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Question n. 1

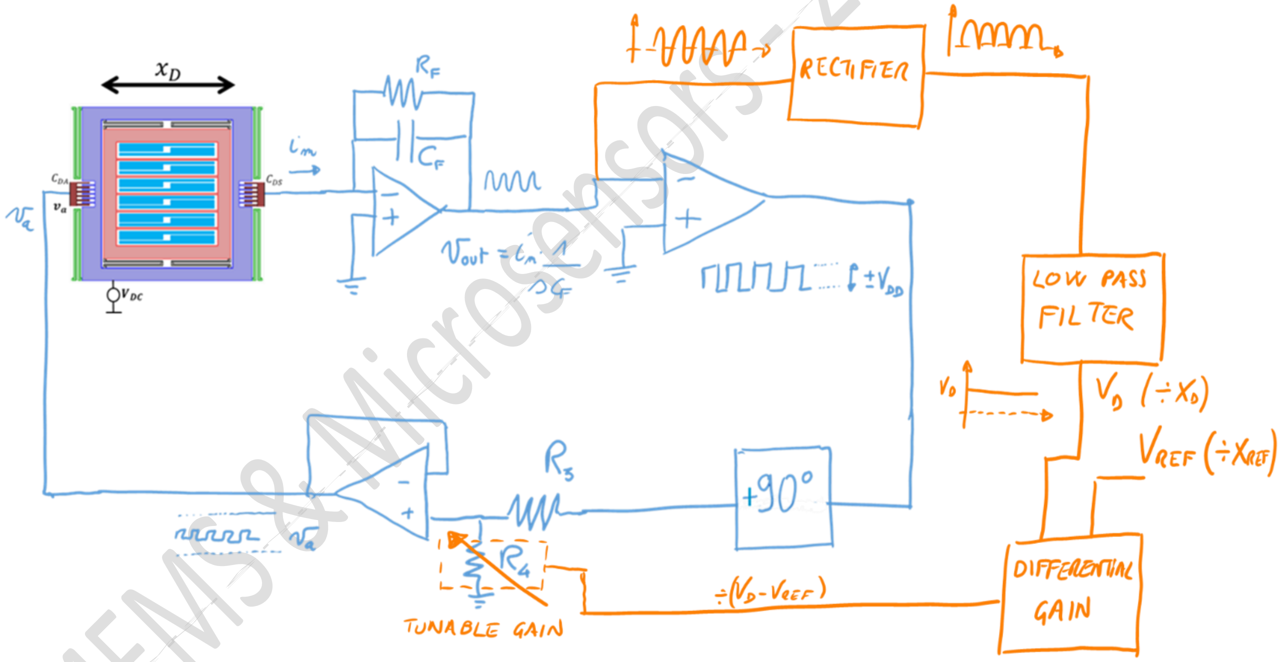
Define the Barkhausen criteria for oscillation and describe how the oscillation start-up occurs. Then, describe and plot the electronic block scheme of a charge-amplifier based drive oscillator of a gyroscope (clearly indicating the +/- inputs of the amplifiers). Finally, indicate how the passive components of the front-end amplifier feedback shall be chosen.

The regime conditions for stable oscillation in a harmonic oscillator at a frequency ω_0 require that the loop gain of the system equals 1 in modulus and 0° in phase, meaning that a sinusoidal waveform comes back to the same point in the loop, after one turnaround, without changes:

$$\begin{cases} |G_{loop}(\omega_0)| = 1 = 0\ dB \\ \phi[G_{loop}(\omega_0)] = 0° \end{cases}$$

At the start-up, however, for the oscillation to grow up to the desired amplitude, the gain is required to be larger than 1, so that, among all noise components in the system, the harmonic component at ω_0 begins growing, up to the regime condition, reached after a nonlinearity adjusts the gain to 1.

$$\begin{cases} |G_{loop}(\omega_0)| > 1 = 0\ dB \\ \phi[G_{loop}(\omega_0)] = 0° \end{cases}$$



A sample drive loop of a gyroscope is sketched, accordingly, in the figure above, and has to satisfy the gain and phase conditions. The overall phase lags are given by the MEMS (0° at resonance), the charge amplifier (-270°) between motional current and output voltage, the inverting comparator (-180° due to inversion), a differentiator (+90°) and a non-inverting buffer (0°). The loop gain is provided by the ratio of the feedback impedance and the MEMS motional resistance ($1/sC_FR_{eq}$), the high-gain of the comparator (a high-gain stage can be equivalently used), the differentiator gain (usually set close to 1) and the voltage divider de-gain (used to limit the applied signal at the driving port).

