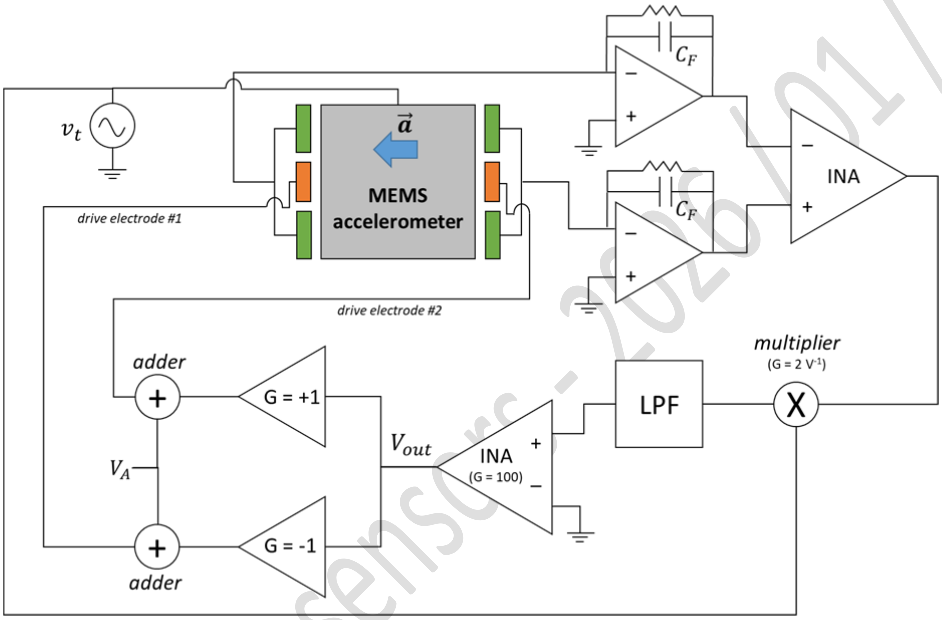
Last Name __Sameod_ **Given Name** ___Iole____ **ID Number** __20260120__

Question n. 1

Describe the idea behind the force-feedback operation of a MEMS accelerometer and the required additional elements on the MEMS. Draw a possible circuit to implement this force-feedback scheme. Derive the expression of the sensitivity (scale-factor) in this mode of operation. Finally, comment on advantages and drawbacks of this negative feedback mechanism.

The concept of "force-feedback" implies the application of a feedback force as a reaction to the rotor motion induced by the acceleration. In turn, the accelerometer is ideally kept in its central position and we measure the acceleration through the voltage required to keep it still.
With respect to a conventional implementation, we will need thus two additional electrodes to be used for the application of the feedback force.
A possible circuit implementation is shown below:



We see that part of the electronics (the rotor modulation voltage, the pair of charge amplifiers, the INA and the demodulation stage with multiplier and low-pass filter) is common to the classic circuit based on rotor modulation.
But the signal at the LPF output is now compared to a "0" reference. The difference, which is our traditional output and thus indicates the motion direction and amplitude of the rotor, is amplified and used to apply a force which – with the proper size so to implement a negative feedback – keeps the device in its rest position (the one corresponding to "0" output).
As the applied force needs to balance the one caused by the external acceleration, the derivation of the scale-factor is simply done with a comparison between these two forces:

$$F_{elec} = \left[\frac{(V_A + V_{out})^2}{2} - \frac{(V_A - V_{out})^2}{2}\right]\frac{C_{0d}}{g} = \frac{2V_A V_{out} C_{0d}}{x_0} = m\, a_{ext}$$

Which yields our scale-factor as:

$$\frac{V_{out}}{a_{ext}} = \frac{m\, g}{2V_A C_{0d}} = \frac{k\, g}{\omega_0^2 2V_A C_{0d}}$$

We thus observe that, like in all feedback-based systems, the transfer is not a function of the forward path, but only of the feedback branch. This gives us some advantages:

- The scale factor remains independent of parameters on the forward path, and their variability;
- The closed-loop bandwidth is not set by the MEMS only, but by the loop transfer;

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- The closed-loop transfer does not show peaks even for Q>>1. So, noise can be reduced!
- Linearity is increased (the system is ideally kept still), and thus vibration rectification is also reduced!

