\frac{C_0}{g} = \frac{5\ V\ 30fF}{1\ \mu m} = 152.5\,\frac{nN}{V}$$

And finally evaluate the equivalent electrical parameters:

$$L_{eq} = \frac{m}{\eta^2} = 696\ H \qquad R_{eq} = \frac{b}{\eta^2} = 86.9\ k\Omega \qquad C_{eq} = \frac{\eta^2}{k} = 36\ aF$$

It is additionally evident a feedthrough capacitance, which we can estimate from the marker at 300265 Hz:



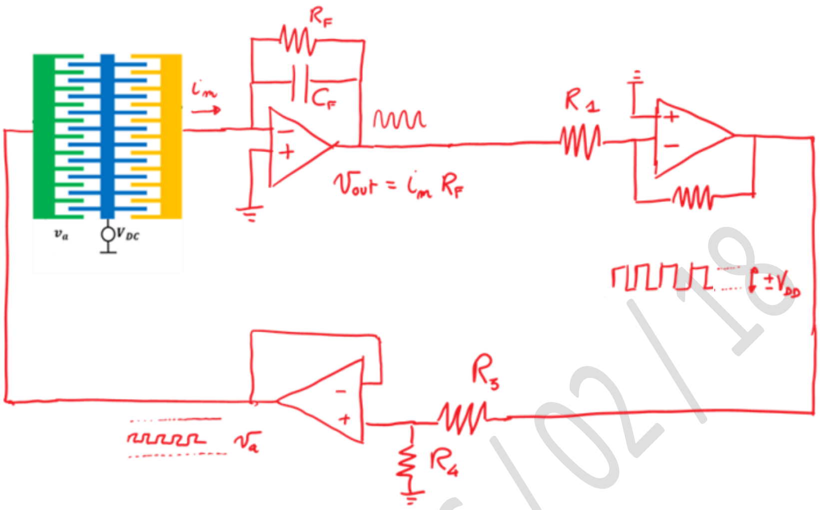
$$|sC_{FT}| = 2\pi f_x C_{FT} = -156.97\ dB \rightarrow \ C_{FT} = \frac{10^{\left(-\frac{156.97}{20}\right)}}{2\pi\ 300265Hz} = 7.5\ fF$$

The model is sketched in the figure aside.

NOTE: you could also calculate R_{eq} from the peak of the transfer function, bypassing the Q calculation. Results may be slightly different because of the graphs resolution, but all solutions will be accepted as valid.

(iii)

A circuit based on a transimpedance amplifier is shown in the figure below. A first stage, featuring a feedback capacitance and resistance, operates in such a way that the pole falls at least one decade after the resonance, which means at least 10 MHz in this case. Additionally, a second stage may be required if the resistive gain of the first stage is not enough to compensate the losses.



Let us see the values required to cope with the position of the pole, using a minimum capacitance:

$$f_{pole} = \frac{1}{2\pi C_{min} R_F} = 10\ MHz$$

$$R_F = 795\ k\Omega$$

This value is, in principle, high enough to compensate the resistive loss of 86.9 $k\Omega$. However, there is also a constraint on the maximum driving voltage and thus we need to insert a de-gaining voltage divider.

To find the actuation voltage, we consider the target drive motion:

$$x_d = \frac{F_d Q}{k} = \frac{\eta v_d Q}{k} = \frac{\eta \frac{4}{\pi} v_{sq} Q}{k} \quad \rightarrow \quad v_{sq} = \frac{x_d k}{\eta \frac{4}{\pi} Q} = 16.4\ mV$$

As the supply is set to 1 V, we need a de-gain divider by

$$G_{degain} = \frac{16.4\ mV}{1\ V} = 0.0164$$

Without the intermediate gain stage, the loop gain at resonance would thus be

$$|G_{loop}(\omega_0)| = \frac{R_F}{R_{eq}} G_{degain} = 0.15$$

Which is lower than one, and will not let the oscillation begin and be sustained. We therefore need effectively also the intermediate gain stage, with a gain at least equal to 1/0.15 = 6.67. We choose e.g. a gain by a factor 30, so that we get:

$$|G_{loop}(\omega_0)| = \frac{R_F}{R_{eq}} G_2\ G_{degain} = 4.5 = 13\ dB$$