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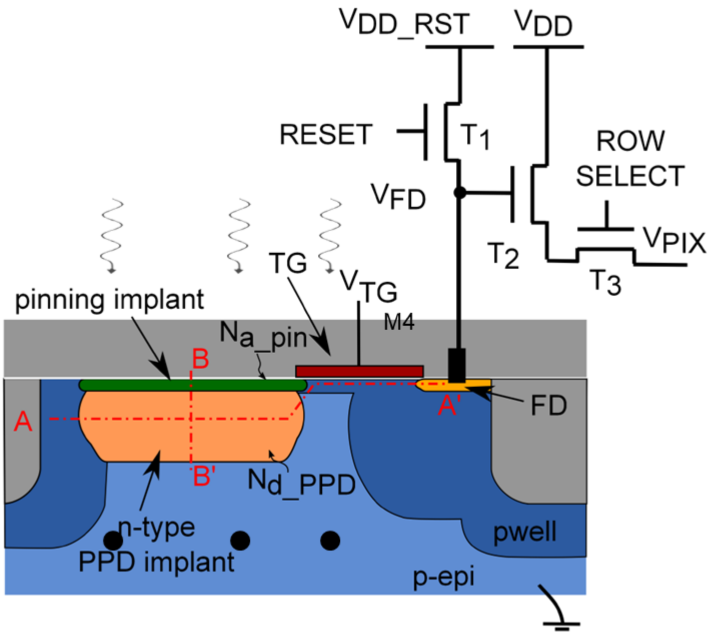
**Question n. 1**

Draw a detailed cross section of a 4-T pixel, and the associated electronic circuit. Carefully describe the operating phases of the circuit, assisting your discussion with waveforms graphs. Finally, comment on advantages and drawbacks over the 3-T architecture.

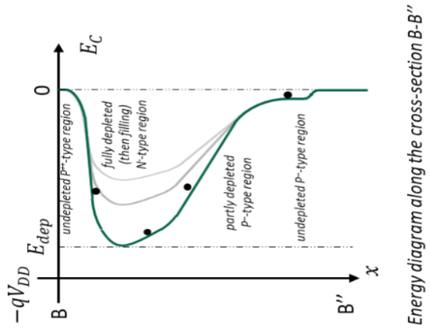


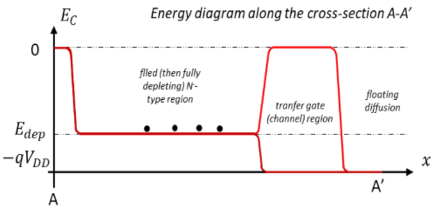
A pinned photodiode consists in a N-type "buried" implant, which is kept separate from surface impurities through a blocking, pinning, heavily doped and ultra-thin P-plus implant. As is, the buried N-type region (PPD) appears to have no electrical (ohmic) contact. In order to access it, it is thus necessary to create a conductive channel between the pinned photodiode implant and a floating diffusion N-type shallow region, which is then accessed through an ohmic contact. The channel can be activated or not, depending on the bias applied to the transmission gate (TG), like in a transistor.

In other words, the buried region can be connected to the floating diffusion (transmission gate voltage high) or can be kept isolated (transmission gate voltage low). Additional lateral p-type wells prevent cross talk between pixels. The floating diffusion is then connected to the equivalent of a 3-transistor architecture, i.e. to the gate of a source follower (T2). This gate can be initialized to the supply voltage through a reset transistor (T1), and the source follower itself can be switched on (i.e. connected to a column current source) through a selection transistor T3.

This specific structure enables photocharge isolation along two spatial directions:

- assuming that the PPD region has been reset to the highest voltage and then left disconnected, generated photoelectrons will be collected within its potential well as long as it is not completely full (figure aside, cross-section B-B');
- at the same time, lateral p-type regions will prevent photoelectrons flowing outside the region, letting the generated charge be confined within the PPD (see the cross-section A-A' in the figures aside), until the transmission gate is brought high;
- in case the TG voltage is set to the high level, it provides a conductive path towards a lower potential region (the FD), letting electrons flow towards it.