As a consequence, the passive components shall be chosen such that:

- At the front-end amplifier, the gain ($1/sC_F$) is ideally large enough to cover the voltage swing at the amplifier output (if this is allowed by available capacitances in the used process);

- the resistance sets the pole position, which ideally shall be set at least two decades before the resonance frequency (usually around 20 kHz, thus at about 200 Hz), so that the undesired phase lags are minimized.

As the drive loop goal in a gyroscope is to maintain a stable oscillation at a reference amplitude, an AGC is added such that a signal proportional to the displacement (e.g. the charge amplifier output) is rectified, filtered, and compared to a target reference with a negative loop, as indicated in orange in the figure.

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Question n. 2

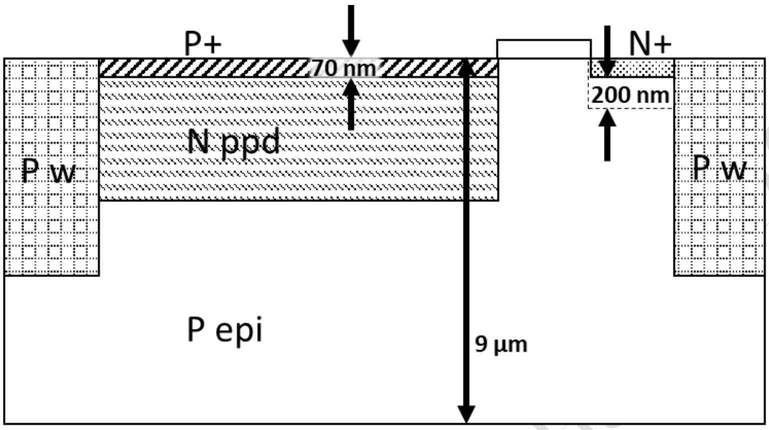
A pinned photodiode is used in a 4T topology at an integration time of 20 ms, with the additional parameters as from the table:

(i) verify that the quantum efficiency equals 0.82 for the provided absorption coefficient, then calculate the responsivity at 555 nm and the conversion gain;

(ii) list all noise sources in terms of electrons rms, and suitably choose the n. of bits of the ADC, assuming that an ideal correlated double sampling occurs.

(iii) with the calculated photocurrent, draw a graph of the SNR vs the integration time, quoting the maximum integration time and all relevant points.

Absorption coefficient	$0.2 \cdot 10^6\ m^{-1}$
P epitaxy thickness	9 µm
Supply voltage	5 V
N+ implant area	$(0.5\ \mu m)^2$
N+ implant depletion	0.2 µm
P+ implant depth	70 nm
Gate capacitance	0.2 fF
Dark current density	$0.02\ fA/\ \mu m^2$
Pixel size	$(2.5\ \mu m)^2$
Fill factor	0.8
Optical power input	10 fW
PRNU percentage	2.5%
DSNU percentage	8.5%



Physical Constants

$\varepsilon_{Si} = 11.7 \cdot 8.85\ 10^{-12}$ F/m
$k_b = 1.38\ 10^{-23}$ J/K;
$q = 1.6\ 10^{-19}$ C;
T = 300 K;
$h = 6.63\ 10^{-34}$ Js;
$c = 3\ 10^8$ m/s;

(i)

