Last Name _Addobbato_ **Given Name** __Pino_____ **ID Number** __20231222__

Question n. 1

Discuss in detail the importance of the amplitude-gain control circuit in a MEMS gyroscope. Draw two possible implementations of such a circuit, as studied in the lectures or exercises of the course. For one of the implementations, write the expression in *s* of the loop gain and draw the AGC loop gain modulus.

i. importance of AGC

A MEMS gyroscope relies on the Coriolis force effect, and it is based on a 2-axis system where a motion is forced on one axis (e.g. the x-axis), and sensed along the other one (e.g. the y-axis), the angular rate being orthogonal to both of them (e.g. along the z-axis). In particular, the displacement in the sense direction y_s per unit input rate Ω can be written as:

$$\frac{y_s}{\Omega} = \frac{x_d}{\Delta\omega}$$

where $\Delta\omega$ is the -3 dB width of the sense-mode peak or the split between modes, depending on whether operation is in mode-matched or mismatch conditions. As a consequence, to guarantee that the sensitivity remains constant over time and environmental changes, it is fundamental that the amplitude of the sinusoidal drive displacement, x_d, is well controlled and kept constant.

The drive mode, which sets this amplitude, is based on a harmonic oscillator, where Barkhausen criteria are satisfied forcing a sinusoidal force F_{drive} at the drive fingers. However, the displacement x_d, in absence of a specific control, will change due to quality factor changes following:

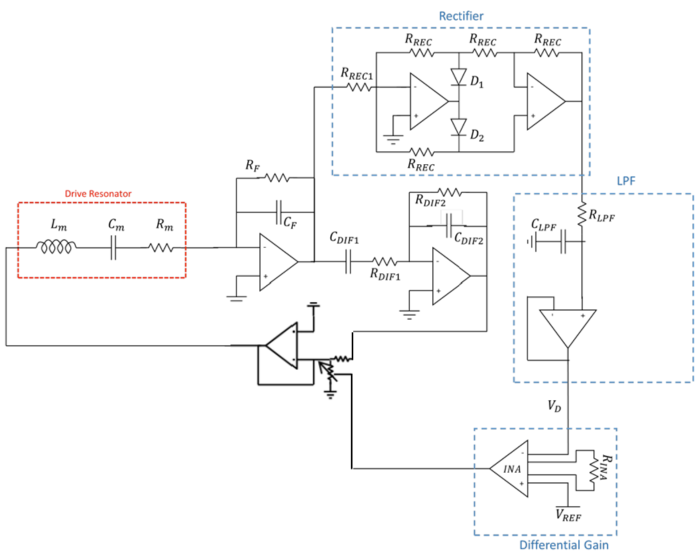
$$x_d = F_{drive} \cdot \frac{Q_d}{k_d}$$

As the quality factor changes (i) from part to part due to process spread and (ii) under temperature changes, it is mandatory to reject drive displacement changes which would, otherwise, change the sensitivity.

ii. possible implementations

For the aforementioned reasons, an amplitude-gain control (AGC) circuit is implemented as a negative feedback loop that controls the output of the charge amplifier, which is, indeed, a signal proportional exactly to the displacement:

$$V_{out,CA} = x_d \cdot \frac{\eta}{C_{Fd}}$$



The AGC circuit takes this AC sinusoidal signal, generates a corresponding DC signal by rectifying it and taking its mean value, and then compares it to a DC voltage reference. The difference is used as the error signal for the negative feedback: the amplitude of the drive signal is indeed changed accordingly, either increased or reduced, so to keep the amplitude of the signal $V_{out,CA}$, and thus of x_d, constant.

