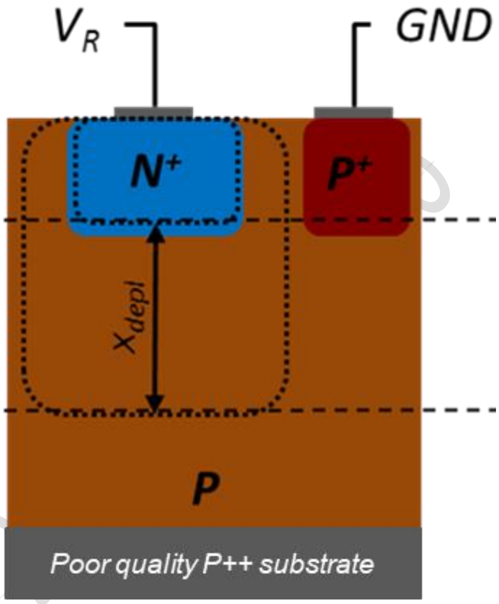
Last Name __Opelettro__ **Given Name** __Nicolò___ **ID Number** __20230206__

Question n. 1

Describe and sketch the basic cell forming a photodiode of a pixel in a 3T architecture. Indicate typical doping values and geometric dimensions, and motivate these numbers. Describe how photocarriers collection occurs in the pixel. Finally, sketch the electrical equivalent model of the photodiode, again indicating the typical values of the electrical parameters.



A 3T cell is based on a standard PN junction. In a planar CMOS process, the diode is usually formed as a N-type heavily-doped implant (10^{18} to 10^{20} cm^{-3}) on a high-quality, lowly-doped (10^{15} to 10^{16} cm^{-3}) P-type epitaxial layer. A P-plus implant is also present to provide a good ohmic biasing for the epitaxial layer. This active portion of the diode is usually grown on a poor-quality substrate.

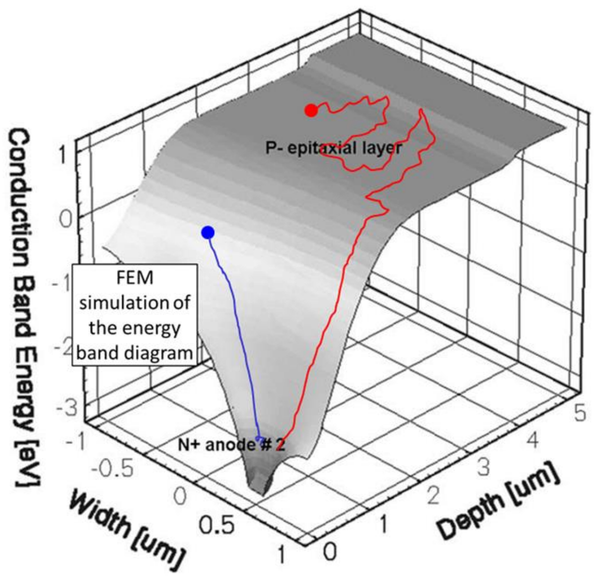
The diode area is usually a reasonable fraction (50% or so) of the total pixel area, which also includes the three transistors and interconnections. Overall, this implies a diode with a side in the order of 1 µm (mobile applications) or a few µm (and a larger FF, higher-end applications).

The epitaxial layer thickness is such that light in the visible range is mostly absorbed in the active portion. Given that "red" light (700 nm wavelength) has a penetration length of several µm, a 10-12 µm thickness is usually a good choice. On the other side, in order to avoid losses in the "blue" range (400 nm), the N-type implant is usually 100-nm deep, or even less.

In operation, the reverse bias depletes the junction by an amount given by

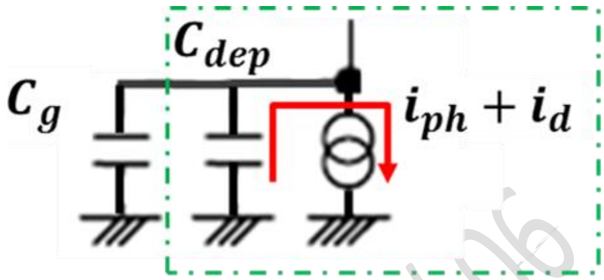
$$x_{dep} = \sqrt{\frac{2\epsilon_0\epsilon_{Si}(V_{rev}+V_{bi})}{q\ N_A}}$$

