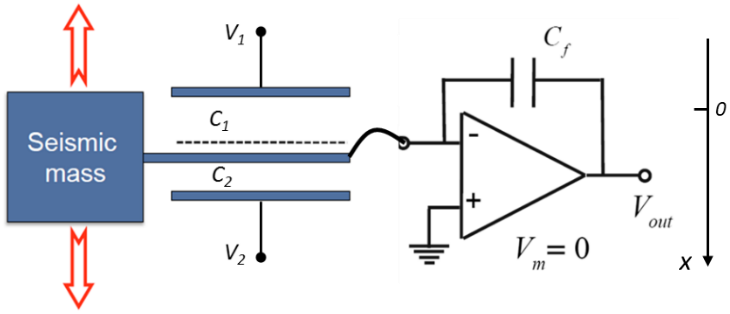
**Last Name** __Dentecompi__ **Given Name** ___Tosca___ **ID Number** ___20240111___

**Question n. 1**

Write the full force balance equation for a differential parallel plate configuration in MEMS accelerometers with a voltage difference between rotor and stators, describing the different terms appearing thereof. Then, discuss the impact of the electrostatic forces in presence and absence of a quasi-static acceleration. Finally, comment on the trade-offs they imply on the sensitivity.



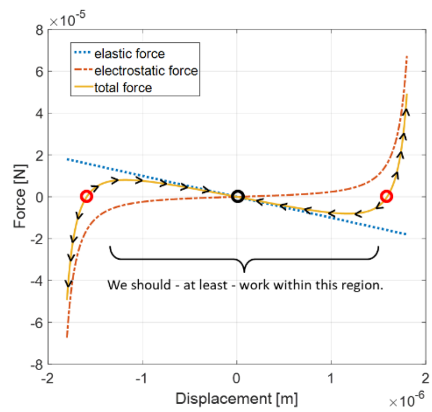

Let us assume a configuration like the one depicted aside, and further assume that the voltages $V_1$ and $V_2$ are set at $\pm V_{DD}$. As the rotor is kept to a virtual ground null reference, there is a net voltage difference towards the parallel plate stators. The expression of the two electrostatic forces, given below, shall be thus included in the full force-balance equation as further below:

$$F_{elec,1} = -\frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g+x)^2} \qquad F_{elec,2} = +\frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g-x)^2}$$

$$m\ddot{x} + b\dot{x} + kx = ma_{ext} + \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g+x)^2}$$

The real electrostatic force thus sums up to the apparent external acceleration inertial force $ma_{ext}$ and to the damping ($-b\dot{x}$) and stiffness ($-kx$) restoring terms. All these forces shall balance, according to the Newton second law, the product of the MEMS mass times the MEMS mass acceleration ($m\ddot{x}$).

Accelerometers usually operate in quasi-stationary conditions and thus, except in case of shocks or vibrations, the dynamic terms associated to the MEMS mass velocity and acceleration can be neglected (the derivative $\dot{x}$ of a low-frequency motion $x$ is low). The equation can be thus analysed in the simplified form below.



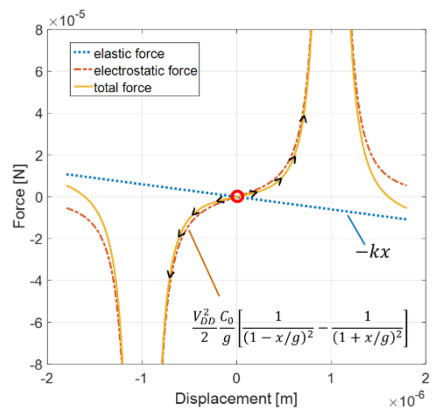

$$kx = ma_{ext} + \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g+x)^2}$$

Let us first see the impact of electrostatic forces on the system even in absence of an external acceleration, so further simplifying the analysis as:

$$kx = \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\,N}{(g+x)^2}$$

If one develops the equation, exploiting a graphical approach, he/she can easily find that this third order system shows:

- either one stable and two unstable equilibrium points (first figure aside);
- or one unstable equilibrium point (and two non-valid solutions, second figure aside).