The action on the drive voltage and thus on the drive force can be implemented in at least two different ways: the first circuit

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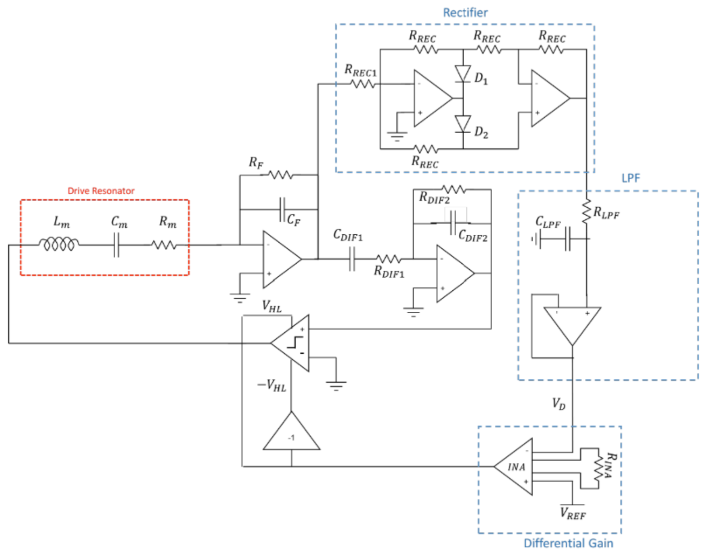
above (seen in lectures) reacts by changing the resistance of a voltage divider which is part of the drive loop, thus changing the drive amplitude.

The second solution, shown aside, changes the positive and negative voltages of a comparator, so to change the square wave voltage applied to the drive port (seen during the exercises).



bias

gain

on

gain

iii. loop gain

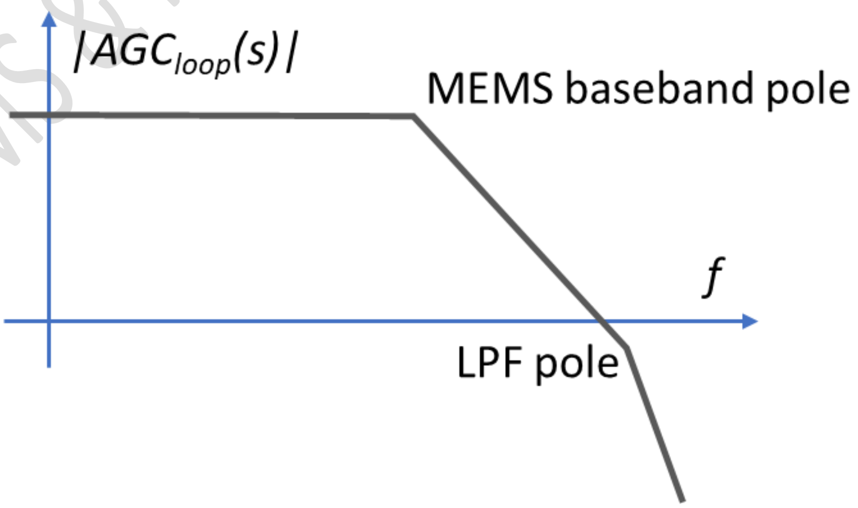
For the second solution we can write the expression of the loop considering the the AGC operates the amplitude of signals.

The contributions to the AGC loop are:

- The MEMS, which appears with its own base-band model (a pole at the resonance frequency divided by twice the Q factor);
- The charge amplifier, whose gain shall be calculated only at the drive frequency and is not a function of s;
- The rectifier and low-pass filter, which overall contribute with a gain of 2/π due to the filtering of a rectified sinewave, and with the LP filter pole;
- The INA gain
- The amplitude of the first harmonic of a square wave, which is 4/π.

$$G_{loop,AGC} = G_{INA}\frac{2}{\pi}\frac{1}{(2\pi f_{rd})\,C_F}\frac{1}{R_{md}}\frac{4}{\pi}\frac{1}{1+s\dfrac{2Q}{\omega_{rd}}}\frac{1}{1+sR_{LPF}C_{LPF}}$$

This is thus a 2-pole system and a sample loop gain is shown below. Note that the pole of the LPF shall be sized to guarantee stability of the system.



Question n. 2

A differential parallel plate MEMS accelerometer is designed for next-generation cube satellites missions, for vibration monitoring during moon landing. The sensor has the specifications in the Table.