which usually, for a few V supply and the mentioned P-type doping, corresponds to 1 µm to 2 µm. This implies that collection by drift, i.e. driven by the electric field streamlines, occurs only for a relatively small portion of the photodiode volume, mostly corresponding to blue/green light, absorbed within the depletion region. All electrons generated at a depth larger than the depletion region, typically move by diffusion. In absence of a strong concentration gradient, this motion is almost random. The higher doping of the substrate is thus used to provide a small barrier that prevents carriers flow into the substrate. The high quality (low defects and impurities) of the epitaxial layer provides an electron lifetime long enough to



guarantee that the random motion by diffusion eventually brings the carriers to the depletion region, allowing their final collection by drift.

The photodiode electrical equivalent model includes essentially three elements: one is representative of the photogeneration of charge, and is thus implemented as a current generator; similarly, a second current generator in parallel to the first one (or just combined into a single generator, as shown aside), is representative of thermal carrier generation (so called dark current). Additionally, the PN-junction depletion capacitance can be sketched, which is in principle a function of the applied voltage (see again the formula of the depletion region):



PN-junction photodiode small-signal model

$$C_{dep} = \frac{\epsilon_0 \epsilon_{Si} A_{pd}}{x_{dep}}$$

In parallel to this capacitance, though not strictly part of the photodiode modelling, we usually have another capacitance which is representative of the gate of the source follower transistor attached to the anode node and of the parasitic brought by interconnections.

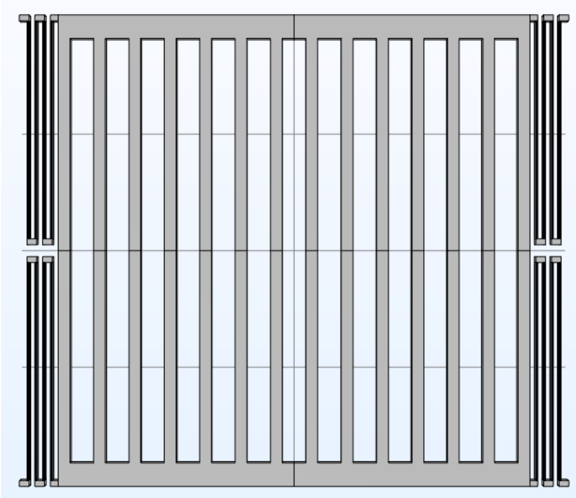
Typical values of the dark current are in the sub-fA range, obviously proportional to the pixel size. The photocurrent usually lies in the fractions of fA to pA range, depending both on the pixel area and the light amount. The depletion capacitance is itself in the fF or sub-fF range, depending on the geometrical parameters and supply voltage.

Last Name __Opelettro__ **Given Name** __Nicolò___ **ID Number** __20230206__

Question n. 2

You have to finalize the design of a MEMS accelerometer to match the reported specification, considering the parameters included in the table.

The accelerometer has nested parallel plates for the readout. The rotor is kept at the virtual ground of an operational amplifier, with the MEMS stators biased at ± 15 V:



C_f	$1\ pF$
gap	$2\ um$
N_{pp}	13
L_{pp}	$400\ um$
N_{fold}	4
h	$30\ um$
m	$30\ nKg$
C_{par}	$20\ pF$

Physical Constants

$\varepsilon_{Si} = 11.7 \cdot 8.85\ 10^{-12}$ F/m

$k_b = 1.38\ 10^{-23}$ J/K;

T = 300 K;

- **(i)** find the resonance frequency in operation, so to reach a target sensitivity of 240 mV/g and find the stiffness of each fold used in the design of the accelerometer;
- **(ii)** find the quality factor to avoid ringing of the mass longer than $0.25\ ms$ and display in a Bode plot the transfer function from the input acceleration to the output voltage of the sensing charge-amplifier;
- **(iii)** size the noise of the operational amplifier used in a CA configuration, considering an overall target noise of $18 \frac{ug}{\sqrt{Hz}}$.

