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Question n. 1

Write the detailed signal-to-noise ratio expression for pixels of a 3T imaging sensors. Then, draw a realistic quoted graph of the SNR vs the quoted photocurrent.

Discuss how spatial noise sources can be compensated and, finally, assuming that FPN sources are eliminated, redraw the same SNR graph.

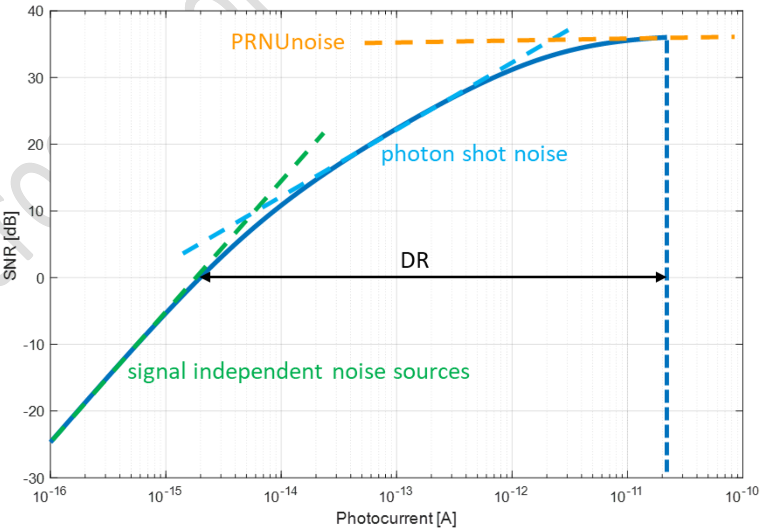
The signal to noise ratio of a 3T topology includes contributions from reset noise, shot noise of both dark and signal current, quantization noise, PRNU and DSNU. It can be written as a ratio of signal charge to rms noise charge (quadratic sum) as:

$$SNR = 20 \cdot log_{10}\left[\frac{i_{ph} \cdot t_{int}}{\sqrt{q(i_{ph}+i_d)\cdot t_{int} + k_B T C + \left[(C_g + C_{fd})\frac{V_{DD}}{2^{N_{bit}}\sqrt{12}}\right]^2 + \left(i_d\, t_{int}\frac{\sigma_{DSNU,\%}}{100}\right)^2 + \left(i_{ph}\, t_{int}\frac{\sigma_{PRNU,\%}}{100}\right)^2}}\right]$$

We observe how for small signal values the signal-dependent terms (signal shot noise and PRNU) are negligible and thus the SNR is proportional to the signal (+20 dB/dec in a log log plot). As the signal increases, the signal shot noise begins to appear with a change in the curve slope, now proportional to the square root of the photocurrent. At even larger signals, PRNU becomes the dominant noise terms and the SNR grows no longer.

Typical photocurrent values for a 3T pixel of a few microns size range between fractions of fA to a few pA. Correspondingly, the SNR will grow up to about 40-45 dB – corresponding to the maximum SNR (after these values, the human eye does not perceive any longer differences in SNR).

A sample graph is thus as reported aside.

