Last Name __Vinacosi___ **Given Name** _Ciro___ **ID Number** _20250120__

Question n. 1

Discuss the origin of quadrature error in gyroscopes. Then explain how ideally such error can be completely nulled at the gyroscope output. Next, explain the reasons why this does not occur and the impact this has on the gyroscope output. Finally, describe all the possible compensation techniques that come to your mind.

The quadrature error is a mechanism by which the sense frames of the gyroscope displace during drive motion, at the resonance frequency of the drive mode, even in absence of any angular rate stimuli. The displacement is, however, in phase with the drive position and not with the drive velocity, so it is in quadrature with respect to the Coriolis force action, from which the name derives.

The origin of such a phenomenon may lie in asymmetricities in the fabricated device, which can be in turn due to (i) design imperfections (e.g. non-symmetric design of the buried interconnections, generating small asymmetricities in the resulting structure), or to (ii) fabrication imperfections (e.g. non-uniform etching of the different springs, non-uniform etching of comb fingers, or skew angle issues deriving from wafer bending during deep reactive ion etching). All the mentioned effects may result in a force component orthogonal to the drive direction (so, in the sensing direction) during drive motion, $|F_q| = k_{ds}\, x$.

As this motion component is in quadrature with respect to the Coriolis generated one, in principle one can apply a synchronous demodulation to filter out the quadrature component. In equations this becomes, before and after demodulation respectively:

$$\Delta V_{out} = 2\,\frac{V_{DC}}{C_{FS}}\frac{C_S}{g}\frac{x_{D,0}}{\Delta\omega_{MS}}\left[\boldsymbol{\Omega}\cos(\omega_D\, t) + \boldsymbol{B_q}\sin(\omega_D\, t)\right] = S\left[\boldsymbol{\Omega}\cos(\omega_D\, t) + \boldsymbol{B_q}\sin(\omega_D\, t)\right]$$

$$V_{dem} = S\left[\Omega\cos(\omega_D t) + B_q\sin(\omega_D t)\right]\cos(\omega_D t) * LPF = G_{LPF}\frac{S}{2}\{\Omega\cos(0) + \mathrm{B_q}\sin(0)\}$$
$$= 2\,\frac{V_{DC}}{C_{FS}}\frac{C_S}{g}\frac{x_{D,0}\Omega}{\Delta\omega_{MS}}$$

Showing that quadrature is completely bypassed. Unfortunately, in presence of demodulation phase errors, cancellation is not complete. Note that such error may arise either in the electronic domain (delays along the drive loop for the demodulation reference, delays along the sense chain for the modulated output) or in the mechanical domain (delays caused by the sense mode transfer function). In such a situation the result of the demodulation operation becomes:

$$V_{dem} = S\left[\Omega\cos(\omega_D t) + B_q\sin(\omega_D t)\right]\cos(\omega_D t + \varphi_{err} + \varphi_n) * LPF \approx$$

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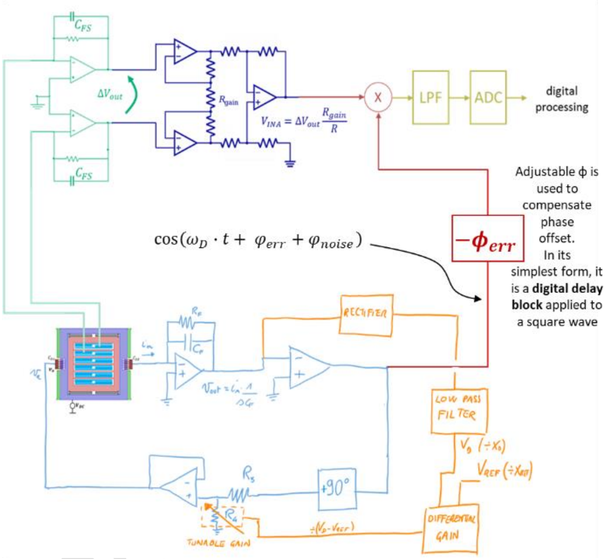
$$\approx S[\Omega\cos(\varphi_{err}+\varphi_n)+B_q\sin(\varphi_{err}+\varphi_n)] = S[\Omega + B_q\cdot\varphi_{err}+B_q\cdot\varphi_n] =$$
$$= S\cdot\Omega\ + S\cdot B_q\cdot\varphi_{err} + S\cdot B_q\cdot\varphi_n$$