The drawbacks can be summarized as:

- There is a need for more electronic blocks, so there will be a little higher consumption.
- Like in all negative feedback systems, the stability is not guaranteed and shall be studied, giving additional design complexity.

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Question n. 2

You are given a MEMS yaw gyroscope, widely used in the consumer market. Based on a tuning-fork architecture, it exploits differential charge amplifiers, both for the sense and the drive mode. The drive motion is controlled by an AGC loop. The device and circuit parameters are given in the table:

- **(i)** calculate the scale-factor and full-scale range of the device in the conditions reported in the table (account for the sense tuning);
- **(ii)** calculate the input-referred noise density in [dps/√Hz];
- **(iii)** your boss decides to use the same device for high-end applications. He asks you to re-define the circuit or operational parameters, so to use the device with a rotor voltage of 10 V without changing the drive displacement and scale factor;
- **(iv)** discuss with your boss the advantages and drawbacks of this solution.

Electro-mechanical parameters for one half of the structure		
Mechanical drive frequency	f_d	$23\ kHz$
Drive mode mass	m_d	$40\ nkg$
Drive quality factor	Q_d	4000
Mechanical sense frequency	f_s	$24\ kHz$
Sense mode mass	m_s	$38\ nkg$
Sense quality factor	Q_s	600
Process thickness	h	$40\ \mu m$
Gap of parallel plates	g_{PP}	$1\ \mu m$
N. of parallel plates (single-ended)	N_{PP}	20
Parallel plate length	L_{PP}	$50\ \mu m$
Gap of comb fingers	g_{CF}	$1.5\ \mu m$
N. of drive comb fingers (single-ended)	N_{CF}	60
N. of drive-detection fingers (single-ended)	N_{CF}	60
Comb finger overlap	L_{OL}	$6\ \mu m$
Operational parameters		
Rotor voltage	V_{rot}	$5\ V$
AGC reference voltage	V_{ref}	$1.5\ V$
System supply voltage	$V_{ss} \div V_{dd}$	$\pm 3.3\ V$
Circuit parameters		
Parasitic charge-amplifier capacitance	C_p	$12\ pF$
Drive feedback capacitance	$C_{f,d}$	$1\ pF$
Drive feedback resistance	$R_{f,d}$	$1\ G\Omega$
INA gain for drive loop	$G_{INA,d}$	2
Sense feedback capacitance	$C_{f,s}$	$0.5\ pF$
Sense feedback resistance	$R_{f,s}$	$1\ G\Omega$
INA gain for sense chain	$G_{INA,s}$	4
Amplifier input voltage noise	$v_{n,CA}$	$4\ nV/\sqrt{Hz}$
Amplifier input current noise	$v_{i,CA}$	$1\ fA/\sqrt{Hz}$
INA input voltage noise	$v_{n,INA}$	$2\ nV/\sqrt{Hz}$

Physical Constants
$k_b = 1.38\ 10^{-23}$ J/K;
$\varepsilon_0 = 8.85\ 10^{-12}$ F/m;
T = 300 K;

(i)

We begin the solution by calculating the transduction for the drive detection. Here note that we have plenty of «factors 2»: one for the two-sided comb fingers, one for the two halves of the structure, and one for the differential sensing of the drive mode. We thus write:

$$\eta_{dd} = V_{rot}\frac{2\epsilon_0 h N_{cf}}{g_{cf}} \cdot 2 \cdot 2 \cdot G_{inad} = 1.1\ 10^{-6}\ \frac{A}{m/s}$$