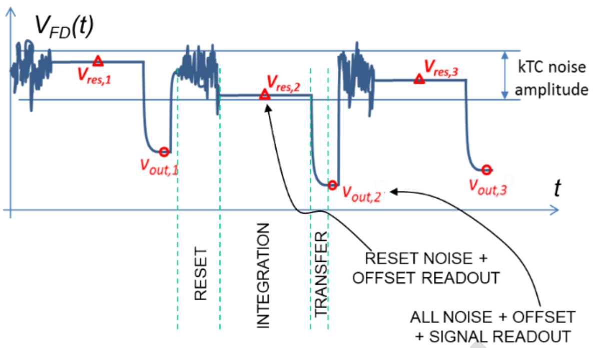

Energy diagram along the cross-section B-B''


Energy diagram along the cross-section A-A'

The pixel operation thus follows the steps described below:

- assume that the pinned PD is fully depleted by the former readout operation (its potential being brought to the maximum level, i.e. the potential-well being the deepest;

*Multi-chance students should skip point n. (iv) of question n. 2 and point n. (iv) of question n. 3*



- RESET: with $V_{TG}$ = 0 V, the sense (floating diffusion) node is reset to $V_{DD}$ by closing the reset (T1=$M_{RES}$). The floating diffusion is thus charged at $v_{FD} \approx V_{DD}$ (with possible noise fluctuations frozen to a random value due to kTC noise);
- INTEGRATION: the reset transistor is then opened, but charge continues to accumulate only in the pinned $N_{d\_PPD}$ region. The sense node voltage therefore does not change (remains flat in the graphs aside);
- TRANSFER: the TG is closed: ($V_{TG}$ risen): charge is wholly transferred to the floating diffusion (which empties the $N_{d\_PPD}$ region), causing a change in the sense node voltage $v_{out} = V_{DD} - \frac{(i_{ph}+i_d)t_{int}}{C_S}$;
- READOUT: the selection transistor closes (T3=$M_{SEL}$), activating the source follower (T2=$M_{RD}$) for the readout.

The mentioned operation provides a few remarkable advantages over the simple 3-transistor operation:

- the capacitance of the sense node (the FD) can be made low, as this small N+ implant is decoupled from the wide buried N- implant used for photons collection. As a consequence, the sense node capacitance (the parallel of the FD junction and the follower capacitance) is now small and dominated by the follower gate, even at large collecting areas (and FF)! Therefore, the overall capacitance at the sense node depends very poorly on the accumulated charge, avoiding typical nonlinearity of 3T topologies.
- the sensing node has a very small active area, so that – as already said – the capacitance is dominated by the source follower. Nevertheless, the FF is almost unchanged. A further consequence is that the conversion gain (output voltage variation with respect to the input signal charge) can be larger than in the 3T topology – even at larger collection areas.
- The presence of the pinning shallow P-type implant has also a positive effect on the dark current. Indeed, surface generated charges are not within the N-type region (as in a 3T) and typically recombine before reaching it. Therefore, the surface generated current can be almost neglected and the overall dark current density decreases typically by a factor > 10.
- A correlated double sampling (CDS) can be operated by subtracting to the readout value another readout, formerly stored during the integration operation. Indeed, as shown in the graphs above, during integration the output voltage does not change (unlike in the 3T architecture). This allows the storing of two samples, the former including offset and kTC noise, the second including offset, kTC noise, dark signal and photo signal. The subtraction thus eliminates electronics offsets (not the dark current related term) and kTC noise, improving the overall noise performance against the 3T topology.
- The three transistors of the conventional 3T architecture can be shared among different pixels formed by pinned photodiode and transfer gate. This enables the reduction of the equivalent n. of transistors to e.g. 1.75-T: 4 photodiodes with 4 transmission gates and 3 additional transistors, so, overall, 7 transistors for 4 pixels.

In the end, a higher dynamic range can be achieved, thanks to the reduction of dark current shot noise and kTC noise. E.g. maximum DRs up to 70-75 dB can be achieved, compared to the typical 60-65 dB of a 3T architecture. Not to corrupt the so achieved DR, the ADCs of 4T architecture usually require more bits (e.g. 12) than a conventional 3T topology (e.g. 10).

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**Question n. 2**

A tuning-fork yaw gyroscope is operated in mode-split conditions, exploiting a drive-loop with amplitude gain control (AGC) and differential charge-amplifier readout. With the parameters reported in the table:

| | | | |
|---|---|---|---|
| Process thickness | h | 20 | um |
| Process gap | g | 1 | um |
| Drive mass (½ structure) | $m_d$ | 4 | nkg |
| Sense mass (½ structure) | $m_s$ | 2 | nkg |
| Sense spring stiffness | $k_s$ | 14 | N/m |
| Drive detection fingers (½ structure) | $N_{cf}$ | 48 | - |
| Drive charge amplifier capacitance | $C_d$ | 1 | pF |
| Sense charge amplifier capacitance | $C_s$ | 0.2 | pF |
| Sense resonance frequency | $f_s$ | 26.6 | kHz |
| Sense quality factor (½ structure) | $Q_s$ | 500 | - |
| Differential PP cells (½ structure) | $N_{pp}$ | 6 | - |
| PP length | $L_{pp}$ | 300 | um |
| Opamp voltage noise density | $S_{Vn}$ | 70 | nV/√Hz |
| Rotor bias voltage | $V_{dc}$ | 5 | V |
| Circuit power supply | $\pm V_{dd}$ | ±5 | V |