(i)
We begin by calculating the rest capacitance of the sensing parallel plates as:

$$C_0 = \frac{\epsilon_0 \cdot h \cdot L_{pp} \cdot N_{pp}}{g} = 690\ fF$$

All the parameters required for the calculation of the electrostatic stiffness are know, and thus we evaluate:

$$k_{el} = -2 \cdot \frac{C_0}{g^2} V_{DD}^2 = -77.7 \frac{N}{m}$$

Given the target sensitivity in operation, we can evaluate the (tuned) resonance frequency as:

$$S = 2 \frac{C_0}{g} \frac{V_{DD}}{C_F} \frac{1}{\omega_0^2}\ 9.81 \frac{m/s^2}{g} = 240 \frac{mV}{g} \rightarrow \omega_0 = \sqrt{2 \frac{C_0}{g} \frac{V_{DD}}{C_F} \frac{9.81}{S}} \rightarrow f_0 = \frac{\omega_0}{2 \cdot \pi} = 3.27\ kHz$$

The mechanical stiffness can be then evaluated by subtracting to the overall stiffness the value of the (negative) electrostatic contribution:

$$k = \omega_0^2 \cdot m - k_{el} = 12.7 \frac{N}{m} - \left(-77.7 \frac{N}{m}\right) = 90.4 \frac{N}{m}$$

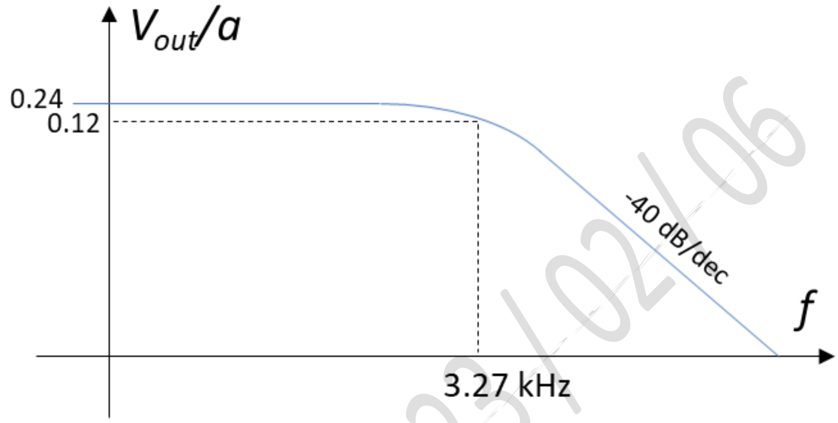
As this stiffness is distributed among four parallel springs, each formed by four folds in series, the stiffness of each fold $k_F = k\left(N_{fold}/N_{spring}\right)$ corresponds to the stiffness found above.

(ii)

We begin by relating the ringdown time of a 2-pole system to the value of the quality factor through the known relationship. We also assume that the ringdown time is five times the time constant τ:

$$\tau = \frac{t_{ring}}{5} = \frac{Q}{\pi f_o} \;\; \rightarrow \;\; Q = \frac{t_{ring}}{5}\pi f_o = 0.51$$

which is, by the way, an optimal value for an accelerometer. The transfer function has thus two real coincident poles. At low frequency it just corresponds to the sensitivity set above. After the poles, it goes down by 40 dB/decade, and no overshoot is found. At resonance it is exactly 6 dB (a factor 2) below the low-frequency value.



(iii)

The thermoemchanical noise contribution, in operation, is calculated as:

$$NEAD = \frac{\frac{\sqrt{4k_BTb}}{m}}{9.8} = \frac{\sqrt{\frac{4k_BT\omega_0}{mQ}}}{9.8} = 15\frac{\mu g}{\sqrt{Hz}}$$

Which thus sets the acceptable electronic noise contribution to the following value:

$$\sigma_{g,el} = \sqrt{\left(18\frac{\mu g}{\sqrt{Hz}}\right)^2 - \left(15\frac{\mu g}{\sqrt{Hz}}\right)^2} = 10\frac{\mu g}{\sqrt{Hz}}$$

This, in the end, allows calculating the operational amplifier voltage noise value as:

$$\sigma_{g,el} = \frac{\sqrt{S_{V_n}\left(1+\frac{C_P}{C_F}\right)^2}}{S} \;\; \rightarrow \;\; \sqrt{S_{V_n}} = \sigma_{g,el} \cdot \frac{S}{\left(1+\frac{C_P}{C_F}\right)} = 110\frac{nV}{\sqrt{Hz}}$$