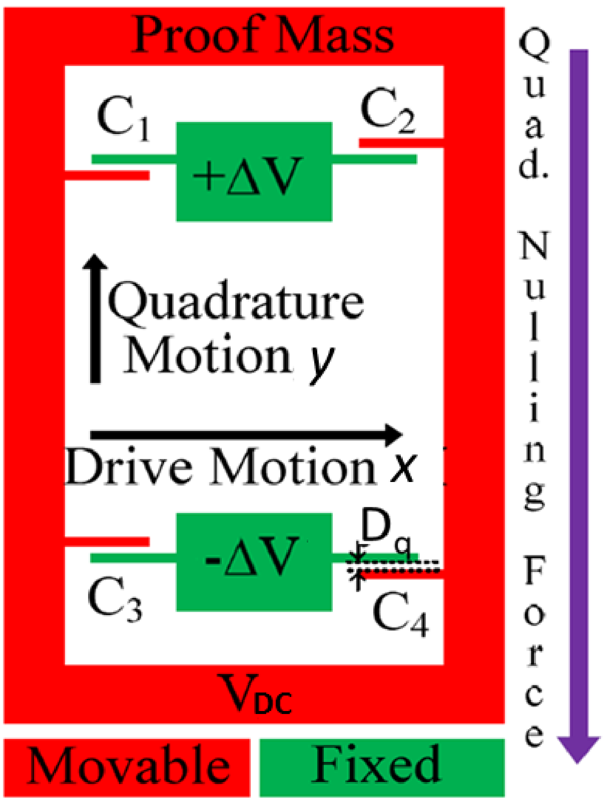
Showing that a noise term and an offset term appear at the output, together with the desired signal. Offset is particularly critical in case of drift of the phase error with temperature, as this would induce an output change not associated to an angular rate.



Different compensation techniques can be conceived:

- Including a phase trimming stage within the path of the reference demodulation signal allows a compensation of the phase error and thus a nulling of the quadrature effects at the output (see the image aside);
- Injecting a signal equal and opposite to the quadrature (not discussed in the lectures, but somewhat mimicking what you studied for feedthrough cancellation);
- Compensating the error at the origin by inducing a force which can be
  - Trimmed in amplitude
  - With a selectable sign
  - Proportional to the drive displacement

  Which can be obtained by including in the Coriolis frame of the gyroscope an architecture of additional electrodes, designed and biased as shown in the image aside.



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Question n. 2

A tuning-fork yaw MEMS gyroscope is operated in mode-split conditions, exploiting a drive-loop with amplitude gain control (AGC) and differential charge-amplifier readout.

Parameters given for half structure		
Voltage supply range	V_{DD}	$0 - 3.3$ V
Rotor voltage	V_{rot}	10 V
Mode-split frequency	Δf_{MS}	500 Hz
Sense resonance frequency	f_s	20 kHz
Sense quality factor	Q	50
Sense mass	m_s	5 nkg
Drive detection comb-finger capacitance (single-ended)	$C_{0,d}$	300 fF
Drive detection comb overlap	L_{ov}	16 μm
Sense parallel-plate capacitance (single-ended)	$C_{0,s}$	500 fF
Sense parallel-plate gap	g	1.8 μm
Drive CA capacitance	$C_{F,d}$	1 pF
Sense CA capacitance	$C_{F,s}$	250 fF
Drive and Sense CA capacitance temperature coefficient	TCC_F	35 ppm/K
Parasitic capacitance	C_p	5 pF
Opamp voltage noise density	S_v	$(15\ \mathrm{nV}/\sqrt{\mathrm{Hz}})^2$

(i) set the value of the AGC reference V_{ref} to achieve a scale factor of 1.2 mV/dps, and compute the full-scale range of the sensor in dps;

(ii) compute the relative variation of the gyroscope's sensitivity $\Delta S/S$ induced by temperature variations affecting the drive and sense CA feedback capacitances;

(iii) evaluate the thermo-mechanical and electronic noise contribution to the input-referred noise power spectral density;

(iv) comment on how to obtain a well-balanced sensor in terms of noise performance.

Physical Constants
$k_b = 1.38 \cdot 10^{-23}$ J/K;
T = 300 K.

(i) The scale factor S of a tuning-fork gyroscope has the following expression:

$$S = 2 \cdot \frac{V_{\mathrm{DC}}}{C_{F,s}} \cdot \frac{2C_{0,s}}{g} \cdot \frac{x_d}{\Delta\omega_{MS}}$$

Note that the voltage supply is unipolar ($V_{\mathrm{DD}} = 0 - 3.3$ V), therefore the stators and the CA outputs must be biased at midrange ($V_{\mathrm{DD}}/2 = 1.65$ V) to allow the gyroscope to deal with both positive and negative input rates; as a result, the DC voltage across the sense capacitance is $V_{\mathrm{DC}} = V_{\mathrm{rot}} - \frac{V_{\mathrm{DD}}}{2} = 8.35$ V.

