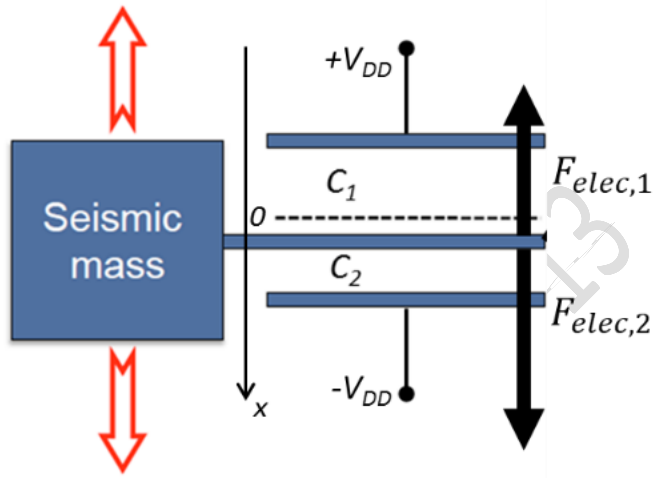
Last Name __MITRENT_______**Given Name** __ADAM_______ **ID Number** __20230113___

Question n. 1

Describe the electrostatic softening phenomenon for a MEMS based parallel-plate accelerometer, clearly indicating and commenting the parameters it depends on, and the associated risks. Describe the consequences of softening issues and the trade/offs that, in turn, arise on the accelerometer design.

The application of an unavoidable electrostatic force between the sensing plates and the rotor in a parallel-plate accelerometer gives rise to a softening effect, resulting from the calculations below. We note that this force is unavoidable, and though below we discuss the case of a constant DC voltage between plates, even in presence of a zero-mean AC voltage, as the force goes with the square of the voltage, a similar effect still arises.



Referring to the scheme aside, the force expression as below:

$$F_{elec} = \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\, N}{(g-x)^2} - \frac{V_{DD}^2}{2}\frac{\varepsilon_0 A\, N}{(g+x)^2}$$

can be simplified through linearization, included in the quasi-stationary expression of the spring mass damper system, and leads to the following result:

$$kx = \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{1}{1-2x/g} - \frac{1}{1+2x/g}\right] = \frac{V_{DD}^2}{2}\frac{C_0}{g}\left[\frac{4x/g}{1-(2x/g)^2}\right] \approx 2V_{DD}^2\frac{C_0}{g^2}x$$

This indicates that a force proportional to the displacement is generated not only by the elastic restoring but also, and with opposite sign, by the electrostatic forces. As a consequence, this is equivalent to a lower overall stiffness, a phenomenon known as electrostatic softening. The effect, as shown by the equation, has a strong dependence on the biasing voltage and on the gap (overall, $1/g^3$).

There is one major risk: as soon as the sizing is such that the equivalent electrostatic stiffness, $-2V_{DD}^2\frac{C_0}{g^2}$, overcomes in modulus the elastic stiffness k_{mec}, the structure will be subject to mechanical instability and the rotor will tend to collide onto either of the parallel plates (with short circuit in between being limited by the presence of mechanical stoppers). This occurs for a biasing voltage value, known as pull-in voltage, equal to:

$$V_{DD,PI} = \sqrt{\frac{g^3\ k_{mec}}{2\ \varepsilon_0 A\, N}}$$

Indeed, mathematically speaking, for such a value, the full equation written above loses any stability points. This maximum voltage may be even lowered when the maximum impinging acceleration occurs, indicating that safe margins need to be taken from that value.

There are numerous comments that can be made about the effects that softening generates in the optimal design of an accelerometer, when taking also into account the expression of the sensitivity:

$$\frac{\Delta V_{out}}{a_{ext}} = 2\frac{dV}{dC}\frac{dC}{dx}\frac{dx}{da} = 2\frac{V_{DD}}{C_f}\frac{C_0}{g}\frac{m}{\left(k_{mec} - 2V_{DD}^2\frac{C_0}{g^2}\right)}$$

- a small gap enhances the sensitivity… but it is unfavourable for pull-in issues…
- a large mass enhances the sensitivity… but take care of bandwidth and area limits…
- a small overall stiffness enhances the sensitivity… but it is unfavourable for pull-in issues and max bandwidth…
- a large bias voltage enhances the sensitivity… but it facilitates pull-in and is limited by the consumption of the IC…
- the in-operation resonance frequency is decreased from the nominal design value, as it shall be now written as $\boldsymbol{\omega_0} = \sqrt{\frac{\boldsymbol{k_{mec}}+\boldsymbol{k_{elec}}}{\boldsymbol{m}}}$
- linearity is limited due to the increased electrostatic forces at decreasing gaps, so under large accelerations.