The quantum efficiency is limited by absorption in regions where photogenerated carriers recombine before reaching the pinned photodiode region. These are the surface, pinning, P+ implant, whose high doping level sets a low carriers lifetime, and the substrate, where the photogenerated carriers recombine due to the low quality of silicon and presence of impurities. As intensity of light travelling through a mean propagates as:

$$I(x) = I(0)e^{-\alpha(\lambda)\cdot x}$$

with $\alpha(\lambda)$ being the absorption coefficient and x the traveling distance, we know that the percentage of absorbed liggt in the useful region over the total impinging light, i.e. the quantum efficiency, is:

$$\eta = \frac{I(0)e^{-\alpha(\lambda)\cdot 70nm} - I(0)e^{-\alpha(\lambda)\cdot 9\mu m}}{I(0)} = e^{-\alpha(\lambda)\cdot 70nm} - e^{-\alpha(\lambda)\cdot 9\mu m} = 0.986 - 0.165 = 0.82$$

The responsivity is related to the quantum efficiency through the energy of photons as:

$$\mathcal{R} = \frac{\eta \cdot q}{hc/\lambda} = \frac{0.82 \cdot 1.610^{-19}\ C \quad 555\ nm}{6.626\ 10^{-34}\ Js \quad 3\ 10^8 m/s} = 0.367\ A/W$$

And finally the conversion gain is found as the electron charge divided by the sum of the capacitances affecting the N-plus region, i.e.

$$CG = \frac{q}{C_{tot}} = \frac{q}{C_{dep_{N^+}} + C_g} = \frac{q}{\frac{\epsilon_{Si} A_{N^+}}{x_{dep_{N^+}}} + C_g} = \frac{q}{0.13\ fF + 0.2 fF} = \frac{1.6\ 10^{-19}\ C/e^-}{0.33 fF} = 486 \frac{\mu V}{e^-}$$

(ii)

After calculating the photocurrent right from the responsivity and the optical input power P, and the dark current gathered by the pinned photodiode:

$$i_{ph} = \mathcal{R} \cdot P = 3.67\ fA$$

$$i_d = j_d A_{pix_{FF}} = 0.1\ fA$$

(note: I am assuming here that the opticla input is meant as the one impinging on the pinned photodiode area. If you assume that is impinging on the all pixel, the FF shall be taken into account. Both solutions are fine)

and assuming that reset noise is perfectly canceled, all other noise contributions are listed below:

$$\sigma_{N_{el}, i_{ph}} = \frac{\sqrt{q i_{ph} t_{int}}}{q} = 21.4\ e^-_{rms}$$

$$\sigma_{N_{el}, i_d} = \frac{\sqrt{q i_d t_{int}}}{q} = 3.54\ e^-_{rms}$$

$$\sigma_{N_{el}, PRNU} = \frac{i_{ph} t_{int} \cdot PRNU_{\%}}{q} = 11.46\ e^-_{rms}$$

$$\sigma_{N_{el}, DSNU} = \frac{i_d t_{int} \cdot DSNU_{\%}}{q} = 1.1\ e^-_{rms}$$

To size the ADC, we have to take into account that signal is not always present: so, in order not to worsen the performance, we shall size the number of bits considering only signal independent noise sources:

$$\sigma_{N_{el}, sig-indep} = \frac{\sqrt{q i_d t_{int} + (i_d t_{int} \cdot DSNU_{\%})^2}}{q} = 3.7\ e^-_{rms}$$

And thus we choose the bit number according to:

$$\frac{LSB_{N_{el}}}{\sqrt{12}} \leq 3.7\ e^-_{rms} \rightarrow \frac{V_{DD}\ C_{tot}}{2^{N_{bit}} \sqrt{12}} \leq q \cdot 3.7\ e^-_{rms} \rightarrow\ \ N_{bit} = \log_2 \frac{V_{DD}\ C_{tot}}{q\ \ 3.7\ e^-_{rms} \sqrt{12}} = 9.6 \rightarrow\ \ N_{bit} = 10$$

(iii)

As there is no reset noise, the only noise contributions that does not depend on integration time is quantization noise. As long as it dominates (only at very low integration times), the SNR grows with 20 dB/dec.

Once shot noise dominates (in this case photon-induced shot is much larger than dark-induced one), the SNR keeps growing with 10 dB/dec. As soon as contributions proportional to the integration time are dominnat (PRNU and DSNU) the SNR grows no longer. This also indicates why the maximum SNR does not match the square root of the maximum number of carriers, because at the longest integration time photon shot noise is not the dominant noise source.