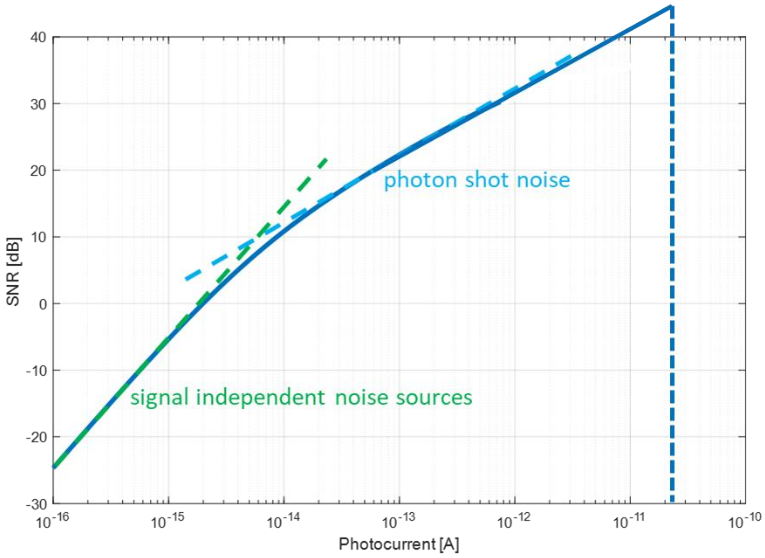


Spatial noise sources can be compensated by using a simple technique: if one takes the same measurement several times and averages the result, temporal noise will be reduced while spatial noise (which is, in practice, the effect of offsets or gain nonuniformities) will remain well visible:

- when the measurement is performed in dark, the remaining value at each pixel output will represent an *offset* that can be stored and later subtracted in the digital domain, for every single pixel as a function of the integration time;
- when the measurement is performed under intense light, the PRNU will be visible. In order to compensate for all possible impinging spectra, the PRNU calibration shall be done, on average, for different reference spectra, typically a reference color chart illuminated by a reference light source. In this situation, the *gain* error inducing the PRNU can be measured, and its deviations from the average can be later compensated in the digital domain.

After calibration, residual DSNU and PRNU will be about 1% to 10% of their original values, due to unavoidably imperfect calibration.

The same graph above, after calibrating for PRNU and DSNU will see thus a (small) SNR increase at low signal values due to the reduction of the DSNU (but other signal independent noise sources may be dominant), and a marked SNR increase at large values due to 2the reduction of PRNU. It may happen that the pixel saturates before residual PRNU becomes visible, as shown in the graph aside. The maximum SNR correspondingly increases.



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Question n. 2

A comb-finger MEMS resonator has to be designed, targeting a resonance frequency of 524.288 kHz.

(i) given the maximum resonator dimensions in the Table, find the required number of fingers and the rotor voltage to cope with the target transduction factor;

(ii) evaluate in details the electrical equivalent model of the resonator (neglecting all parasitic capacitances) and draw its transfer function (modulus and phase) from small-signal input voltage to output current (resonator admittance);

(iii) now consider all parasitic capacitances: redraw as accurately as possible the resonator equivalent electrical model and its Bode plots of the electrical admittance. If you find any issue, propose a solution that restores the correct system operation.

Process thickness	30 µm
Minimum gap	1 µm
Minimum finger size	1.4 µm
Resonance frequency	524288 Hz
Modal mass	2 nkg
Quality factor	10000
Transduction factor	277 nN/V
Rotor voltage range	5-15 V
Maximum resonator dimension	250 µm
Parasitic capacitance 1 (drive port to sense port)	2 pF
Parasitic capacitance 2 (drive port to ground)	0.2 pF
Parasitic capacitance 3 (sense port to ground)	0.2 pF

Physical Constants

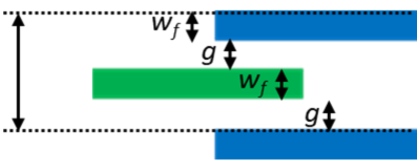
ε_0 = 8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

T = 300 K;

(i)

The finger pitch can be easily calculated as twice the minimum gap + twice the minimum finger width. From this rough calculation, we can then evaluate how many fingers fit in the overall resonator dimension.



$$N_{CF} = \frac{d_{max}}{CF_{pitch}} = \frac{d_{max}}{2(g + w_f)} = \frac{250\mu m}{2(1\mu m + 1.4\mu m)} = 52$$

We can now evaluate the capacitance change per unit displacement and the required rotor voltage as:

$$\frac{dC}{dx} = \frac{2\epsilon_0 h N_{CF}}{g} \rightarrow V_{rot} = \frac{\eta}{dC/dx} = 10\,V$$

(ii)