(i) evaluate the analog accelerometer sensitivity (in units of [V/g]) and the digital accelerometer sensitivity (in units of [levels/g]);

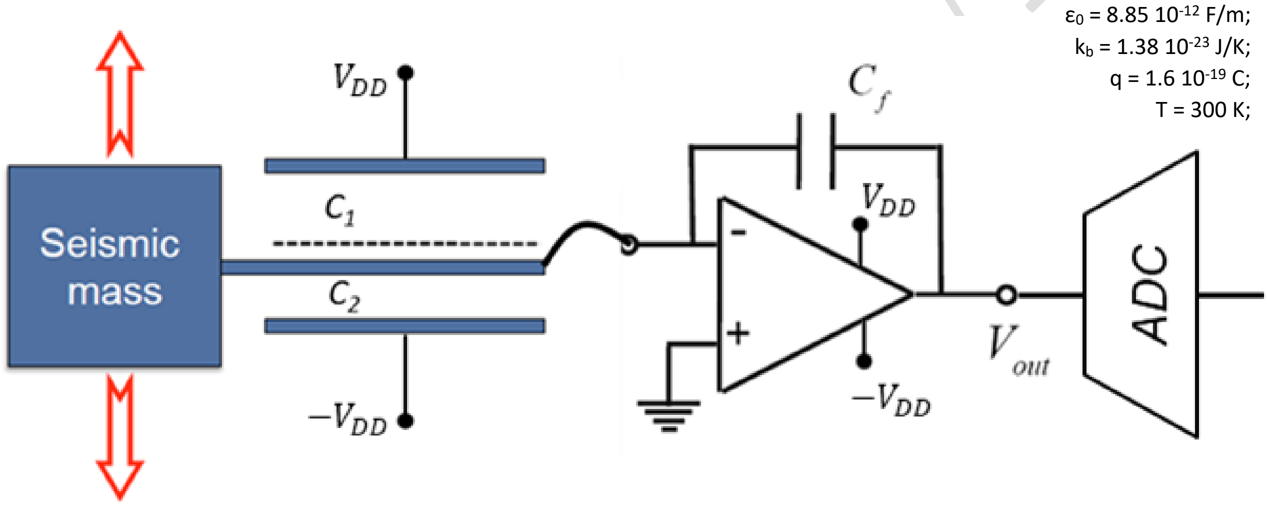
(ii) evaluate the accelerometer noise, input referred in terms of acceleration (in units of [g/√Hz]);

(iii) assume that there is a native mechanical offset of the accelerometer of 75 nm towards the plate n. 1. Graph the % linearity error within the full-scale range of the input acceleration.

Process thickness	100 µm
Process gap	1.5 µm
Stator and Amplifier bias voltage	±5 V
Mechanical stiffness	121 N/m
Sensor effective area	$(615\ \mu m)^2$
Parallel-plate length	485 µm
Parallel-plate number	14
Full-scale range	± 16 g
ADC number of bits	16
Damping coefficient	10^{-5} kg/s
Amplifier voltage noise	20 nV/√Hz
Parasitic capacitance	5 pF

Physical Constants

ρ_{Si} = 2370 kg/m^3;
ε_0 = 8.85 10^{-12} F/m;
k_b = 1.38 10^{-23} J/K;
q = 1.6 10^{-19} C;
T = 300 K;



(i) The analog sensitivity of the accelerometer is computed as the ratio of the output voltage of the charge-amplifier and the input acceleration. The text specifies the input range of the accelerometer to be $\pm 16\ \hat{g}$; assuming that the feedback capacitance of the amplifier stage will be sized to exploit the full voltage dynamics $\pm V_{\mathrm{DD}}$, we can compute the sensitivity as:

$$SF_a = \frac{V_{\mathrm{DD}}}{a_{\mathrm{FSR}}} = \frac{5\ \mathrm{V}}{16\ \hat{g}} = 0.3125 \frac{\mathrm{V}}{\hat{g}}$$