*Multi-chance students should skip point n. (iii) of question n. 2 and point n. (iii) and (iv) of question n. 3*

It is thus obvious that we shall avoid the second situation: in this case, indeed, the accelerometer would be unusable as, as soon as anything displaces the mass from the ideal point at x = 0, the mass tends to collapse towards one electrode. The reason behind this behaviour is that the electrostatic force for small displacement is larger than the elastic force. Said in other words, the positive slope of the linearized electrostatic force is larger than the negative (restoring) slope $-k$ of the elastic force ($-kx$). To avoid this situation, it is thus mandatory to remain in a condition such that:

$$kx > 2V_{DD}^2 \frac{\varepsilon_0 A\, N}{g^3} x = 2V_{DD}^2 \frac{C_0}{g^2} x$$

In turn, this sets a limit to the maximum voltage we can apply to the stators for the readout. This voltage is known as pull-in voltage, and its value is:

$$V_{DD,PI} = \sqrt{\frac{g^3\ k}{2\ \varepsilon_0 A\, N}}$$

Even remaining below the pull-in voltage, the biasing of the stators gives a contribution that can be seen as an equivalent electrostatic stiffness term, with a sign which is however opposite to the mechanical stiffness and tends to reduce it:

$$\mathrm{k_{elec}} = -2V_{DD}^2 \frac{C_0}{g^2} \quad \rightarrow \quad k_{tot} = k_{mech} + k_{elec}$$

The resonance frequency of the accelerometer in operation, biased with the mentioned voltages, will be thus accordingly reduced to:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{\left(k_{mech} - 2V_{DD}^2 \frac{C_0}{g^2}\right)}{m}}$$

As in presence of an external acceleration, the travel range of the mass before the unstabe equilibrium point further reduces, reasonable margins in the stator bias voltage shall be taken, from the pull-in voltage, when operating the accelerometer.

Given that the sensitivity of an accelerometer is:

$$\frac{\Delta V_{out}}{a_{ext}} = 2 \frac{dV}{dC} \frac{dC}{dx} \frac{dx}{da} = 2 \frac{V_{DD}}{C_f} \frac{C_0}{g} \frac{m}{\left(k_{mech} - 2V_{DD}^2 \frac{C_0}{g^2}\right)}$$

We see a lot of trade-offs arising due to pull-in phenomena:

- a small gap enhances the sensitivity, but is unfavorable for pull-in issues;
- a small overall stiffness enhances the sensitivity but is unfavorable for pull-in issues and max bandwidth;
- a large bias voltage enhances the sensitivity but facilitates pull-in and is limited by the consumption of the IC;
- A large area or number of parallel plates enhance the sensitivity, but are unfavorable for pull-in issues;

All these trade-offs make the optimal design of an accelerometer a rather challenging problem.

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**Question n. 2**

An imaging sensor has the PTC shown in the image. Some of the process parameters are known and shown in the Table aside. Other parameters need to be estimated:

| | |
|---|---|
| Supply Voltage | 3 V |
| Process quantum efficiency | 0.8 |

**(i)** derive the conversion factor from photons to digital numbers, the number of bits of the ADC, and estimate the integration capacitance from the full-well charge;