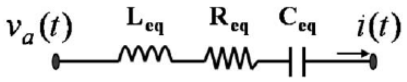
The electrical equivalent model requires the knowledge of the three mechanical-domain parameters and of the transduction factor (which is known):

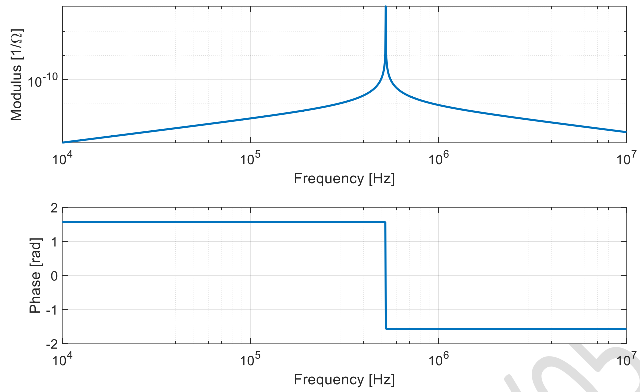
$$L_{eq} = \frac{m}{\eta^2} \quad R_{eq} = \frac{b}{\eta^2} \quad C_{eq} = \frac{\eta^2}{k}$$

As we know the mass, we can evaluate the stiffness and the damping coefficient from resonance and quality factor as:

$$k = (2\pi f_0)^2 \cdot m = 21703\frac{N}{m} \quad b = \frac{(2\pi f_0)\cdot m}{Q} = 6.58 \cdot 10^{-7}\frac{kg}{s}$$

And thus we found the three parameters as:

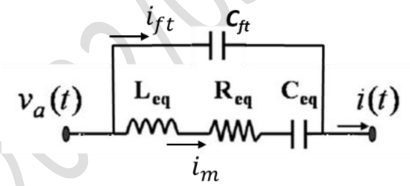




$L_{eq} = 26\ kH, R_{eq} = 8.6\ M\Omega,\ \ C_{eq} = 3.5\ aF$

Putting them in the admittance graph between input voltage and output current, we get the graph aside.

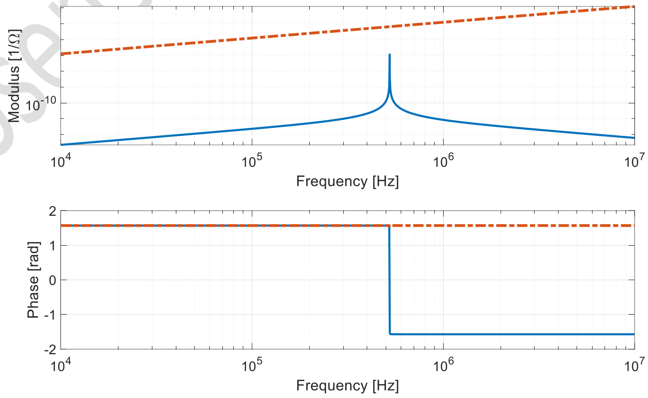
(iii)



The only parasitic that affects significantly the response is the feedthrough contribution between drive and sense port, while the other contributions can be neglected. The equivalent electrical model sees now this contribution in parallel to the admittance shown above.

We can easily compare the admittance value at resonance with respect to the former situation:

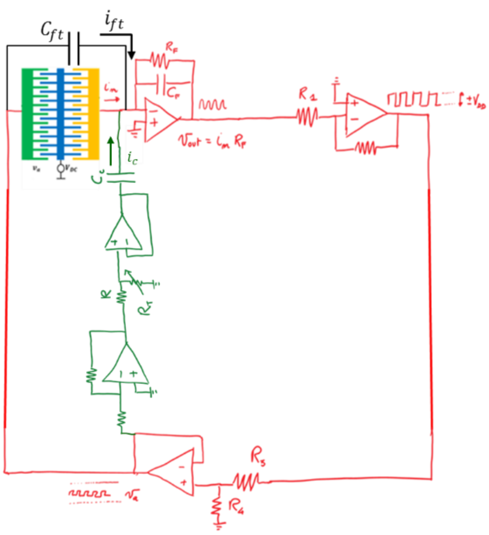
$$Z(\omega_0) = 1/R_{eq} + sC_{ft} = \frac{1}{8.6\ M\Omega} + j2\pi f_0 C_{ft} = 1.16 \cdot 10^{-7} S + j6.58 \cdot 10^{-6} S$$