The drive displacement can be at this point quantified, as we have the AGC circuit, as:

$$x_d = \frac{V_{ref} C_{fd} \frac{\pi}{2}}{\eta_{dd}} = 2\ \mu m$$

We are then asked to verify the tuning of the sense mode, to correctly predict the mismatch value:

$$C_{0s} = \frac{\epsilon_0 h N_{pp} L_{pp}}{g} = 708\ fF \ \rightarrow\ k_{el} = -\frac{2C_{0s}}{g_{pp}^2} V_{rot}^2 = -35.4\frac{N}{m}$$

And the sense resonance is therefore down-tuned to:

$$k_{mech,s} = (2\pi f_{0s})^2 m_s = 864\frac{N}{m} \ \rightarrow\ k_{tot,s} = k_{mech,s} + k_{el} = 828.7\frac{N}{m}$$

$$f_{s,tun} = \frac{1}{2\pi}\sqrt{\frac{k_{tot,s}}{m_s}} = 23.503\ kHz$$

We now have all the parameters for a correct calculation of the scale factor, which yields (the last factor converts rad/s into dps):

$$SF = 2\frac{C_{0s}}{g_{pp}}\frac{V_{rot}}{C_{fs}}\frac{x_d}{2\pi\Delta f}\ G_{inas}\frac{\pi}{180°} = 650\frac{\mu V}{dps}$$

Given the voltage supply, we immediately get also the FSR in dps:

$$FSR = \pm\frac{V_{dd}}{SF} = \pm 5075\ dps$$

(ii)

We begin from thermomechanical noise, for which we need the damping coefficient, calculated as:

$$b_s = 2\pi f_{s,tun}\frac{m_s}{Q_s} = 9.35 \cdot 10^{-6}\frac{kg}{s}$$

Given the tuning fork architecture, the expression of the NERD has a root-of-2 factor at the denominator, and thus we get (with the conversion from international system into dps):

$$\sigma_{\Omega_{\text{therm}}} = \frac{1}{x_d 2\pi f_d m_s}\sqrt{\frac{k_b T b_s}{2}}\frac{180°}{\pi} = 0.7\frac{mdps}{\sqrt{Hz}}$$

As a second noise source, we go through the voltage noise density of the two charge amplifiers and get:

$$\sigma_{\Omega_{\text{vn}}} = \frac{\sqrt{2v_{n,CA}^2\left(1+\frac{C_p}{C_{fs}}\right)^2}\,G_{inas}}{SF} = 0.43\ \frac{mdps}{\sqrt{Hz}}$$

We do the same procedure for the current noise sources, which are the amplifiers current noise and the feedback resistors noise:

$$\sigma_{\Omega_{\text{in}}} = \frac{\sqrt{2i_{n,CA}^2\left(\frac{1}{2\pi f_d C_{fs}}\right)^2}\ G_{inas}}{SF} = 0.12\ \frac{mdps}{\sqrt{Hz}}$$

$$\sigma_{\Omega_{\text{Rf}}} = \frac{\sqrt{2\left(\frac{4k_b T}{R_f}\right)\left(\frac{1}{2\pi f_d C_{fs}}\right)^2}\ G_{inas}}{SF} = 0.49\ \frac{mdps}{\sqrt{Hz}}$$

The total noise is the quadratic sum of all noise sources, and reads:

$$\sigma_\Omega = \sqrt{\sigma_{\Omega_{\text{therm}}}^2 + \sigma_{\Omega_{\text{vn}}}^2 + \sigma_{\Omega_{\text{in}}}^2 + \sigma_{\Omega_{\text{Rf}}}^2} = 1\frac{mdps}{\sqrt{Hz}}$$

(iii)

To hold the same drive motion when increasing the rotor, we need to change the AGC reference voltage, as the transduction indeed changes to twice the former value:

$$\eta_{dd_{10V}} = 10V\frac{2\epsilon_0 h N_{cf}}{g_{cf}} \cdot 2 \cdot 2 \cdot G_{inad} = 2.2\ 10^{-6}\frac{A}{m/s}$$