To set the scale factor to the desired value, we shall set the drive displacement to:

$$x_d = \frac{S}{2 \cdot \frac{V_{\mathrm{rot}} - \frac{V_{\mathrm{DD}}}{2}}{C_{F,s}} \cdot \frac{2C_{0,s}}{g} \cdot \frac{1}{\Delta\omega_{MS}}} = \frac{1.2\ \mathrm{mV/dps}}{2 \cdot \frac{10\ \mathrm{V} - 1.65\ \mathrm{V}}{250\ \mathrm{fF}} \cdot \frac{2 \cdot 500\ \mathrm{fF}}{1.8\ \mu\mathrm{m}} \cdot \frac{1}{2\pi \cdot 500\ \mathrm{rad/s}}} = 5.8\ \mu\mathrm{m}$$

We can now compute the required AGC reference voltage V_{ref}:

$$V_{\mathrm{ref}} = \frac{2}{\pi} \cdot 2 \cdot \frac{V_{\mathrm{rot}} - \frac{V_{dd}}{2}}{C_{F,d}} \cdot \frac{2C_{0,d}}{L_{\mathrm{ov}}} \cdot x_d = \frac{2}{\pi} \cdot 2 \cdot \frac{10\ \mathrm{V} - 1.65\ \mathrm{V}}{1\ \mathrm{pF}} \cdot \frac{2 \cdot 300\ \mathrm{fF}}{16\ \mu\mathrm{m}} \cdot 5.8\ \mu\mathrm{m} = 2.3\ \mathrm{V}$$

The full-scale range of the sensor is set by the maximum allowed voltage swing at the output of the sense charge-amplifiers:

$$S = \frac{\Delta V_{\mathrm{out}}}{\Delta\Omega} = \frac{\frac{V_{\mathrm{DD}}}{2}}{\Omega_{\mathrm{FSR}}} = 1.2\,\frac{\mathrm{mV}}{\mathrm{dps}} \quad \Rightarrow \quad \Omega_{\mathrm{FSR}} = \pm 1375\ \mathrm{dps}$$

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(ii) In order to evaluate the impact of temperature-induced CA feedback capacitance variations, let us write the complete expression of the scale factor:

$$S = 2 \cdot \frac{V_{\mathrm{rot}} - \frac{V_{\mathrm{DD}}}{2}}{C_{F,s}} \cdot \frac{2C_{0,s}}{g} \cdot \frac{1}{\Delta\omega_{MS}} \cdot \frac{V_{\mathrm{ref}}}{\frac{2}{\pi} \cdot 2 \cdot \frac{V_{\mathrm{rot}} - \frac{V_{dd}}{2}}{C_{F,d}} \cdot \frac{2C_{0,d}}{L_{\mathrm{ov}}}} = \frac{C_{F,d}}{C_{F,s}} \cdot \frac{C_{0,s}}{g} \cdot \frac{L_{\mathrm{ov}}}{C_{0,d}} \cdot \frac{V_{\mathrm{ref}}}{\Delta\omega_{MS}} \cdot \frac{\pi}{2}$$

This way, the dependency on the drive and sense CA feedback capacitances is highlighted. We can compute the relative variation of the scale factor:

$$\frac{\Delta S}{S} = -\frac{\Delta \mathrm{C_{F,s}}}{C_{F,s}} + -\frac{\Delta \mathrm{C_{F,d}}}{C_{F,d}} = -\mathrm{TC}C_{F,s} \cdot \Delta T + \mathrm{TC}C_{F,d} \cdot \Delta T = 0$$

Since the drive and sense CA capacitance have the same temperature coefficient, the

(iii) The thermo-mechanical noise contribution to the input-referred noise can be computed as:

$$\mathrm{NERD}^2 = 2 \cdot \frac{k_{\mathrm{B}} T}{m_s \omega_s Q_s} \cdot \frac{1}{x_d^2} = 2 \cdot \frac{1.38 \cdot 10^{-23}\ \mathrm{J/K} \cdot 300\ \mathrm{K}}{5\ \mathrm{nkg} \cdot 2\pi \cdot 20\ \mathrm{krad/s} \cdot 50} \cdot \frac{1}{(5.8\ \mathrm{\mu m})^2} = \left(5.06\ \frac{\mathrm{mdps}}{\sqrt{\mathrm{Hz}}}\right)^2$$