One can conclude that, due to such effects, it is not easy to enhance the sensitivity in PP axels without acting on the process.

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Question n. 2

An imaging pixel for high temperature applications is characterized, at a short integration time, through the photon transfer curve (PTC) shown below in terms of digital numbers (DN), with separate contributions of photon shot noise and FPN identified through different experiments. Assuming a unitary quantum efficiency:

(i) calculate the gain K from input photons to digital number and the number of bits of the ADC;

(ii) calculate % PRNU noise, quantization noise, and read noise in electrons rms. Then calculate the full well charge in electrons rms, the dynamic range in dB;

(iii) calculate the maximum SNR, in dB, with two different approaches;

(iv) finally, estimate the integration capacitance and the maximum pixel voltage swing (assume to operate at 600 K).

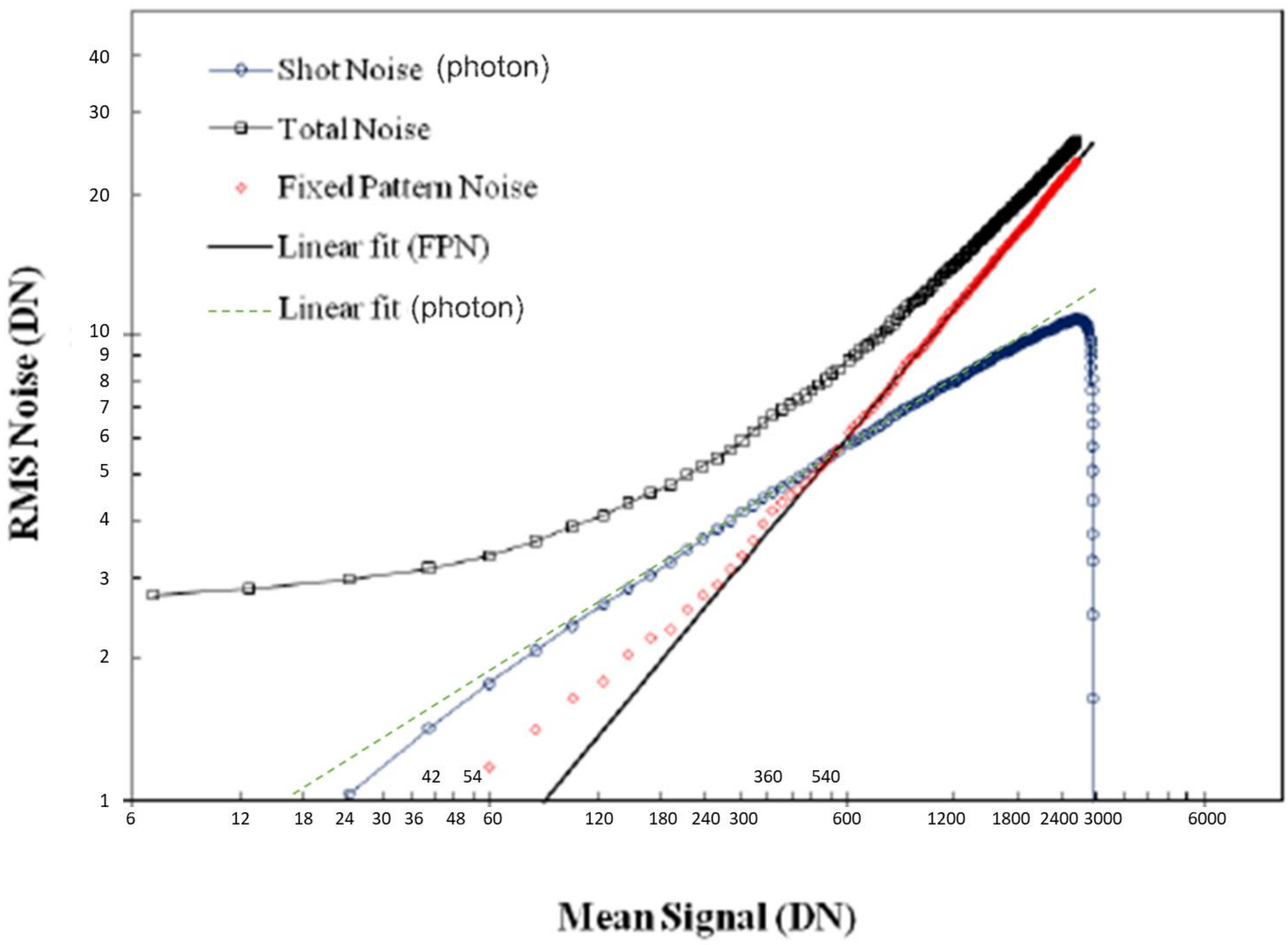
Physical Constants

$\varepsilon_{Si} = 11.7 \cdot 8.85\ 10^{-12}$ F/m

$k_b = 1.38\ 10^{-23}$ J/K;

$q = 1.6\ 10^{-19}$ C;

$T = 600$ K;



(i)

According to the PTC relationship, the gain K shall be calculated looking at the region where shot noise dominates. In this specific graph, there is not such a region completely dominated by photon shot noise, but we can do even better, as we directly have the individual photon shot noise contribution (in blue circles). We find that:

$$K = \frac{\sigma_B^2}{B} = \frac{\sigma_B^2}{B}(\sigma_b = 1) = \frac{1}{17} = 0.06\,\frac{\text{DN}}{\text{ph}}$$

Additionally, as the maximum signal level is around 3000, assuming a well designed ADC we can expect that the maximum number of levels is the power of 2 just above this number, i.e.:

$$N_{bit} \geq \log_2 3000 = 11.55 \ \rightarrow \ \ 12\ bit$$

(ii)

Looking now at the portion of the PTC dominated by PRNU, we can evaluate (in any point, but I'll choose the one with noise equal to 1) that:

$$\sigma_{PRNU,DN} = \sigma_{PRNU,\%} \cdot DN \ \rightarrow \ \ \sigma_{PRNU,\%} = \frac{\sigma_{PRNU,DN}}{DN} = \frac{1}{80} = 1.25\%$$

It is also easy to evaluate quantization noise, as considering 1 DN as the LSB we direclty get:

$$\sigma_{quant,DN} = \frac{1DN}{\sqrt{12}} = 0.29\ DN$$

As this is negligible w.r.t. read noise in the flat region, and as the sensor is working at short integration times, we can expect that the dominant noise source at low signal sis kTC noise and thus:

$$\sigma_{kTC,DN} = 2.8\ DN$$

We can turn this noise terms into electrons rms through the gain K, so to achieve:

$$\sigma_{quant} = \frac{\sigma_{quant,DN}}{K} = 4.6\ e^-_{rms}$$

$$\sigma_{kTC} = \frac{\sigma_{kTC,DN}}{K} = 44.8\ e^-_{rms}$$

Given the full well charge of 3000 DN and the minimum noise of 2.8 DN, the DR is easily calculated to be around 60 dB. Given the unitary quantum efficiency, the FWC expressed in electrons is just calculated as:

$$FWC = \frac{FWC_{DN}}{K} = \frac{3000\ DN}{0.06\ DN/ph} = 48000\ ph = 48000\ e^-$$

(iii)

The maximum SNR can be calculated graphically just before saturation of the black curve as:

$$SNR_{max} = 20\log_{10}\frac{2700}{28} = 39\ dB$$

As an alternative, as we know that PRNU noise dominates, the maximum SNR can be expressed as:

$$SNR_{max} = 20\log_{10}\frac{DN_{max}}{\sigma_{PRNU,\%} \cdot DN_{max}} = 20\log_{10}\frac{1}{\sigma_{PRNU,\%}} = 38\ dB$$

Results from the two techniques are rather consistent.