From which we can see that the feedthrough contribution dominates across almost the entire range, and even at the resonance frequency. The peak will be thus barely visible in this situation, as shown by the new graph.

The MEMS admittance contribution is negligible on the entire frequency range – as it is negligible at resonance where it has the largest modulus.

A solution to this issue is the adoption of a feedthrough compensation block, which injects in the sense port a trimmed current equal and opposite to the feedthrough one. A scheme of a possible circuit that implements this solution is shown in the figure.



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Question n. 3

You need to design the electronics for a wide-dynamic-range MEMS accelerometer for consumer applications, with the specifications in the Table.

(i) choose the DC bias of the positive input of the front-end stage, the modulation voltage amplitude, the parameters of the passive components forming the front-end stage feedback, and the INA gain;

(ii) assuming 3/4 of the consumption ascribed to the INA, demodulation and ADC, with the remaining 1/4 ascribed to the front-end, evaluate the electronic noise and check the compatibility with the specifications in the Table;

(iii) suitably choose the number of bits of the ADC and indicate a suggested output data rate.

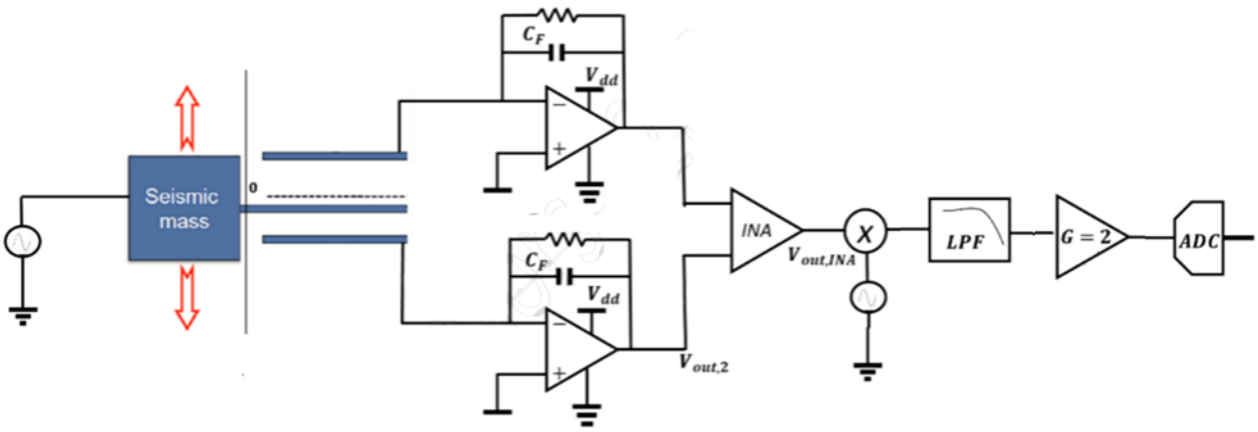
IC voltage supply	3.3 V
IC maximum consumption	40 μA
Minimum integrated capacitance	30 fF
Accelerometer FSR	± 32 g
Accelerometer resolution	10 μg/√Hz
Bandwidth	400 Hz
Mechanical gain (differential)	10 fF/g
Mechanical offset range (differential)	± 50 fF
Modulation frequency	100 kHz
Parasitic capacitance	5 pF
K_n MOS factor	1 mA/V²

Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$k_b = 1.38\ 10^{-23}$ J/K;

T = 300 K;



(i)

There is no reason not to choose the reference voltage at the positive input of the amplifier to be at half the supply dynamic, so at $V_{DD}/2$ = 1.65 V. This ensures the same swing for both positive and negative accelerations (in absence of offset).