The electronic contribution is due to the voltage noise of the sense charge-amplifiers used for differential readout:

$$S_{\Omega,\mathrm{eln}} = \frac{2 \cdot S_v \left(1 + \frac{C_p + 2C_{0,s}}{C_{F,s}}\right)^2}{S^2} = \frac{2 \cdot \left(15\ \mathrm{nV}/\sqrt{\mathrm{Hz}}\right)^2 \left(1 + \frac{5\ \mathrm{pF} + 2 \cdot 500\ \mathrm{fF}}{250\ \mathrm{fF}}\right)^2}{(1.2\ \mathrm{mV/dps})^2} = \left(0.44\ \frac{\mathrm{mdps}}{\sqrt{\mathrm{Hz}}}\right)^2$$

(iv) The thermo-mechanical contribution to the input-referred noise can be made equal to the electronic contribution by increasing the Q-factor of the sense mode (i.e. by reducing the capping pressure of the device).

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Question n. 3

A 12-Mpixel smartphone camera is used to take a frontal picture of a red painting ($\lambda_{\text{red}} = 650\ \text{nm}$):

- **(i)** compute the distance required to fit the whole painting into the picture. What is setting the limit to the sensor's resolution?
- **(ii)** compute the maximum photocurrent generated in a red pixel;
- **(iii)** compute the SNR for an integration time of 10 ms;
- **(iv)** For the same integration time, compute the DR.

Power illuminating the painting	P_{scn}	3 W
Painting reflectance (isotropic)	R_{scn}	25%
Painting width	W_{scn}	2.4 m
Painting height	H_{scn}	1.5 m
Lens diameter	D_{lens}	4 mm
Lens aperture	$F\#$	4
Sensor width	W_{sens}	10 mm
Sensor height	H_{sens}	7.5 mm
Fill factor	FF	0.46
Red micro-lens transmittance	T_{red}	0.8
Quantum efficiency	η	0.65
Dark current density	J_d	$50\ \text{aA}/\mu\text{m}^2$
Depletion region depth	x_d	1 µm
Pixel supply voltage	V_{dd}	2 V

Physical Constants

ε_{si} = 8.85 10^{-12} F/m *11.7;
h = 6.62 10^{-34} J/Hz ;
k_b = 1.38 10^{-23} J/K;
c = 3 10^{8} m/s ;
q = 1.6 10^{-19} C;
T = 300 K.

(i) Let us fit the width of the scene into the width of the sensor. The resulting magnification factor m is:

$$m = \frac{W_{\text{sens}}}{W_{\text{scn}}} = \frac{10\ \text{mm}}{2.4\ \text{m}} = 0.0042$$

Assuming the distance between the lens and the sensor to be much smaller than the distance d_{scn} between lens and painting, we can compute the latter through the magnification factor:

$$d_{\text{scn}} = \frac{f}{m} = \frac{D_{\text{lens}} \cdot F\#}{m} = \frac{16\ \text{mm}}{0.0042} = 3.84\ \text{m}$$

In order to determine the limit to the sensor's resolution, we shall evaluate the size of the Airy diffraction disc corresponding to the given lens aperture and compare it with the size of a pixel:

$$d_{\text{Airy}} = 2.44 \cdot \lambda \cdot F\# = 6.3\ \mu\text{m}$$

$$l_{pix} = \sqrt{\frac{W_{\text{sens}} \cdot H_{\text{sens}}}{N_{\text{pix}}}} = \sqrt{\frac{10\ \text{mm} \cdot 7.5\ \text{mm}}{12 \cdot 10^6}} = 2.5\ \mu\text{m}$$

Since $d_{\text{Airy}} > l_{\text{pix}}$, we can conclude that the resolution is limited by diffraction.

(ii)

In order to evaluate the photocurrent, we need to compute the optical power impinging on a single pixel. First, we have to compute the area of the scene corresponding to one pixel:

$$A_{\text{pix,scn}} = \frac{A_{\text{pix}}}{m^2} = \frac{l_{\text{pix}}^2}{m^2} = \frac{(2.5\ \mu\text{m})^2}{(0.0042)^2} = (600\ \mu\text{m})^2$$

All of the photons reflected by this area, <u>if captured by the lens</u>, are focused on a single pixel.