**(i)** consider the figure below: size the drive springs and tuning-fork stiffness to obtain an in-phase drive resonance frequency at 20 kHz and an anti-phase drive resonance frequency at 26 kHz;

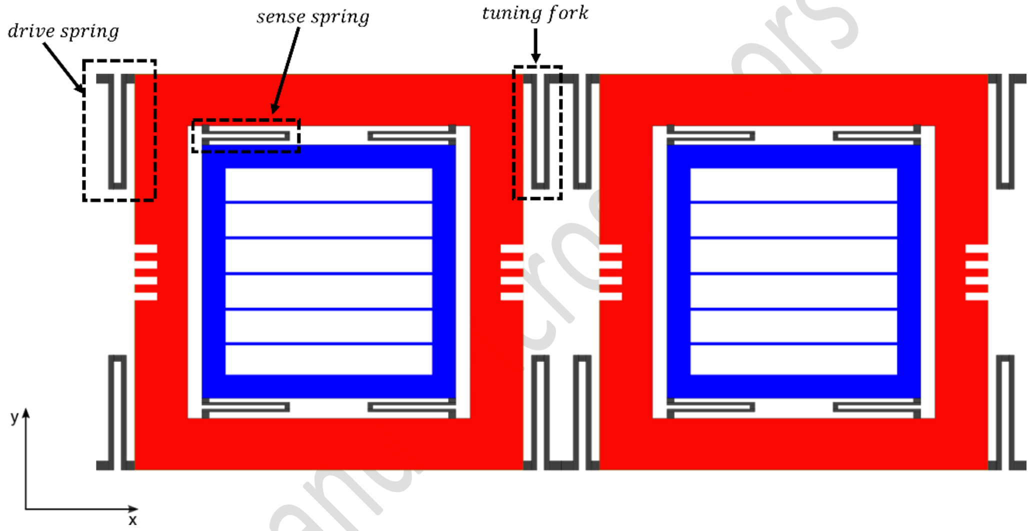
**(ii)** size the AGC reference voltage needed to obtain a controlled drive amplitude of 8 um;

**(iii)** evaluate the overall sensitivity in [V/dps] and the full-scale range of the sensor;

**(iv)** evaluate the noise equivalent rate density (NERD) in [dps/√Hz], and, knowing that the overall sensor noise is equal to 2 mdps/√Hz, calculate the parasitic capacitance at the sense amplifier input node (neglect noise from the feedback resistance).

**Physical Constants**

$\rho_{Si}$ = 2370 kg/m$^3$;
$\varepsilon_0$ = 8.85 $10^{-12}$ F/m;
$k_b$ = 1.38 $10^{-23}$ J/K;
q = 1.6 $10^{-19}$ C;
T = 300 K;



(i) The drive springs of the structure contribute in setting both the in-phase and anti-phase drive modes stiffness, while the tuning fork acts only during the anti-phase drive motion, as during the in-phase motion they are not excited. Therefore, having two drive springs and two tuning fork springs connected to ½ structure:

$$f_{D.ip} = \frac{1}{2\pi}\sqrt{\frac{k_D \cdot 2}{(m_D + m_S)}} \qquad f_{D,ap} = \frac{1}{2\pi}\sqrt{\frac{k_D \cdot 2 + k_{TF} \cdot 2}{(m_D + m_S)}}$$

from which:

$$k_D = \left(2\pi f_{D,ip}\right)^2 \cdot \frac{m_D + m_S}{2} = 47.37\ N/m$$

$$k_D = \left(\left(2\pi f_{D,ap}\right)^2 \cdot (m_D + m_S) - 2k_D\right)/2 \ = 32.69\ N/m$$

*Multi-chance students should skip point n. (iv) of question n. 2 and point n. (iv) of question n. 3*