**(ii)** for the maximum input signal level, derive the photon shot noise and the PRNU in terms of electrons rms;

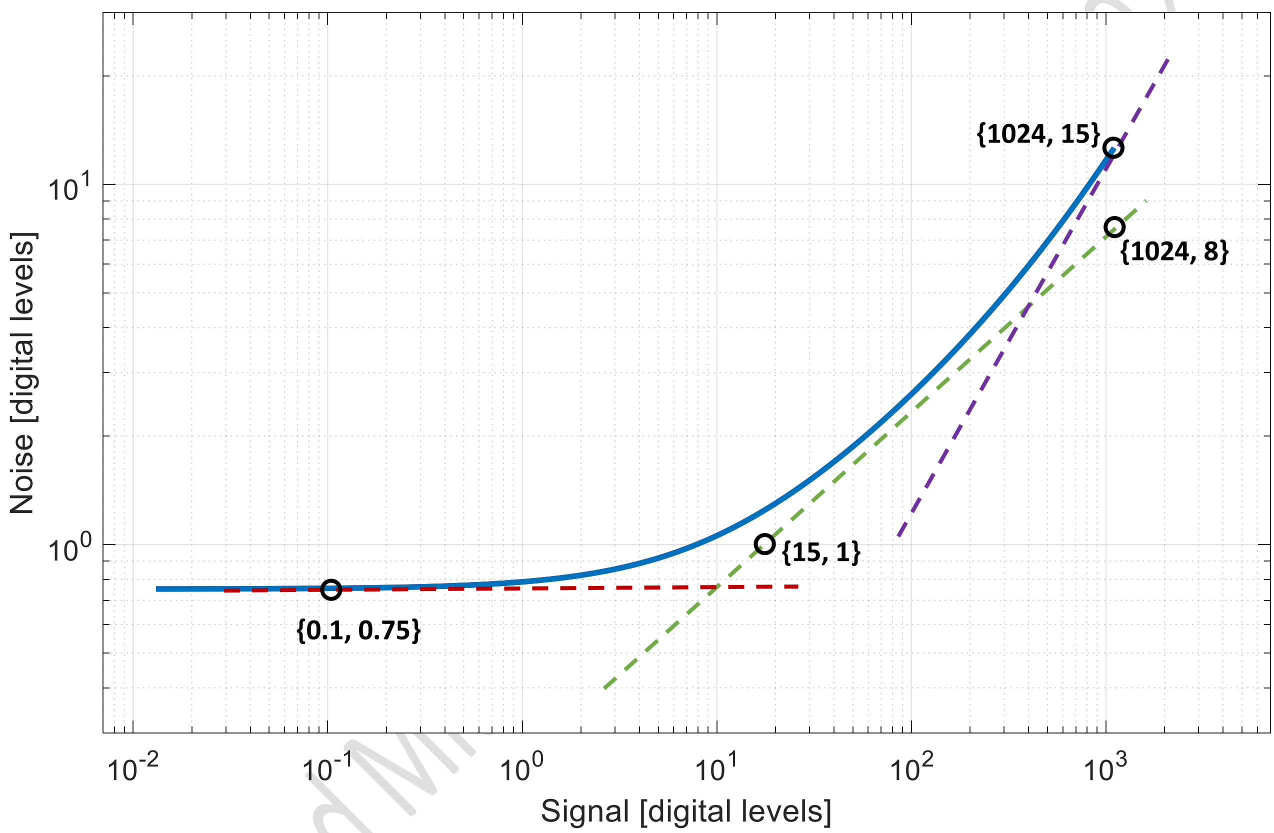
**(iii)** are dark current related noise sources dominant or negligible? Is the ADC well sized?

**Physical Constants**

$k_b$ = 1.38 $10^{-23}$ J/K;

q = 1.6 $10^{-19}$ C;

T = 300 K;




(i)

Consider the PTC gain between photons and digital numbers, expressed as:

$$K = \frac{\sigma_B^2}{B}$$

We start by looking at the intercept of photon shot noise (slope of +10 dB/dec) with $\sigma_B^2 = 1$. To this purpose, we observe that the signal independent noise weighs about 0.75 levels, so we have to look at a point where total noise weighs about $\sqrt{0.75^2 + 1} = 1.25$ levels. This corresponds to B = 15 digital levels, approximately, and for this coordinate photon shot noise is indeed unitary. The PTC gain results thus to be:

$$K = \frac{1}{15} = 0.067 \frac{DN}{ph}$$

To infer the n. of bit of the ADC, the easiest way is to look at the maximum digital number, which is in the order of 1000. As $2^{10}$ = 1024, it is easy to understand that the user number of bits is here 10, which is in the typical range for imaging sensors.

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To estimate the integration capacitance, we can turn this number into electrons, and then extract the capacitance from the equations below:

$$Q_{max} = C_{int} V_{DD} = \frac{DN_{max}}{K} \eta q \quad \rightarrow \quad C_{int} = \frac{2^{N_{bit}}}{K} \frac{\eta q}{V_{DD}} = \frac{1024\, DN}{0.067 \frac{DN}{ph}} \cdot \frac{0.8 \frac{e^-}{ph}}{3V} 1.6\ 10^{-19} \frac{C}{e^-} = 0.66\, fF$$

(ii)

For the maximum signal, it is reasonable to assume that PRNU dominates. This is visible when drawing the different contributions, with different slopes, as shown in the graph. On the y-axis of the graph we thus estimate a PRNU noise of 15 digital levels, which corresponds to:

$$\sigma_{PRNU,q} = \frac{15\, DN}{K} \eta = \frac{15\, DN}{0.067 \frac{DN}{ph}} 0.8 \frac{e^-}{ph} = 180\, e^-$$

To find the value of shot noise at this signal level, we can either:

- read the y-axis value of shot noise on the dashed line with slope of 10 dB/dec, or
- take tha value already calculated for a x-axis signal of 15 and increase it by the square root of 1024 over 15 (indeed, shot noise grows with the signal square root).