Calculations of relevant points in the graph are given below:

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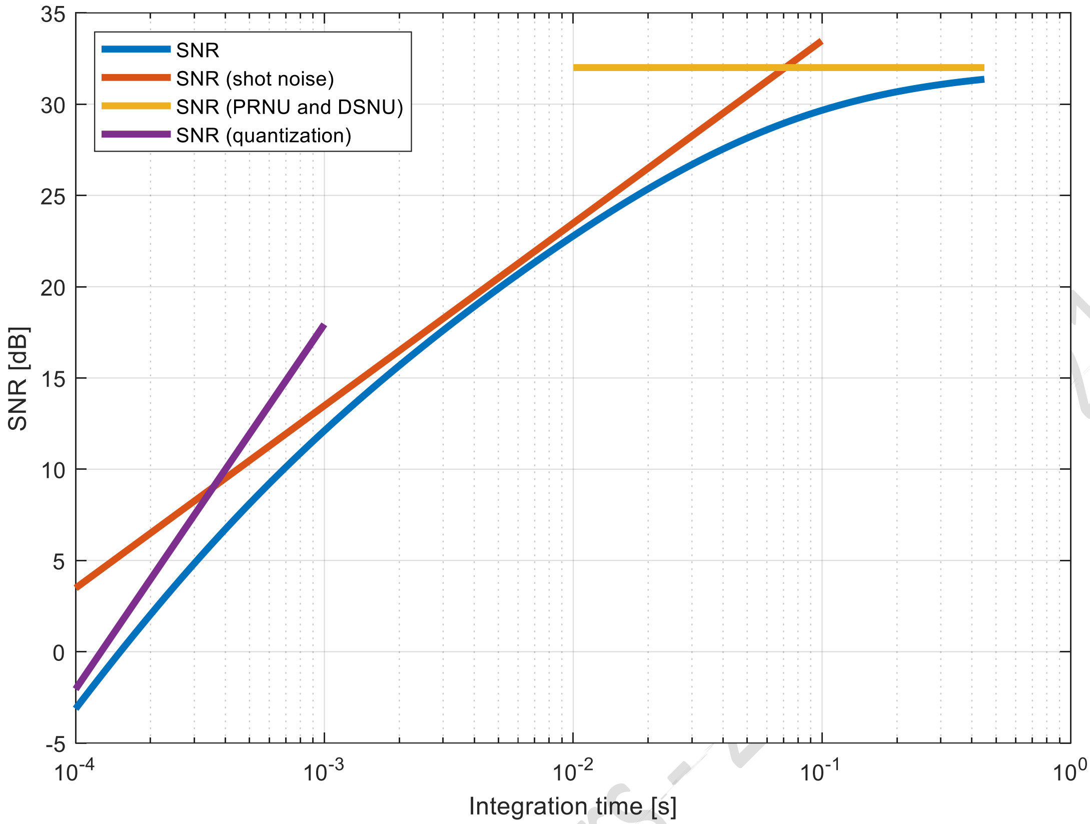

- cross-over time between quantization and shot:

$$\frac{V_{DD}\,C_{tot}}{2^{N_{bit}}\sqrt{12}} = \sqrt{q i_{ph} t_{int}} \;\rightarrow\; t_{int} = 367\ \mu s$$

- cross-over time between shot and PRNU:

$$\sqrt{q i_{ph} t_{int}} = i_{ph} t_{int} PRNU_{\%} \;\rightarrow\; t_{int} = \frac{q}{i_{ph}\ PRNU_{\%}^2} = 69.8\ ms$$

- maximum integration time:

$$i_{ph} t_{int} = V_{DD} C_{tot} \rightarrow\; t_{int} = \frac{V_{DD} C_{tot}}{i_{ph}} = 449\ ms$$

The maximum SNR is dominated by PRNU and thus reads:

$$SNR_{max} = 20 \log_{10} \frac{V_{DD} C_{tot}}{i_{ph} t_{int_{max}}\ PRNU_{\%}} = 32\ dB$$

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Question n. 3

Consider a comb-finger based MEMS accelerometer, where the rotor voltage is modulated with an AC signal at 40 times the MEMS resonance frequency, and the readout occurs through a pair of charge amplifiers. Given the parameters in the table:

(i) calculate the rest capacitance of each stator and the differential sensitivity at the charge amplifiers output;

(ii) calculate the maximum acceptable parasitic capacitance such that the system is well balanced in terms of noise.