Similarly, there is no reason not to choose a sinewave at the input that swings the maximum dynamic, i.e. $v_{mod} = V_{DD}/2$ = 1.65 V. This indeed maximizes the signal before electronic noise is introduced in the readout chain.

At this point, we choose a feedback capacitance such that – operating in charge amplifier mode – the front-end output is half of the dynamic. With the differential INA gain, this will cover the full voltage dynamics at the INA output (it is here reasonably assumed that also the INA is biased between 0 V and V_{DD}, with a common mode at $V_{DD}/2$).

Given the transfer function from input to INA output, let's assume to choose an INA unitary gain and verify whether we can adopt a capacitance that fits within the specifications. We force the equation below to yield the half dynamics output for a full-scale acceleration and offset (the other half dynamics will be covered by accelerations and offsets of the other sign):

$$\frac{V_{INA}}{\Delta C + C_{OS}} = 2\frac{v_{mod}}{C_F}G_{INA} = \frac{1.65\,V}{32g \cdot 10\frac{fF}{g} + 50fF} = 4.46\frac{mV}{fF} \rightarrow \quad C_F = \frac{2 \cdot 1.65V \cdot 1}{4.46\frac{mV}{fF}} = 740\,fF$$

This value can be integrated and poses no issues. So, we choose a resistance such that the pole is two orders of magnitude before the operating frequency, which is the modulation frequency at 100 kHz:

$$R_F = \frac{1}{2\pi(f_{mod}/100)C_F} = 215\,M\Omega$$

The INA gain is confirmed to be unitary. Larger resistances to make their noise negligible can also be accepted.

Other sizing which are reasonable will be accepted as a correct solution.

Note that the resulting scale factor with this configuration is not 1.65V/32g because we accounted for possible offsets. It is instead 1.65V/37g = 44.6 mV/g (50 fF of offset correspond to 5 g through the 10 fF/g conversion).

(ii)

The front-end can dissipate 10 µA, thus 5 µA per operational amplifier, thus 2.5 µA per input transistor branch. The amplifiers noise can be thus calculated at the front-end differential output (which is the INA output given the unitary gain) as four uncorrelated noise sources:

$$\sqrt{S_{V_{n,out}}} = \sqrt{4 \cdot \frac{4k_B T\gamma}{g_m}}\left(1 + \frac{C_P}{C_F}\right) = \sqrt{4 \cdot \frac{4k_B T\gamma}{2\sqrt{k_n I_{MOS}}}}\left(1 + \frac{C_P}{C_F}\right) = 163\frac{nV}{\sqrt{Hz}}$$

This turns into an input referred noise of:

$$\sqrt{S_{a_{n_{eln}}}} = \frac{\sqrt{S_{V_{n,out}}}}{SF} = 3.66\,\frac{\mu g}{\sqrt{Hz}}$$

This is in line with the specifications and leaves room for a similar or even larger thermomechanical noise.

(iii)

Assuming that the final noise, including thermomechanical noise, is effectively 10 $\frac{\mu g}{\sqrt{Hz}}$, this yields an overall noise integrated across the sensing bandwidth of:

$$\sigma_a = 10\,\frac{\mu g}{\sqrt{Hz}} \cdot \sqrt{400Hz} = 200\,\mu g_{rms}$$

Given the full-scale – including the offset – the required number of levels and bits turns out to be:

$$N_{levels} = \frac{\pm(32g + 5g)}{200\,\mu g_{rms}} = \frac{2 \cdot 37g}{200\,\mu g_{rms}} = 370000 \rightarrow \quad 2^{N_{bit}} > N_{levels} \rightarrow \quad N_{bit} = 19$$

The suggested output data rate (ODR) is just twice the maximum bandwidth, to cope with the Nyquist theorem.

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