Last Name __Opelettro__ Given Name __Nicolò___ ID Number __20230206__

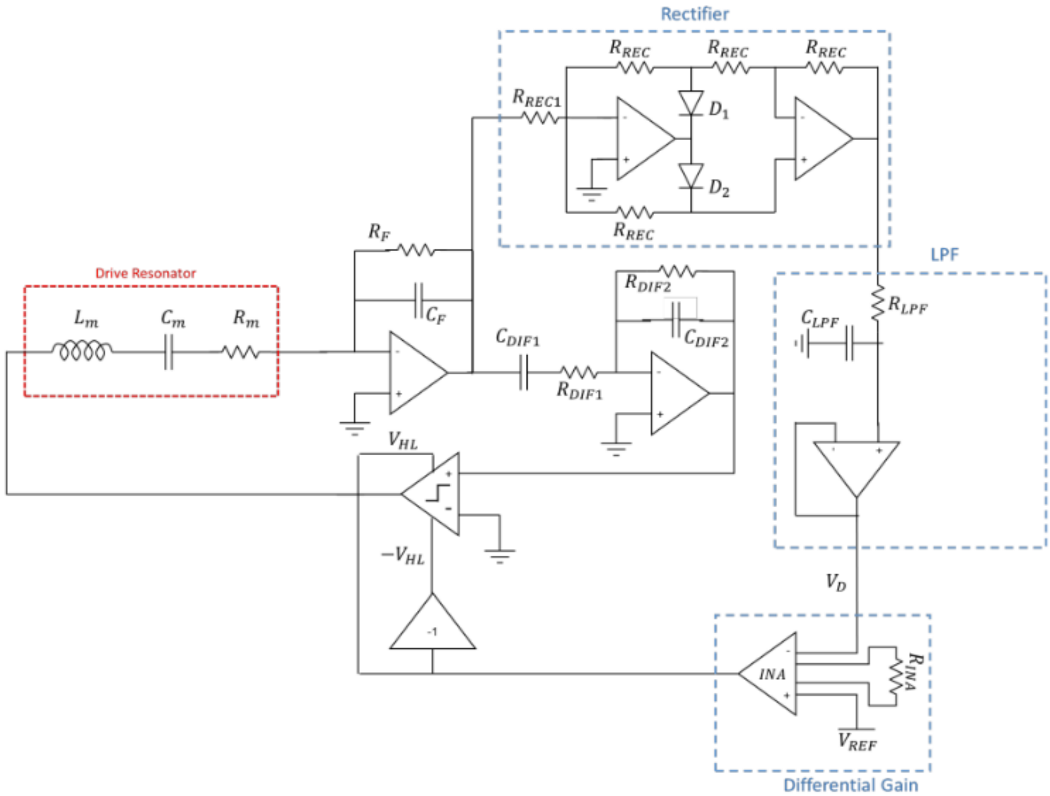
Last Name __Opelettro__ **Given Name** __Nicolò___ **ID Number** __20230206__

Question n. 3

Consider the drive loop of a mode-split tuning-fork MEMS gyroscope operating at a drive frequency of 25 kHz, as shown in the figure below. Given the parameters in the table:

R_{eq}	1.96 MΩ
C_{eq}	0.32 fF
L_{eq}	125 kH
V_{DD}	±5 V
$\Delta f = f_s - f_d$	600 Hz
x_d	10 um
FSR	±1000 dps
η_{da}	200 nN/V
η_{dd}	200 nA/(m/s)
R_{DIF2}	1 MΩ

- **(i)** optimize the charge amplifier passive components and the 90D passive components (target a unitary gain for the latter), and set the V_{ref} voltage to target a $10\ um$ drive displacement;
- **(ii)** evaluate the sensitivity (in [V/dps]); then, evaluate the $TC_{\Delta f}$ (temperature coefficient of the mode-split value, in ppm/K) and the corresponding sensitivity variation for a temperature increase of 40° C from room temperature (27° C);
- **(iii)** evaluate the drive quality factor (Q_d) at room temperature and express its relation with temperature; which is the maximum amplitude of the drive square wave, considering a variation of ±40° C?