The digital sensitivity is the ratio of the full-scale numerical output of the ADC and the full-scale input acceleration; it can be evaluated by multiplying the analog sensitivity we just computed by the gain [levels/V] of the 16-bit ADC:

$$G_{\mathrm{ADC}} = \frac{1}{\mathrm{LSB}} = \frac{1}{\left(\frac{\mathrm{FSR_{ADC}}}{2^n}\right)} = \frac{2^n}{2V_{\mathrm{DD}}} = \frac{2^{16}}{10\ \mathrm{V}} = 65536 \frac{\mathrm{levels}}{\mathrm{V}}$$

$$SF_d = SF_a \cdot G_{\mathrm{ADC}} = 0.3125 \frac{\mathrm{V}}{\hat{g}} \cdot 65536 \frac{\mathrm{levels}}{\mathrm{V}} = 2048 \frac{\mathrm{levels}}{\hat{g}}$$

(ii) The input-referred acceleration noise can be computed as the sum of the thermo-mechanical and electronic noise contributions.

The expression for the input-referred thermo-mechanical noise is:

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$$NEAD = \sqrt{\frac{4k_bTb}{m^2}}$$

We need to compute the mass:

$$m = \rho_{\mathrm{Si}} \cdot (A_{\mathrm{eff}}h) = 2370\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot [(615\ \mu\mathrm{m})^2 \cdot 100\ \mu\mathrm{m}] = 89.6\ \mathrm{nkg}$$

and we can then evaluate the expression for the NEAD:

$$\mathrm{NEAD} = \sqrt{\frac{4k_bTb}{m^2}} = \sqrt{\frac{4 \cdot 1.38 \cdot 10^{-23}\frac{J}{K} \cdot 300\ \mathrm{K} \cdot 10^{-5}\frac{\mathrm{kg}}{\mathrm{s}}}{(89.6\ \mathrm{nkg})^2}} = 462.8\frac{\mathrm{n\hat{g}}}{\sqrt{\mathrm{Hz}}}$$

The value is quite low, because of the large mass of the sensor. The electronic noise contribution can be evaluated at the output of the charge-amplifier and input-referred by dividing by the analog sensitivity computed in point (i):

$$S_{a,\mathrm{in}} = \frac{S_{\mathrm{V,out}}}{SF_a} = \frac{S_{V,\mathrm{oa}}\left[1 + \frac{C_{\mathrm{p}} + 2C_0}{C_{\mathrm{F}}}\right]}{SF_a}$$

where $S_{\mathrm{V,oa}}$ is the amplifier voltage noise, $C_{\mathrm{p}} + 2C_0$ is the overall parasitic capacitance between the virtual ground of the charge-amplifier and ground, and C_{F} is the feedback capacitance. Note that the rest capacitance C_0 of the parallel-plate electrodes adds a non-negligible contribution to the parasitics:

$$C_0 = \frac{\varepsilon_0 h L_{\mathrm{pp}} N_{\mathrm{pp}}}{g} = \frac{8.85 \cdot 10^{-12}\frac{\mathrm{F}}{\mathrm{m}} \cdot 100\ \mu\mathrm{m} \cdot 485\ \mu\mathrm{m} \cdot 14}{1.5\ \mu\mathrm{m}} = 4\ \mathrm{pF}$$

The feedback capacitance C_F should be sized to exploit the full voltage dynamics of the amplifier:

$$V_{out}(a_{\mathrm{FSR}}) = 2\frac{V_{\mathrm{DD}}}{C_F}\frac{dC}{dx}\frac{1}{\omega_0^2}a_{\mathrm{FSR}} = V_{\mathrm{DD}}$$

In order to size C_{F}, we need to compute the capacitive variation per unit displacement $\frac{dC}{dx}$ and the radial resonance frequency ω_0 of the accelerometer first:

$$\frac{dC}{dx} = \frac{C_0}{g} = \frac{4\ \mathrm{pF}}{1.5\ \mu\mathrm{m}} = 2.67\ \frac{\mathrm{fF}}{\mathrm{nm}}$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_{\mathrm{m}} - k_{\mathrm{el}}}{m}} = \sqrt{\frac{k_{\mathrm{m}} - \frac{2V_{\mathrm{DD}}^2 C_0}{g^2}}{m}} = \sqrt{\frac{121\frac{\mathrm{N}}{\mathrm{m}} - \frac{2 \cdot (5\ \mathrm{V})^2 \cdot 4\ \mathrm{pF}}{(1.5\ \mu\mathrm{m})^2}}{89.6\ \mathrm{nkg}}} = 2\pi \cdot 3\ \frac{\mathrm{krad}}{\mathrm{s}}$$

which corresponds to a resonance frequency $f_0 = 3$ kHz.