Whatever the method, we find:

$$\sigma_{shot,ph,q} = \frac{8\, DN}{K} \eta = 96\, e^- \qquad \sigma_{shot,ph,q} = 1\, DN \sqrt{\frac{1024}{15}} \frac{1}{K} \eta = 99\, e^-$$

(iii)

we can estimate whether signal-independent noise is dominated by kTC or not, as we already evaluated the integration capacitance in point (i). We just write:

$$\sigma_{kTC_{DN}} = \frac{\sqrt{k_B T C_{int}}}{q} \frac{1}{\eta} K = 10.3\, e^- \frac{1}{0.8 \frac{e^-}{ph}} 0.067 \frac{DN}{ph} = 0.57\, DN$$

Quantization noise in terms of digital levels is just:

$$\sigma_{quant_{DN}} = \frac{LSB}{\sqrt{12}} = \frac{1\, DN}{\sqrt{12}} = 0.29$$

Their quadratic sum corresponds to 0.64 DN which is close to the value of signal-independent noise that we read on the y-axis of the flat region (in the order of 0.75 DN). The ADC is thus well sized, and dark-signal related terms are, in this scenario, reasonably lower than kTC noise.

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**Question n. 3**

A tuning-fork MEMS yaw gyroscope with single-ended, identical drive-actuation and drive-detection ports has the parameters given in the Table (data given for half the structure):

| | |
|---|---|
| Full-scale Range | ±2500 dps |
| Supply voltage | ±1.8 V |
| Rotor voltage | 8 V |
| AGC reference voltage | 1.5 V |
| Process Gap | 2.2 µm |
| Drive comb overlap | 16 µm |
| Drive resonance frequency | 30 kHz |
| Sense mode stiffness | 820 N/m |
| Sense capacitance (single ended) | 320 fF |
| Drive capacitance (single ended) | 100 fF |
| Drive/Sense feedback capacitance | 255 fF |
| Drive feedthrough capacitance | 2 fF |
| Parasitic capacitance | 8 pF |
| Amplifier voltage noise | 10 nV/√Hz |

**(i)** find the mode split value, and then verify that the electrostatic stiffness is reasonably negligible for this situation;

**(ii)** size the feedback resistor of the sense charge amplifier so to provide a phase delay in the order of 0.5°, and find the input referred contributions to angular rate noise density due to the electronics;

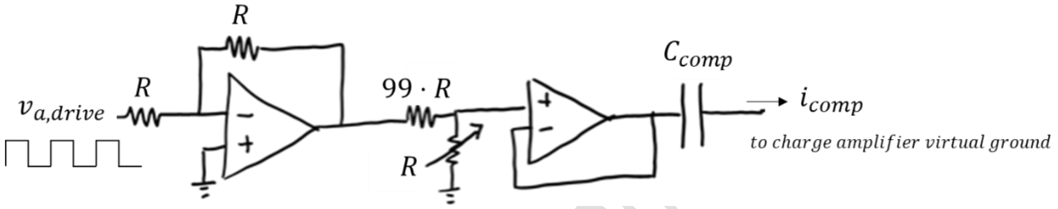
**(iii)** considering the feedthrough compensation circuit below for the drive mode, size the value of the capacitance $C_{comp}$;

**(iv)** with the found value of $C_{comp}$, what happens if the first resistance of the divider is erroneously designed at 98·R?