Resonance frequency	2.5 kHz
Process thickness	40 µm
Comb finger gap	1.6 µm
Comb finger overlap	8 µm
N. of fingers per stator	68
Rotor mass	5 nkg
Rotor AC peak voltage	1 V
Feedback capacitance	200 fF
Feedback resistance	1 GΩ
Damping coefficient	10^{-5} kg/s
Amplifier noise	5 nV/√Hz

Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$k_b = 1.38\ 10^{-23}$ J/K;

T = 300 K;

(i)

As this is a comb-based accelerometer, the formula for the capacitance is given as:

$$C_0 = \frac{2\epsilon_0 N_{CF} h(L_{ov} + x)}{g}$$

Which at rest (x=0) yields 241 fF per stator.

The differential sensitivity at the amplifier output, assuming the positive input of the amplifier biased to ground, and a peak signal of amplitude V_{DD}=1V applied at the rotor, can be written as:

$$S = \frac{\Delta V_{out}}{\Delta a} = 2\frac{V_{DD}}{C_F}\frac{C_0}{L_{ov}}\frac{1}{\omega_0^2} = 2\frac{1}{200fF}\frac{241fF}{8\ \mu m}\frac{1}{(2\pi\ 2.5\ kHz)^2} = 1.2\frac{mV}{m/s^2} = 12\frac{mV}{g}$$

Note again that this is a comb-based accelerometer, thus (a) we have a different transduction factor (C_0/L_{ov} instead of C_0/g of parallel plate solutions) and (b) we do not have electrostatic tuning effects. Additionally, the fact that we are modulating the signal around 100 kHz does not change the sensitivity formula as this only modulates the current term proportional to the voltage derivative, instead of the one proportional to the capacitance derivative.

(ii)

We first calculate thermomechanical and feedback resistor noise contributions, input referred in g units as accelerations:

$$\sigma_{a,MEMS} = \frac{\sqrt{4k_B T b}}{m}\frac{1}{9.8\frac{\frac{m}{s^2}}{g}} = 8.3\frac{\mu g}{\sqrt{Hz}}$$

$$\sigma_{a,RES} = \frac{\sqrt{2\cdot\frac{4k_B T}{R_F}}\frac{1}{(2\pi\ 100\ kHz\ C_F)}}{S} = 3.8\frac{\mu g}{\sqrt{Hz}}$$

So that the available noise budget of the amplifier for a balanced system (electronic noise equals mechanical noise) is:

$$\sigma_{a,AMPL} = \sqrt{\left(8.3 \frac{\mu g}{\sqrt{Hz}}\right)^2 - \left(3.8 \frac{\mu g}{\sqrt{Hz}}\right)^2} = 7.3 \frac{\mu g}{\sqrt{Hz}}$$

By equating the quantity above to the amplifier noise, we found out the maximum acceptable parasitic capacitance:

$$\sigma_{a,AMPL} = \frac{\sqrt{2\, S_{V_n}}\left(1 + \frac{C_P}{C_F}\right)}{S} \approx \frac{\sqrt{2\, S_{V_n}}\, \frac{C_P}{C_F}}{S} \quad \rightarrow \quad C_P = \frac{\sigma_{a,AMPL}\, S\, C_F}{\sqrt{2\, S_{V_n}}} = \frac{7.3 \frac{\mu g}{\sqrt{Hz}} \cdot 12 \frac{mV}{g}\ 200 fF}{\sqrt{2}\, 5 \frac{nV}{\sqrt{Hz}}} = 2.5\, pF$$

As this value is much larger than the feedback capacitance, the assumption of neglecting the (1+…) term is reasonably valid.

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