We can now compute C_F as:

$$C_{\mathrm{F}} = \frac{2V_{\mathrm{DD}}\frac{dC}{dx}\frac{1}{\omega_0^2}}{SF_a} = \frac{2 \cdot 5\ \mathrm{V} \cdot 2.67\ \frac{\mathrm{fF}}{\mathrm{nm}} \cdot \frac{1}{2\pi \cdot 3\ \mathrm{krad/s}}}{0.3125\frac{\mathrm{V}}{\hat{\mathrm{g}}}} = 2.35\ \mathrm{pF}$$

Finally, the input-referred electronic noise contribution can be computed:

$$S_{a,\text{in}} = \frac{S_{\text{V,out}}}{SF_a} = \frac{20\ \frac{\text{nV}}{\sqrt{\text{Hz}}} \cdot \left[1 + \frac{5\ \text{pF} + 2 \cdot 4\ \text{pF}}{2.35\ \text{pF}}\right]}{0.3125\ \frac{\text{V}}{\hat{\text{g}}}} = 418.32\ \frac{\text{n}\hat{\text{g}}}{\sqrt{\text{Hz}}}$$

The overall input-referred noise of the accelerometer is given by the quadratic sum of the two contributions:

$$S_{a,\text{TOT}} = \sqrt{\text{NEAD}^2 + S_{a,\text{in}}^2} = 623.8 \frac{\text{n}\hat{\text{g}}}{\sqrt{\text{Hz}}}$$

(iii) The linearity error in absence of mechanical offset has the following expression:

$$\varepsilon_{\%} = 100 \cdot \frac{x_{max} \cdot \left[\frac{(x_{max}/g)^2}{1-(x_{max}/g)^2}\right]}{x_{\text{max}}\left[\frac{1}{1-(x_{\text{max}}/g)^2}\right]} = 100 \cdot (x_{max}/g)^2$$

where x_{max} is the displacement of the mass under a full-scale acceleration:

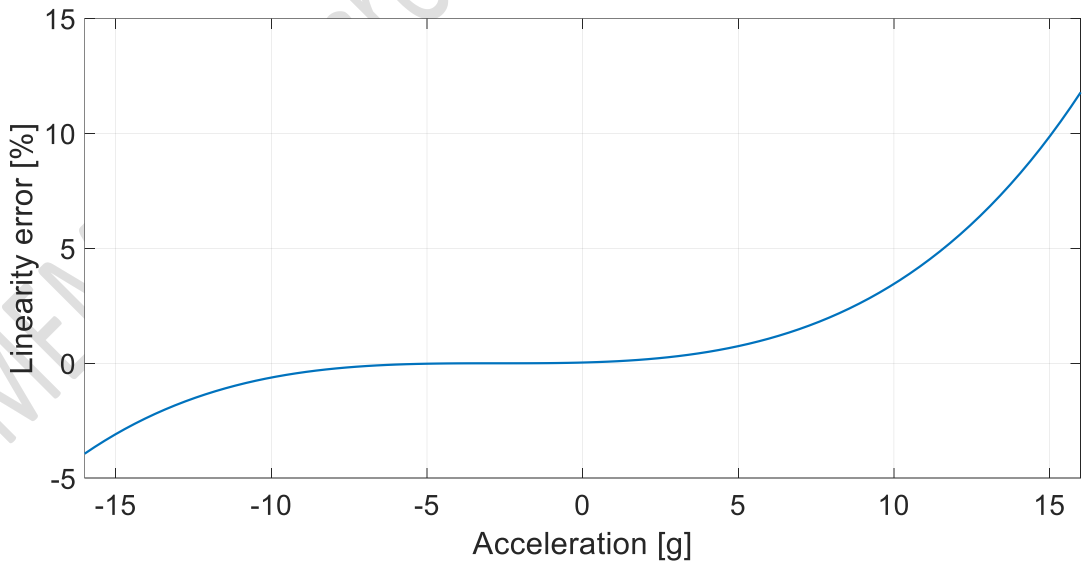
$$x_{\text{max}} = \frac{a_{\text{FSR}}}{\omega_0^2} = 440\ \text{nm}$$