**Physical Constants**

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m;
$k_b = 1.38\ 10^{-23}$ J/K;
T = 300 K;



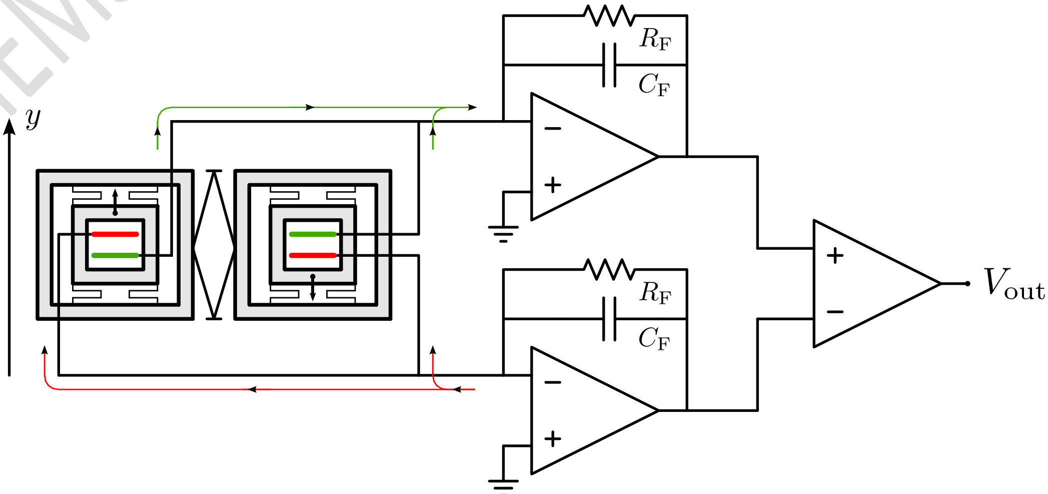



(i)

The mode split value $\Delta\omega_{MS}$ can be computed by inverting the formula for the sensitivity of a mode-split gyroscope:

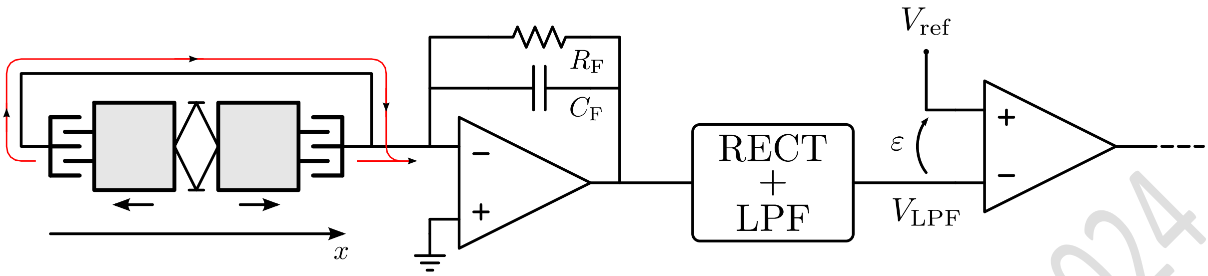
$$\frac{\Delta V_{\text{out}}}{\Delta\Omega} = 2 \cdot 2\frac{V_{\text{rot}}}{C_F} \cdot \frac{C_s}{g} \cdot \frac{x_d}{\Delta\omega_{MS}}$$

Note the factor 2 due to the two sense frames in the tuning-fork structure injecting twice the sense current in the front-end, as shown in the schematic below.




*Multi-chance students should skip point n. (iii) of question n. 2 and point n. (iii) and (iv) of question n. 3*

We need to compute the drive oscillation amplitude $x_d$ first. Let us refer to the schematic below, which depicts the two drive masses coupled by a tuning-fork spring, the single-ended (one per mass) comb-finger drive-detection electrodes, the charge amplifier front-end of the drive oscillator, and the AGC chain up to the comparator stage.




The negative feedback of the AGC loop nulls the voltage error $\varepsilon$ at the comparator input, making the output voltage $V_{\mathrm{LPF}}$ of the LPF equal to the AGC reference voltage $V_{\mathrm{ref}}$:

$$V_{\mathrm{LPF}} = \frac{2}{\pi} \cdot 2\frac{V_{\mathrm{rot}}}{C_F} \cdot \frac{C_d}{L_{ov}} \cdot x_d = V_{\mathrm{ref}}$$

Note the $\frac{2}{\pi}$ gain of the rectifier + LPF stage, and the expression for the $\frac{dC}{dx}$ of the comb-finger drive-detection electrodes, which depends on the comb overlap $L_{\mathrm{ov}}$:

$$\frac{dC_d}{dx} = \frac{d}{dx}\left(\frac{2\varepsilon_0 h N_{\mathrm{cf}} \cdot (L_{\mathrm{ov}} + x)}{g}\right) = \frac{2\varepsilon_0 h N_{\mathrm{cf}}}{g} = \frac{C_d}{L_{ov}}$$