The optical power P_{ref} reflected by the area $A_{\text{pix,scn}}$ of the painting is just a fraction of the power P_{scn} impinging on the painting:

$$P_{\text{ref}} = P_{\text{scn}} \cdot \frac{A_{\text{pix,scn}}}{A_{\text{scn}}} \cdot R_{\text{scn}} = 3\ \text{W} \cdot \frac{(600\ \mu\text{m})^2}{2.4\ \text{m} \cdot 1.5\ \text{m}} \cdot 0.25 = 75\ \text{nW}$$

The painting behaves as an isotropic reflector, therefore the reflected power P_{ref} is evenly spread in the half-space. We can compute the corresponding intensity as:

$$I_{\text{ref}} = \frac{P_{\text{ref}}}{\Omega_{\text{half-space}}} = \frac{75\ \text{nW}}{2\pi} = 11.94\ \text{nW/sr}$$

The optical power reflected by the area $A_{\text{pix,scn}}$ of the painting and collected by the lens is confined into the solid angle Ω_{lens} seen from the painting towards the lens:

$$\Omega_{\text{lens}} = \frac{\pi \left(\frac{D_{\text{lens}}}{2}\right)^2}{d_{\text{scn}}^2} = \frac{\pi (2\ \text{mm})^2}{(3.84\ \text{m})^2} = 852.2\ \text{nsr}$$

We can thus compute the optical power P_{pix} impinging on the pixel (also accounting for the transmittance of the red colour filter) as:

$$P_{\text{pix}} = I_{\text{ref}} \cdot \Omega_{\text{lens}} \cdot T_{\text{red}} = 11.94 \frac{\text{nW}}{\text{sr}} \cdot 852.2\ \text{nsr} \cdot 0.8 = 8.14\ \text{fW}$$

Finally, the photocurrent can be computed:

$$i_{\text{ph}} = q \cdot \frac{P_{\text{pix}}}{\frac{hc}{\lambda}} \cdot \eta = 2.77\ \text{fA}$$

(iii) Let us compute the signal in terms of number of photogenerated charges for the given integration time:

$$N_{\text{el}} = \frac{i_{\text{ph}} t_{\text{int}}}{q} = 173\ \text{el}$$

The sensor is affected by shot noise and reset noise. The shot noise due to the photocurrent is readily computed:

$$\sigma_{i_{\text{ph}}} = \frac{\sqrt{q i_{\text{ph}} t_{\text{int}}}}{q} = 13.2\ \text{el}_{\text{rms}}$$

The shot noise can be evaluated knowing the dark current density and the active area of the pixel:

$$\sigma_{i_{\text{d}}} = \frac{\sqrt{q J_d A_{pix} FF t_{\text{int}}}}{q} = 3.0\ \text{el}_{\text{rms}}$$

To compute the reset noise, we first need to evaluate the integration capacitance. Since no data on the gate capacitance of the source follower are available, we shall approximate it as the photodiode capacitance only:

$$C_{\text{pd}} = \varepsilon_{\text{Si}} \cdot \frac{A_{\text{pix}} \cdot FF}{x_d} = 11.9 \cdot 8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}} \cdot \frac{(2.5\ \mu\text{m})^2 \cdot 0.46}{1\ \mu\text{m}} = 291\ \text{aF}$$

The corresponding reset noise is:

$$\sigma_{\text{kTC}} = \frac{\sqrt{k_{\text{B}} T C_{\text{pd}}}}{q} = 6.8\ \text{el}_{\text{rms}}$$

The overall noise is given by the quadratic sum of the contributions, and the SNR results:

$$\mathrm{SNR} = \frac{173\ \mathrm{el}}{\sqrt{(13.2\ \mathrm{el_{rms}})^2 + (3.0\ \mathrm{el_{rms}})^2 + (6.8\ \mathrm{el_{rms}})^2}} = 21.2\ \mathrm{dB}$$

(iv) To evaluate the dynamic range, we need to compute the full-well charge (i.e. the maximum input signal that the sensor can handle):

$$\mathrm{FWC} = Q_{\max}\frac{C_{\mathrm{pd}}V_{\mathrm{dd}}}{q} = 3640\ \mathrm{el_{rms}}$$

The minimum detectable signal can be found by setting $\mathrm{SNR} = 1$ (neglecting the photocurrent shot noise):

$$Q_{\min} = \frac{\sqrt{qJ_d A_{pix} FF t_{\mathrm{int}} + k_{\mathrm{B}} T C_{\mathrm{pd}}}}{q} = 7.5\ \mathrm{el_{rms}}$$

Finally, the dynamic range can be evaluated:

$$DR = \frac{\mathrm{FWC}}{Q_{\min}} = 53.8\ \mathrm{dB}$$