(ii) Considering a differential readout of the drive motion, the induced motional current in each amplifier is given by the following:

$$i_m = \frac{2\varepsilon_0 h N_{CF}}{g} \cdot V_{DC} \cdot \left(2\pi f_{D,ap}\right) \cdot x_D = 111.03\, nA$$

and the differential output is:

$$V_{out,D} = 2 \cdot i_m \cdot \frac{1}{2\pi f_{D,ap} C_d} = 1.36\, V$$

and the AGC reference is thus given by the output of the rectification+LPF stage of the AGC:

$$V_{ref,AGC} = V_{out,D} \cdot \frac{2}{\pi} = 0.865\, V$$

(iii) The sensitivity of the system, considering a differential sense readout, can be expressed as below:

$$SF = \frac{x_D}{\Delta\omega} \cdot 2 \cdot \frac{2\varepsilon_0 L_{PP} N_{PP} h}{g^2} \cdot \frac{V_{DC}}{C_s} \cdot \frac{\pi}{180} = 1.2 \frac{mV}{dps}$$

And the FSR can be obtained by evaluating which input range cover the whole $\pm V_{DD}$ supply range:

$$FSR = \pm \frac{V_{DD}}{SF} = \pm 4237\, dps$$

(iv) From the expression of the quality factor, the damping coefficient can be retrieved:

$$b_S = \frac{\omega_S m_S}{Q_S}$$

and from this, the noise equivalent rate density is calculated as:

$$NERD = \frac{\sqrt{k_B T b_S}}{\sqrt{2}} \cdot \frac{1}{x_D m_S \omega_{D,ap}} \cdot \frac{180}{\pi} = 815.45 \frac{\mu dps}{\sqrt{Hz}}$$

where the $\sqrt{2}$ at the denominator is given by the fact that the gyroscope is made of two masses. At this point, knowing that the total input referred white noise, neglecting the CA feedback resistors, is given by the quadratic sum of the NERD and the contribution given by the Opamps input noise:

$$S_{opamp,tot} = \sqrt{S^2_{noisw,w,tot} - NERD^2} = 1.8 \frac{mdps}{\sqrt{Hz}}$$

from which, taking into account the differential readout:

$$C_P = C_S \cdot \left(\frac{S_{opamp,tot} \cdot SF}{\sqrt{2} \cdot S_{Vn}} - 1\right) = 4.15\, pF$$

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**Question n. 3**

|  |  |
|---|---|
| Process thickness | 30 µm |
| W spring | 5 µm |
| L spring | 500 µm |
| Rotor voltage | 15 V |
| Drive square wave pk-to-pk voltage | 0.1 Vpp |
| Feedback capacitance | 200 fF |
| Number of drive comb finger | 60 |
| Number of sense comb finger | 60 |
| Resonator mass | 10 nkg |
| Temperature range | -40/120°C |
| Minimum CA output signal | 1 V |
| Q at 27°C | 5000 |
| Number of tuning electrodes | 8 |
| Gap of tuning electrodes | 5 µm |

You are required to design a MEMS oscillator as shown in the Figure, with specifications given in the Table.

**(i)** find the mechanical resonance frequency of the structure;

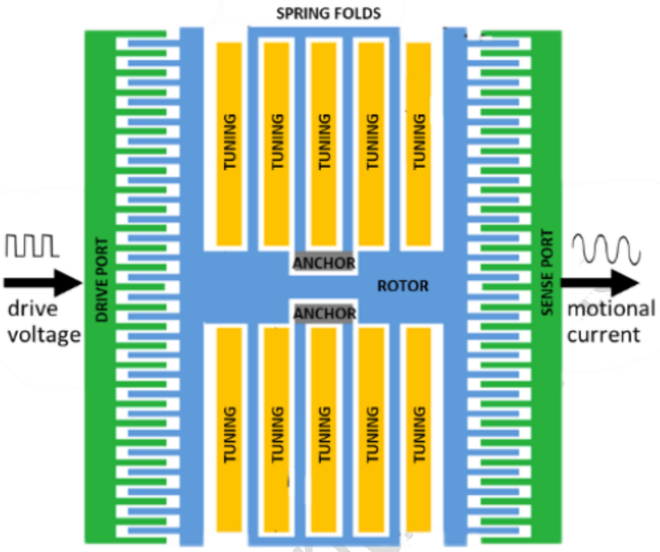
**(ii)** the output of the CA shall be a sinusoidal voltage with at least 1V of amplitude: find the maximum gap of the comb fingers to satisfy this condition;

**(iii)** for the found gap value, derive the equivalent electrical resonator model at 27 °C;

**(iv)** consider now process deviations for the etching resolution of $\sigma_x$=$\pm 0.1$ µm. At ambient temperature, without changing the springs, redesign the resonator so that it is possible to trim each device to match the nominal $f_0$ of point (i). Then find the maximum tuning voltage for the tuning plates, needed to compensate a $\pm 3\sigma$ etching.