We can solve for the drive oscillation amplitude $x_d$:

$$x_d = \frac{V_{\mathrm{ref}} \cdot \frac{\pi}{2} \cdot C_F L_{\mathrm{ov}}}{2V_{\mathrm{rot}} C_d} = \frac{1.5\ \mathrm{V} \cdot \frac{\pi}{2} \cdot 255\ \mathrm{fF} \cdot 16\ \mu\mathrm{m}}{2 \cdot 8\ \mathrm{V} \cdot 100\ \mathrm{fF}} = 6\ \mu\mathrm{m}$$

We can now derive the mode split value $\Delta\omega_{MS}$ from the sensitivity, assuming the full voltage dynamics $\pm V_{\mathrm{dd}}$ to be exploited:

$$\Delta\omega_{MS} = 2 \cdot 2\frac{V_{\mathrm{rot}}}{C_F} \cdot \frac{C_s}{g} \cdot \frac{x_d}{\left(\frac{V_{\mathrm{dd}}}{\Omega_{FSR}}\right)} = \frac{2 \cdot 2 \cdot 8\ \mathrm{V} \cdot 320\ \mathrm{fF} \cdot 6\ \mu\mathrm{m}}{255\ \mathrm{fF} \cdot 2.2\ \mu\mathrm{m} \cdot \left(\frac{1.8\ \mathrm{V}}{2500\ \mathrm{dps}}\right)} = 2\pi \cdot 423 \frac{\mathrm{rad}}{\mathrm{s}}$$

corresponding to:

$$\Delta f_{MS} = 423\ \mathrm{Hz}$$

Let us now check the value of the electrostatic stiffness of the sense mode:

$$|k_{el}| = \frac{2V_{rot}^2 C_s}{g^2} = \frac{2 \cdot (8\ \mathrm{V})^2 \cdot 320\ \mathrm{fF}}{(2.2\ \mu\mathrm{m})^2} = 8.46 \frac{\mathrm{N}}{\mathrm{m}}$$

which is almost a factor 100 lower than the mechanical stiffness $k_s$.

(ii)

The phase delay due to the low-frequency pole of the charge amplifier at the drive resonance frequency $f_d$ is given by:

$$\Delta\varphi = 90° - \mathrm{arctg}\left(\frac{f_d}{f_p}\right) = 90° - \mathrm{arctg}(f_d \cdot 2\pi R_F C_F)$$

By setting $\Delta\varphi = 0.5°$ we can derive the value of the feedback resistance $R_F$:

$$R_F = \frac{\text{tg}(90° - \Delta\varphi)}{2\pi f_d C_F} = \frac{\text{tg}(89.5°)}{2\pi \cdot 30\ \text{kHz} \cdot 255\ \text{fF}} = 2.38\ \text{G}\Omega$$

The output voltage noise density of a single charge amplifier around the drive frequency $f_d$ has contributions from the opamp and the feedback resistance:

$$S_{v,\text{oa}}(f_d) \simeq S_v \cdot \left(1 + \frac{C_p + 2C_s}{C_F}\right) = 0\frac{nV}{\sqrt{Hz}} \cdot \left(1 + \frac{8\ \text{pF} + 2 \cdot 320\ \text{fF}}{255\ \text{fF}}\right) = 348.8\ \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

$$S_{v,R_F}(f_d) \simeq \sqrt{\frac{4k_bT}{R_F}} \cdot \left(\frac{1}{\omega_d C_F}\right) = \sqrt{\frac{4 \cdot 1.38 \cdot 10^{-23}\ \frac{\text{J}}{\text{K}} \cdot 300\ \text{K}}{2.38\ \text{G}\Omega}} \cdot \left(\frac{1}{2\pi \cdot 30\ \frac{\text{krad}}{\text{s}} \cdot 255\ \text{fF}}\right) = 54.9\ \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

The overall output noise of each charge amplifier is:

$$S_{v,\text{CA}}(f_d) = \sqrt{S_{v,\text{oa}}(f_d)^2 + S_{v,R_F}(f_d)^2} = 353\ \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