Thus, the AGC reference should also double to:

$$V_{ref_{new}} = \eta_{dd_{10V}}\frac{x_d}{C_{fd}}\frac{2}{\pi} = 3\ V$$

Now that the drive motion is kept constant, to hold the sensitivity constant we need to first check the new resonance frequency of the sense mode, and then we need to adjust one parameter, e.g. the sense feedback or the sense INA gain. The newly tuned sense frequency becomes, with the same procedure as above:

$$k_{el_{10V}} = -141.6\frac{N}{m} \rightarrow k_{tot,s_{10V}} = 722.5\frac{N}{m} \rightarrow f_{s,tun_{10V}} = 21.946\ kHz$$

Looking at the scale factor expression:

$$SF = 2\frac{C_{0s}}{g_{pp}}\frac{V_{rot}}{C_{fs}}\frac{x_d}{2\pi\Delta f}\ G_{inas}\frac{\pi}{180°}$$

we note that the rotor value is doubled, the Δf value has instead increased from about 500 Hz to about 1 kHz, so approximately doubled as well. Thus, we do not need to change anything in the sense chain, to preserve the same scale-factor.

(iv)

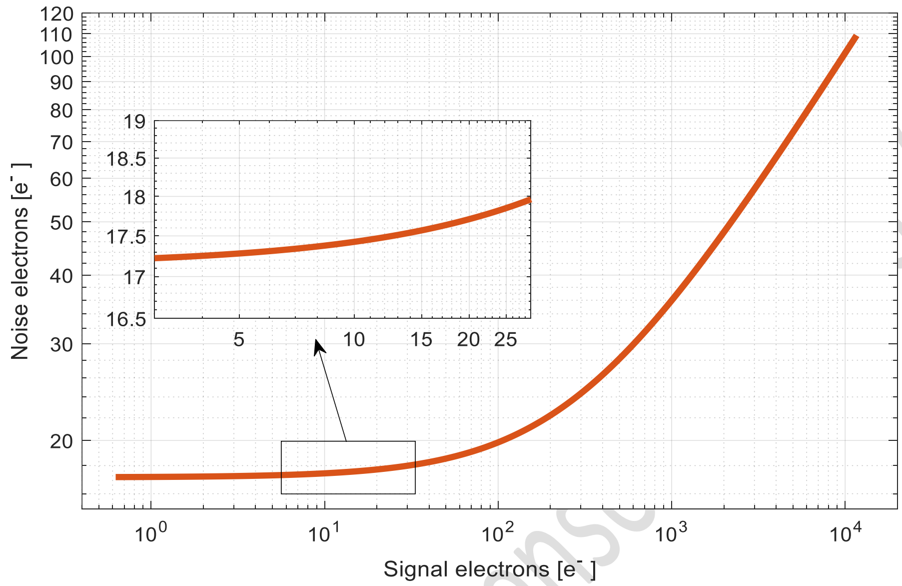
We keep the same scale-factor but now we have a mode-split value which is twice larger, so we can afford twice the maximum bandwidth, at the cost of an increase in the rotor voltage (a little higher consumption). Noise remains substantially unchanged.

Question n. 3

An old-generation 3T CMOS image sensor features the parameters listed in the table aside.

- **(i)** find the maximum number of electrons which can be accommodated in the photodiode;
- **(ii)** discuss if the claimed maximum dynamic range is in line with the calculation you can derive from the table data;
- **(iii)** you are also given a PTC for the old-generation device. Could you estimate the integration time at which it was captured and the true maximum voltage swing of the pixel output?

Pixel area	(3 µm)^2
Fill factor	35 %
Micro-lenses	No
Dark current density	0.05 fA/ (µm)^2
Depletion layer width	1.5 µm
Gate capacitance	0.7 fF
N. of bits	9
Supply voltage	3.6 V
Range of integration times	0.2 ms – 5 s
Maximum dynamic range	62 dB



Physical Constants
$q = 1.6\ 10^{-19}$ C
$k_b = 1.38\ 10^{-23}$ J/K
T = 300 K (if not specified)
$\varepsilon_0 = 8.85\ 10^{-12}$ F/m
$\varepsilon_{Si} = 11.7\ \varepsilon_0$

The next sensor generation is targeting an improvement in the dynamic range by 2 dB. Your colleague suggests to add micro-lenses, reduce the dark current by a factor 10, or increase by 1 bit the ADC resolution.