(iv)

Finally, from the kTC value we ca evaluate the capacitance as:

$$\sigma_{kTC} = \frac{\sqrt{kTC}}{q} \rightarrow \ \ C = \ 6.2\ fF$$

Once C is known, it is easy to calculate the voltage swing from the maximum number of electrons:

$$V_{DD} = \frac{FWC \cdot q}{C} = 1.2\ V$$

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Question n. 3

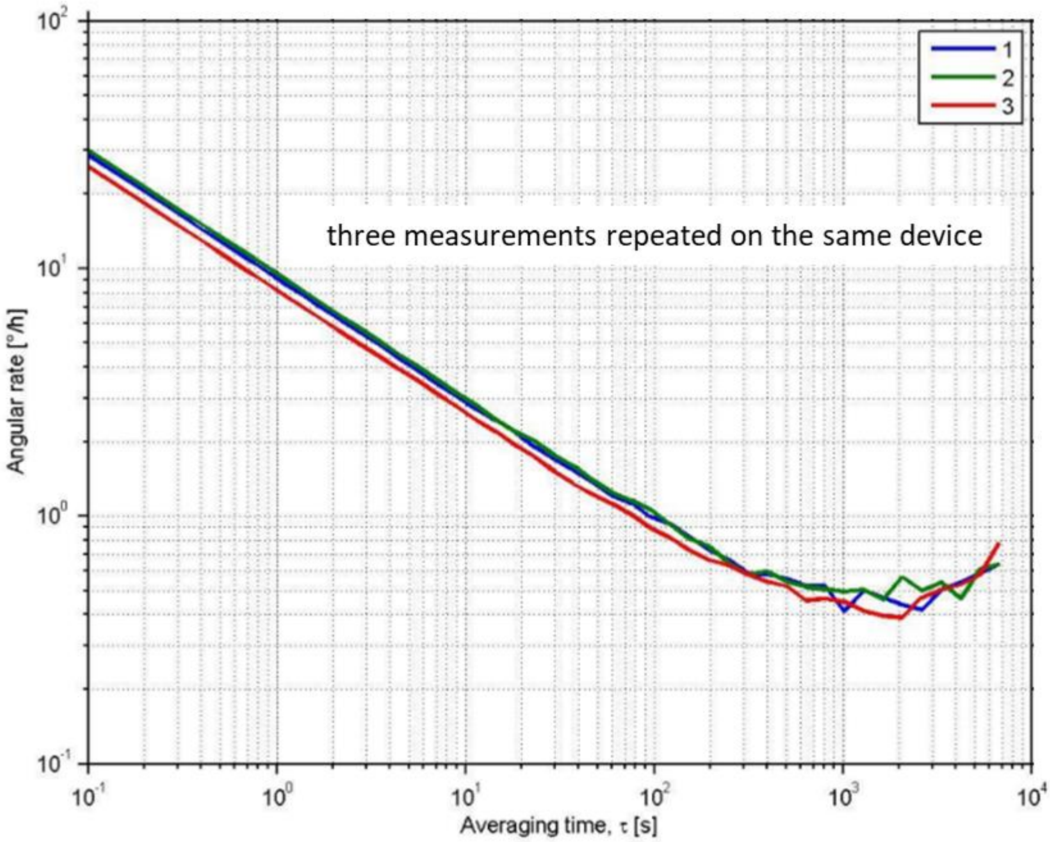
A high-performance z-axis gyroscope is characterized through the Allan variance technique, with the results shown in the figure (note that the vertical axis is in °/h, with 1°/h = 1/3600 dps):

(i) find the input referred white noise, the input referred 1/f noise, and plot a power spectral density, in dps/√Hz, of the gyroscope output, from 0.1 mHz to 100 Hz;

(ii) the gyroscope is used for navigation purposes in experiments with a duration of 300 s. Indicate the typical angle error obtained at the end of such experiments;

(iii) given the parameters in the table, verify whether white noise is given by the sensor or the electronics.

Resonance frequency	25 kHz
Supply	0 - 3 V
Full-scale range	±3000 dps
Amplifier voltage noise	20 nV/√Hz
Feedback capacitance	250 fF
Parasitic capacitance	10 pF
INA gain	1
Sense damping (1/2 structure)	$8\cdot10^{-6}$ kg/s
Drive displacement	4 µm
Sense mass (1/2 structure)	5 nkg



Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$k_b = 1.38\ 10^{-23}$ J/K;

T = 300 K;

(i)

White noise is related to the angle random walk that can be easily estimated on an Allan variance graph by looking at the y-axis value of the point at 1s on the portion of the curve with a -10 dB/dec slope. In this specific case, this yields about 9 °/h = 2.5 mdps. The white noise value is found then as:

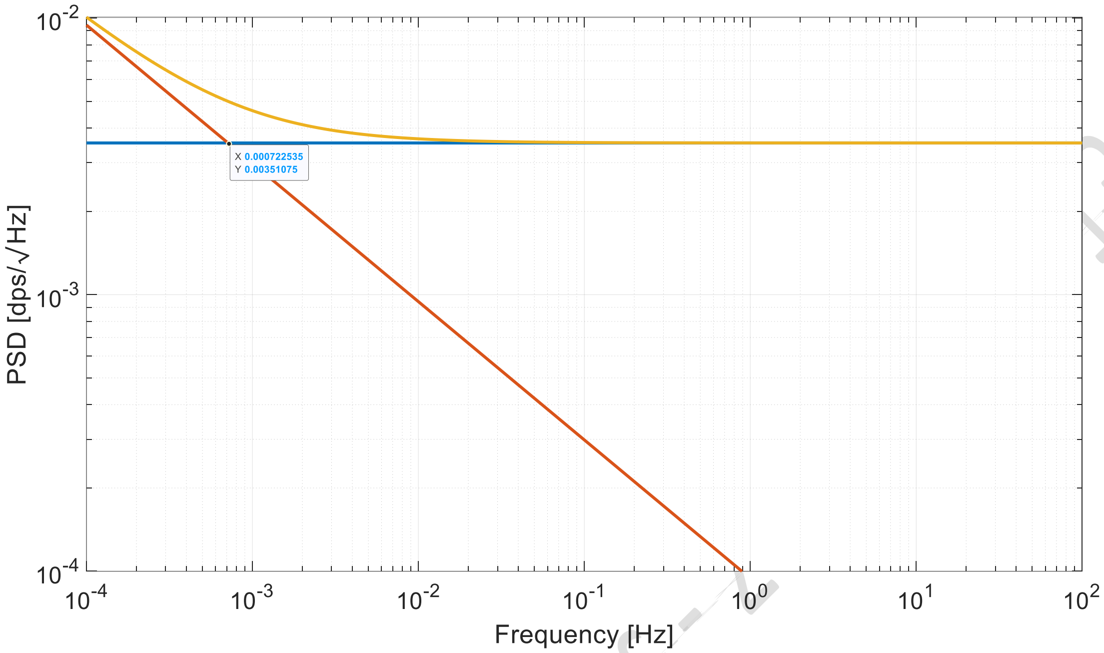
$$\sigma_\Omega = 2.5\ \frac{mdps}{\sqrt{Hz}}\sqrt{2} = 3.5\ \frac{mdps}{\sqrt{Hz}}$$

1/f noise is instead related to the flat part of the Allan variance graph, here corresponding to about 0.4 °/h = 111 µdps: the 1/f noise coefficient becomes:

$$\alpha_n = \frac{(0.4\ \mu dps)^2}{2\cdot\ln 2} = 8.9\cdot10^{-9}\ s^2$$

The derived PSD can be thus written and graphed as:

$$\sqrt{S_\Omega} = \sqrt{\left(3.5\frac{mdps}{\sqrt{Hz}}\right)^2 + \frac{8.9 \cdot 10^{-9}\, s^2}{f}}$$



(ii)

For an observation interval of 300 s we find a typical noise of 0.6 °/h corresponding to about 165 μdps. This simply means that after 300 s, the most probable error in angle estimation is:

$$\epsilon_\theta = \sigma_{dps}(300\, s) \cdot 300\, s = 0.05°$$

(iii)

The expression of noise from the thermomechanical and the electronics domains are reported below:

$$\sigma_{dps,MEMS} = \frac{1}{\sqrt{2}}\frac{1}{x_d f_0 m_s}\sqrt{k_B T b_s}$$

$$\sigma_{dps,AMPL} = \frac{\sqrt{2S_{V_n}}\left(1+\frac{C_P}{C_F}\right)}{SF}$$

Once the scale factor (*SF*) is calculated as V_{DD}/FSR, all other parameters are known from the table and yield 2.3 mdps/√Hz for both the contributions, indicating that the system is well balanced, and that the quadratic sums yields 3.3 mdps/√Hz, a value quite close to the one observed in the root Allan variance.