The overall voltage noise at the output of the difference amplifier (whose noise contribution we shall neglect) is given by the quadratic sum of the noise contributions of the two charge amplifiers:

$$S_{v,\text{out}} = \sqrt{S_{v,\text{CA1}}^2 + S_{v,\text{CA2}}^2} = \sqrt{2}S_{v,\text{CA}} = 499\ \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

We can input-refer this voltage noise by dividing by the gyroscope sensitivity:

$$S_{\Omega,\text{in}} = \frac{S_{v,\text{out}}}{\left(\frac{V_{\text{dd}}}{\Omega_{FSR}}\right)} = 693\ \frac{\mu\text{dps}}{\sqrt{\text{Hz}}}$$

(iii)

The compensation capacitance should be sized so that the compensation current $i_{\text{comp}}$ matches the feedthrough current $i_{ft}$:

$$i_{\text{comp}} = \frac{R}{99 \cdot R + R}\omega_d C_{\text{comp}} v_d$$

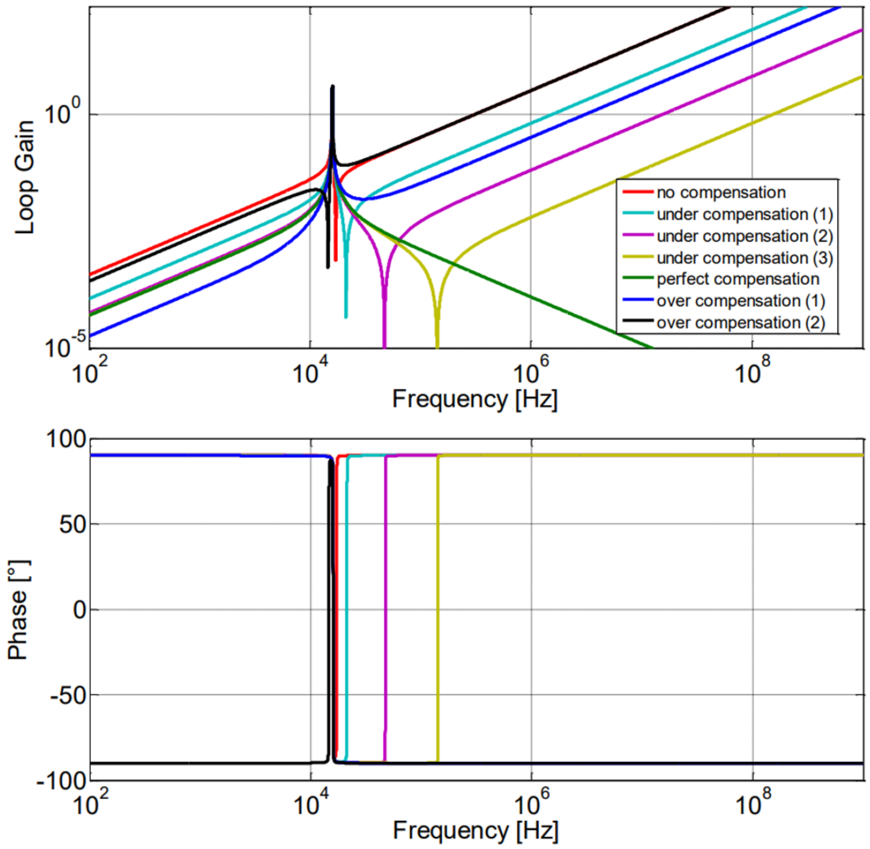
$$i_{ft} = \omega_d C_{ft} v_d$$

By equating the two currents and solving for $C_{\text{comp}}$ we get:

$$C_{\text{comp}} = 100 \cdot C_{ft} = 200\ \text{fF}$$

(iv)

If the voltage divider is sized with $98 \cdot R$, a higher-than-expected voltage is applied to the compensation capacitance, resulting in an over-compensation of the feedthrough. As a result, the anti-peak in the equivalent admittance of the MEMS drive mode is shifted below the resonance frequency.


Loop Gain
10^0
10^-5
10^2
10^4
10^6
10^8
Frequency [Hz]
no compensation
under compensation (1)
under compensation (2)
under compensation (3)
perfect compensation
over compensation (1)
over compensation (2)
Phase [°]
100
50
0
-50
-100
10^2
10^4
10^6
10^8
Frequency [Hz]