The effect of a mechanical offset x_{os} is just a horizontal translation of the curve, as shown in the plot below. Note that, due to the mechanical offset, the linearity error is asymmetric in the range of input accelerations ($\pm 16\ \hat{\text{g}}$):

$$\varepsilon_{\%}(+16\ \hat{\text{g}}) = 100 \cdot \left(\frac{x_{\text{max}} + x_{\text{os}}}{g}\right)^2 = 11.8\%$$

$$\varepsilon_{\%}(-16\ \hat{\text{g}}) = 100 \cdot \left(\frac{-x_{\text{max}} + x_{\text{os}}}{g}\right)^2 = 3.9\%$$

A qualitative graph is shown below:



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Question n. 3

An imaging sensor for lunar soil black/white pictures is based on a 3-transistor architecture, featuring the parameters in the table.

- **(i)** draw the anode voltage as a function of time during one acquisition at the maximum photon flux, assuming that the camera mounts pixel-level micro-lenses;
- **(ii)** list all noise sources, expressed in terms of electrons rms, choose the number of bits for the ADC, and calculate the dynamic range;
- **(iii)** draw a graph of the SNR as a function of the photocurrent for the given integration time at the minimum temperature, highlighting the four most relevant values on the x and y axis.

Maximum photon flux	$2\cdot10^{17}$ ph/s/m^2
Pixel size	$(7\ \mu m)^2$
Fill factor	83%
Quantum efficiency	0.75
Dark current density	10 aA/$(\mu m)^2$
T range	140 - 400 K
Depletion region width	1.8 µm
Parasitic gate capacitance	0.3 fF
Supply voltage	5 V
Integration time	5 ms
Reset time	0.5 ms
$\%_{PRNU}$	1 %
$\%_{DSNU}$	5.5 %

Physical Constants

ε_0 = 8.85 10^{-12} F/m;
ε_{Si} = 11.7
k_b = 1.38 10^{-23} J/K;
q = 1.6 10^{-19} C;

(i)

The integration on a 3T pixel occurs on the parallel of the depletion capacitance and of the parasitic gate capacitance:

$$C_{dep} = \epsilon_0 \epsilon_{Si} \frac{l_{pix}^2\, FF}{x_{dep}} = 2.34\ fF \rightarrow \quad C_{int} = C_{dep} + C_g = 2.64\ fF$$

Note that the depletion capacitance includes only the active photodiode area and not the entire area. The signal, instead, thanks to the microlenses, is entirely collected as:

$$i_{ph} = \phi\, \eta\, q\, l_{pix}^2 = 1.18\ pA$$

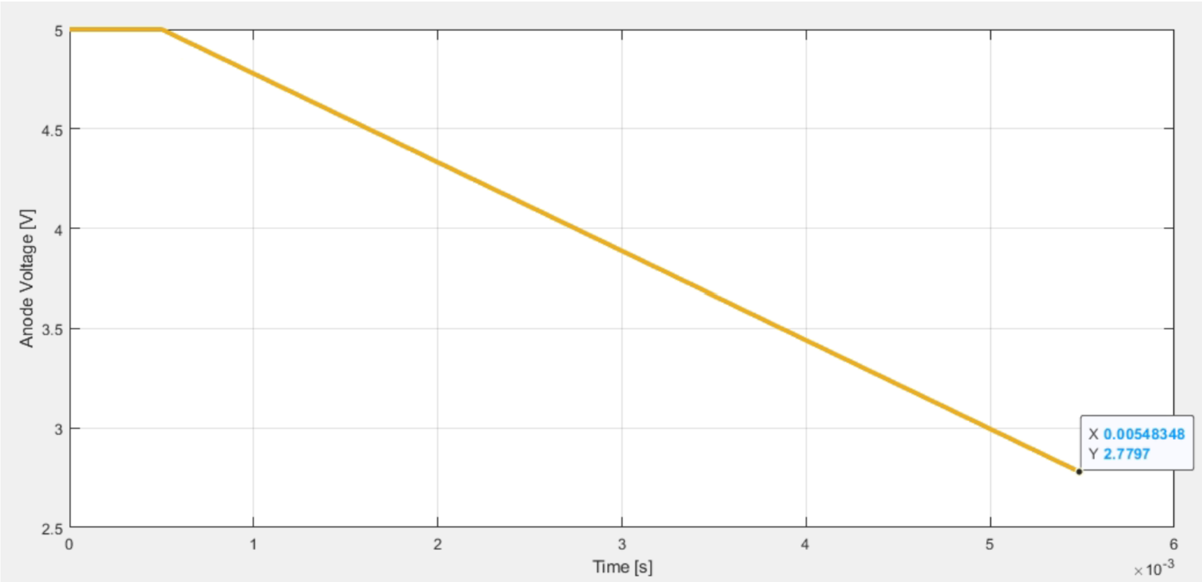
This is a relatively large photocurrent, consistent with the fact that we are calculating it for the maximum input signal. The dark current is instead calculated as:

$$i_d = j_d\, l_{pix}^2\, FF = 0.41\ fA$$

gathered just underneath the photodiode active area, and thus accounting for the FF. The signal at the anode node during integration decreases from the reset voltage, approximately in a linear manner:

$$V_{pix}(t) = V_{DD} - \frac{i_{ph} \cdot t}{C_{int}}$$

In the graph below this signal is translated on the horizontal axis to account for the reset time.



And the value at the chosen integration time is indicated as 2.78 V.

(ii)

All noise sources can be calculated from the expressions below:

$$\sigma_{shot,d} = \sqrt{q\, i_d\, t_{int}}/q = 3.57\ e^-_{rms}$$

$$\sigma_{kTC\_T_{min}} = \sqrt{k_B\, T_{min}\left(C_g + C_{dep}\right)}/q = 14.1\ \ e^-_{rms}$$

$$\sigma_{DSNU} = i_d\, t_{int}\frac{\sigma_{DSNU,\%}}{100}/q = 0.7\ \ e^-_{rms}$$

$$\sigma_{shot,ph} = \sqrt{q\, i_{ph}\, t_{int}}/q = 191.7\ \ e^-_{rms}$$

$$\sigma_{PRNU} = i_{ph}\, t_{int}\frac{\sigma_{PRNU,\%}}{100}/q = 367.5\ \ e^-_{rms}$$

The fact that signal dependent noise sources are largely dominant is consistent with the fact that we are calculating noises for the maximum signal value. In sizing the number of bits of the ADC, however, we shall consider that the camera will be often shooting at low input signals, and we cannot corrupt the performance at low signal levels. We thus choose to size the n. of bits such that quantization noise equals the lowest possible signal-independent noise sources, i.e. at the lowest integration time (shot and DSNU negligible) and at the lowest temperature:

$$\sigma_{ADC} = \frac{\left(C_g + C_{dep}\right) V_{DD}}{2^{N_{bit}}\sqrt{12}}\frac{1}{q} = \sigma_{kTC_{T_{min}}} \quad \rightarrow \quad N_{bit} = \log_2\left(\frac{V_{DD}}{\sqrt{12}}\frac{C_{int}}{q}\frac{1}{\sigma_{kTC_{T_{min}}}}\right) = 11$$

The corresponding quantization noise turns out to be 11.6 electrons rms.

The DR can be thus calculated as:

$$N_{el,max} = \frac{V_{DD}C_{int}}{q} = 82486 \ \rightarrow \ DR = 20\log_{10}\frac{N_{el,max}}{\sqrt{\sigma^2_{ADC} + \sigma^2_{shot,d} + \sigma^2_{kTC} + \sigma^2_{DSNU}}} = 72.9\ dB$$

Note that the DR is relatively large for a 3T pixel. This is due to the large area and the large supply voltage.