**Physical Constants**

$\varepsilon_0$ = 8.85 $10^{-12}$ F/m;
$k_b$ = 1.38 $10^{-23}$ J/K;
E = 179 GPa ;



*Multi-chance students should skip point n. (iv) of question n. 2 and point n. (iv) of question n. 3*

(i) The mechanical resonance frequency is calculated with the usual expression:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the overall stiffness, which can be calculated for the top half of the structure and then multiplied by 2: it is a series of a parallel of two spring. This leads to the following result:

$$k = \frac{2 \cdot 2}{2}\, Eh \left(\frac{W}{L}\right)^3$$

where W and L are respectively the width and the length of each beam. So, the resonance is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{2\, Eh \left(\frac{W}{L}\right)^3}{m}} = 5{,}216\ kHz$$

(ii) The worst-case condition to reach the 1V output amplitude is at the highest temperature: the Q-factor is the lowest and the displacement of the resonator is itself the lowest. So, it is possible to extract the Q-factor at the maximum temperature, from the Q-factor at 300K:

$$Q(T_{max}) = \ Q(T_0) \cdot \frac{\sqrt{T_0}}{\sqrt{T_{max}}} = 4368$$

The amplitude of the sinusoid at the output of the CA is:

$$V_{CA} = \frac{\eta_s \dot{x}}{\omega_0 C_{ca}} = \frac{\eta_s \omega_0 x}{\omega_0 C_{ca}} = \frac{\eta_s}{C_{ca}} F_{elec} \cdot \frac{Q_{max}}{k} = \eta_D \eta_s V_{drive} \cdot \frac{Q_{max}}{k} \cdot \frac{1}{C_{ca}}$$

Adding that $\eta_D = \eta_s = \eta = V_{rot} N_{cf} \cdot 2 \cdot \frac{\epsilon_0 h}{g}$, and then inverting the formula, the maximum allowable gap to retrieve at least a 1V signal at the output of the CA becomes:

$$g = \frac{V_{rot} N_{cf} \cdot 2 \cdot \epsilon_o h}{\sqrt{\frac{V_{CA}\, k\, C_{CA}}{Q_{max}\, V_{drive}}}} = 5.44\ \mu m$$

(iii) The equivalent electrical model of the resonator is described by the following equation:

$$\frac{i_m(s)}{V_a(s)} = \frac{1}{L_{eq}s + R_{eq} + \frac{1}{sC_{eq}}}$$

with

$$L_{eq} = \frac{m}{\eta^2} = 1.3\ MH$$
$$R_{eq} = \frac{b}{\eta^2} = 8.5\ M\Omega$$
$$C_{eq} = \frac{\eta^2}{k} = 0.72\ fF$$

(iv) Since the tuning effect is capable only to downshift the frequency, the redesign of the structure has to upshift all the gaussian distribution so that it is possible to trim all the devices to the resonance frequency of point (i). The change of the resonance frequency without changing the springs can be done by adding mass on the structure. The frequency variability is

$$\frac{\sigma_f}{f} = \frac{3}{2} \cdot \frac{\sigma_w}{W}$$

where $\sigma_W$ is associated to the variability of the spring width, so it is twice the reported etching resolution.

The new target frequency is calculated so that the $3 \cdot \sigma_f$ coincide with the distance between the new frequency and the original one:

$$f_{new} - f_0 = 3 \cdot \sigma_f$$

The variability of the new resonance frequency can be described as:

$$f_{new} - f_0 = 3 \cdot \frac{3}{2} \frac{\sigma_W}{W} \cdot f_{new}$$

and by inverting the expression, is possible to retrieve the new target resonance frequency:

$$f_{new} = \frac{f_0}{1 - 3 \cdot \frac{3}{2} \frac{\sigma_W}{W}} = 6.36\ kHz$$

The maximum tuning voltage that must be applied to the parallel plate is the one that downshifts the highest frequency of the gaussian, that we consider as the $+3 \cdot \sigma_f$. The maximum frequency to down-tune is:

$$f_{max} = f_{new} + 3\sigma_{f_{new}}$$

which correspond to a change in stiffness that is required for the downtuning, considering that the stiffness remains the same:

$$\Delta k = 2\ k \cdot \frac{f_{max} - f_0}{f_{max}} = 8.53 \frac{N}{m}$$

By equating this stiffness change to the electrostatic softening, the maximum difference from the rotor voltage can be written as:

$$\Delta V_{tun} = \sqrt{\Delta k \cdot \frac{gap_{tuning}^{3}}{2\epsilon_0 L h N_{tuning}}} = 22.4V$$