Physical Constants

ε_0 = 8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

T = 300 K;

(i)

Optimisation of the front-end consists in setting the gain such that the full voltage dynamic is exploited, and the pole such that the stage operates effectively as a charge amplifier without introducing significant phase lags. Therefore, we set the two following constraints:

$$\begin{cases} \dfrac{\eta_{dd} \cdot x_d \cdot \omega_d}{C_F \cdot \omega_d} = V_{DD} \\ \dfrac{1}{2\pi C_F R_F} = \dfrac{f_d}{100} \end{cases} \rightarrow \begin{cases} C_F = \dfrac{\eta_{dd} \cdot x_d}{V_{DD}} = 400\ fF \\ R_F = \dfrac{1}{2\pi C_F (\frac{f_d}{100})} = 1.6\ G\Omega \end{cases}$$

For what concerns the second stage, its overall phase lag needs to bring the initial phase of -270° (referred to the drive signal) to -360°=0°, i.e. a shift of -90° is needed. Given the inverting nature of this amplifier configuration (-180°), we need to use it in its derivative portion, so setting the poles well beyond resonance. We thus set the following conditions on the gain and poles position:

$$\begin{cases} C_{D1} = \dfrac{1}{2\pi f_d R_{D2}} = 6.36\ pF \\ R_{D1} = \dfrac{1}{2\pi(100\ f_d)C_{D1}} = 10\ k\Omega \\ C_{D2} = \dfrac{1}{2\pi(100\ f_d)R_{D2}} = 63.6\ fF \end{cases}$$

Finally, in order to reach the target displacement, we set the value of the reference voltage to:

$$V_{ref} = \frac{\eta_{DD} x_d}{C_F}\frac{2}{\pi} = 3.18\ V$$

(ii)

Once the system is optimized, the sensitivity will be easily set at the voltage full-scale divided by the rate full-scale, so to 5V/1000dps = 5 mV/dps.

As the scale factor is proportional to the inverse of the mode-split value, the dependence on temperature can be evaluated as:

$$\frac{d(\Delta\omega_{MS})/_{d\mathrm{T}}}{\Delta\omega_{MS}} = \frac{d(\omega_S - \omega_D)/_{d\mathrm{T}}}{\omega_{S,0} - \omega_{D,0}} = \frac{d\left(\omega_{S,0}(1+\alpha\Delta T) - \omega_{D,0}(1+\alpha\Delta T)\right)/_{d\mathrm{T}}}{\omega_{S,0} - \omega_{D,0}}$$

$$= \frac{(\omega_{S,0} - \omega_{D,0})\, d\left(1+\alpha(T-T_0)\right)/_{d\mathrm{T}}}{\omega_{S,0} - \omega_{D,0}} = \alpha = -30\ ppm/K$$

$$\frac{\frac{dS}{S}}{dT} = -\frac{\frac{d(\Delta\omega_{MS})}{\Delta\omega_{MS}}}{dT} = 30\ ppm/K$$

For a temperature variation of 40°C, this corresponds to a scale factor change of

$$dS = 30\frac{ppm}{K}\ 5\frac{mV}{dps}\ 40\ K = 6\frac{\mu V}{dps}$$

(iii)

In order to evaluate the quality factor, we can easily pass through the electrical equivalent parameters, indeed:

$$Q_{d0} = \frac{\omega_d m_d}{b_d} = \frac{\omega_d L_{eq}\eta^2}{R_{eq}\eta^2} = \frac{\omega_d L_{eq}}{R_{eq}} = 10000$$

And we know that its dependence on temperature goes with the inverse square root as:

$$Q_d(T) = Q_{d0}\sqrt{\frac{T_0}{T}}$$

Last Name __Opelettro__ **Given Name** __Nicolò___ **ID Number** __20230206__

For a ±40 °C (260K to 340 K) change from ambient temperature we get $Q_d(260K) = 10761$ and $Q_d(340K) = 9410$, respectively.

The maximum amplitude of the square wave will thus occur when the Q is the minimum, i.e. for the 340 K situation. Here, we evaluate the amplitude of the drive square wave as:

$$x_d = v_d \frac{4}{\pi} \frac{Q_d(340K)}{k_d} \eta_{Da} = v_d \frac{4}{\pi} \frac{Q_d(340K)}{\eta_{da}^2 / C_{eq}} \eta_{da} \rightarrow \quad v_d = 0.52\,V$$