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(iii)

The four points of interest for the SNR graph are in this situation:

- The point of saturation (maximum SNR);
- The point where PRNU equals shot noise;
- The point where shot noise equals signal-independent noise sources
- The point where the SNR equals 0 dB

They can be calculated using the formulas below:

$$i_{ph,max} = \frac{C_{int}V_{DD}}{t_{int}} = 2.64\ pA$$

$$i_{ph,cross_1} = \frac{q t_{int}}{\left(t_{int}\sigma_{PRNU,\%}\right)^2} = 320\ fA$$

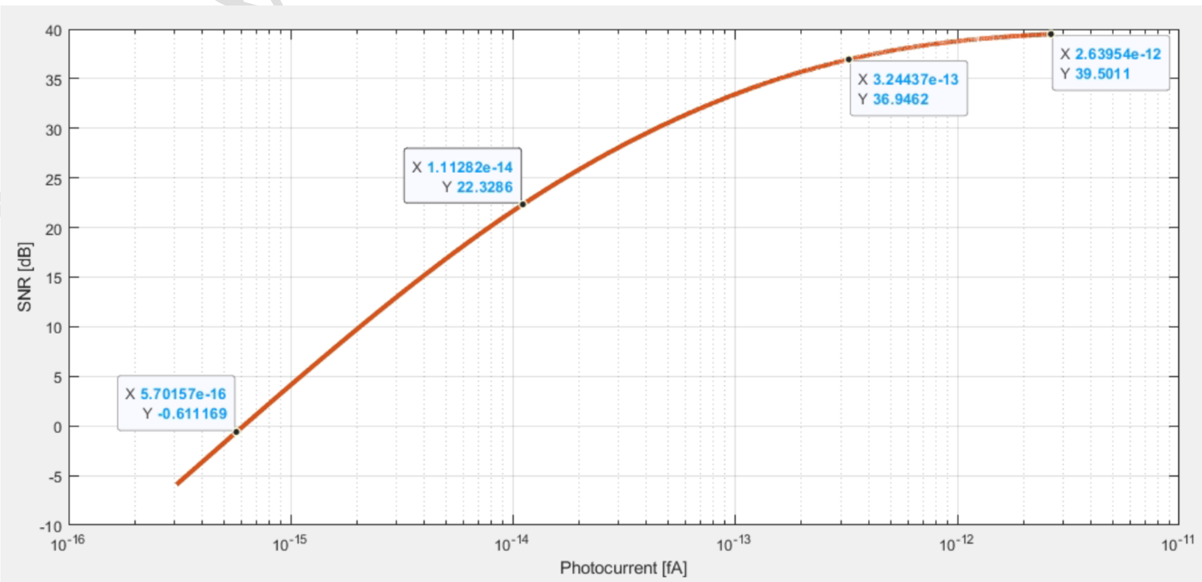
$$i_{ph,cross_2} = \left(\sigma_{kTC}^2 + \sigma_{shot,d}^2 + \sigma_{DSNU}^2 + \sigma_{quant}^2\right)\frac{q}{t_{int}} = 11.1\ fA$$

$$i_{ph,min} = \frac{q}{t_{int}}\sqrt{\sigma_{kTC}^2 + \sigma_{shot,d}^2 + \sigma_{DSNU}^2 + \sigma_{quant}^2} = 0.6\ fA$$

We already know that the SNR for the last situation is 0 dB. The other SNRs can be calculated using the well-known SNR formula, leading then to the graph below where we observe three regions.

$$SNR = 20\log_{10}\frac{i_{ph}t_{int}/q}{\sqrt{\sigma_{ADC}^2 + \sigma_{shot,d}^2 + \sigma_{kTC}^2 + \sigma_{DSNU}^2 + \sigma_{shot,ph}^2 + \sigma_{PRNU}^2}}$$

In the first one, the SNR grows linearly with the signal as noise is dominated by the four signal-independent sources. In the second one, the growth decreases in slope due to the appearance of shot noise. Finally, there is no longer any improvements, due to the dominant PRNU term.



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