- **(iv)** verify if any of the proposed solutions is effective.

(i)

We begin by calculating the pixel area and photodiode area as:

$$A_{pix} = l_{pix}^2 = (9 \cdot 10^{-12})m^2 \rightarrow A_{pd} = FF \cdot A_{pix} = (3.15 \cdot 10^{-12})m^2$$

From which we can calculate the depletion capacitance and thus the total integration capacitance:

$$C_{dep} = \frac{\epsilon_{Si} A_{pd}}{x_{dep}} = 8.85 \cdot 10^{-12} \frac{F}{m} 11.7 \frac{A_{pd}}{1.5\ \mu m} = 0.22\ fF \rightarrow C_{int} = C_g + C_{dep} = 0.92\ fF$$

And the total number of electrons we can ideally accommodate in the photodiode well is:

$$N_{el_{max}} = \frac{C_{int} V_{dd}}{q} = 20643\ e^-$$

(ii)

To calculate the DR from the table we check all the noise sources, in electrons, and then find the DR as we already have the maximum number of electrons.

Note that the maximum DR will occur at the shortest integration time, so we use that value for the calculations below:

$$\sigma_{Nel,dark} = \frac{\sqrt{q\ i_d\ t_{int}}}{q} = \frac{\sqrt{q\ J_d A_{pd}\ t_{int}}}{q} = 0.4\ e^-$$

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$$\sigma_{Nel,reset} = \frac{\sqrt{k_B\ T\ C_{int}}}{q} = 12.2\ e^-$$

$$\sigma_{Nel,quant} = \frac{\frac{V_{dd}}{2^{N_{bit}}} \frac{1}{\sqrt{12}}\ C_{int}}{q} = 11.6\ e^-$$

So that the found DR (quadratic sum of noise components) becomes:

$$DR = 20\log_{10}\frac{N_{el_{max}}}{N_{el_{min}}} = 20\log_{10}\frac{N_{el_{max}}}{\sqrt{\sigma^2_{Nel,dark} + \sigma^2_{Nel,reset} + \sigma^2_{Nel,quant}}} = 20\log_{10}\frac{20643}{16.8}$$

$$= 61.8\ dB$$

Which is in line with the claims by the manufacturer.

(iii)

From the PTC, we observe two main differences with respect to our ideal calculations:

- first, the plateau for signal-independent noise is found at 17.2 electrons, which is not in line with the theoretical calculations which would give 16.5 electrons. This means that the dark induced noise is larger and we can estimate the integration time from

$$\sigma_{Nel,dark} = \sqrt{17.2^2 - \sigma^2_{Nel,reset} - \sigma^2_{Nel,quant}} = 3.5\ e^- \rightarrow t_{int} = \frac{\sigma^2_{Nel,dark}\ q^2}{q\ J_d A_{pd}} = 12\ ms$$

- second, the maximum charge is not 20643 electrons, but a little less, in the range of 13000, approximately. This is about one half of the value, meaning that the effective voltage is:

$$V_{dd,eff} = \frac{13000 \cdot q}{C_{int}} = 2.35\ V$$

Indicating a loss (e.g. due to overdrive voltages of MOS, or linearity limits) by about 1.25 V.

(iv)

The dark current noise is irrelevant for the maximum DR (see the numbers above), so reducing it is not a solution. Likewise, the presence or absence of micro-lenses has no impact on the maximum DR (they impact on the SNR, but it is not our target). Instead, quantization noise is rather relevant in the DR calculation above, and reducing it by 1 bit (a factor 2) is effective, as we now get

$$\sigma_{Nel,quant} = \frac{\frac{V_{dd}}{2^{10}} \frac{1}{\sqrt{12}}\ C_{int}}{q} = 5.8\ e^- \rightarrow DR_{new} = 63.7\ dB$$

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