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Question n. 1

Discuss the characteristics of the human eye that are imitated by a CMOS digital imaging system, in terms of (i) optics, (ii) photodetectors, (iii) color discrimination and (iv) processing. While analyzing the different sub-points, possibly provide quantitative numerical examples to draw a parallelism between human eye and requirements of modern digital cameras.

Quantification of perceived light intensity and color can be physically defined only accounting for the characteristics of the human eye, which is thus unavoidably a source of inspiration in the development of imaging systems from several points of view.

Beginning from the optical part, each human eye features a variable opening (the iris), with an aperture automatically spanning from 1mm to 4 mm, depending on light intensity. At the same time, it features a deformable lens whose curvature radius automatically adjust the focus towards the plane of photoreceptors. The distance between this plane and the lens is in the order of about 15 mm, yielding thus an F number (ratio of focal length and aperture) spanning from 1 to less than 2. The field of view where visual attention is concentrated corresponds roughly to 50°. Modern cameras aim at imitating – in average conditions – these latter parameters, generating a similar FOV and similar F numbers, though this is sometimes made difficult by the limited available space, especially in mobile imaging. The impossibility to have deformable lenses also generates a challenge in terms of adjusting the focal length. For semi-professional cameras, a larger available space enables using systems of multiple lenses, while in mobile cameras the use of multiple cameras, each with a different fixed lens, is combined in postprocessing to yield the variable f.

In terms of photoreceptors, the human eye features about 5-7 million cones per eye, located in the region of maximum visual attention, and about 100-150 million rods, located in the peripheral vision area. Both types lie on the curved retinal surface. These photoreceptors respond with a certain probability function to the impinging wavelength. However, once a photon is absorbed, the electrical stimulus sent by the cone to the brain through the optical nerve is always the same: we can thus say that our photoreceptors work as quantum sensors (i.e. sensitive to the quanta, not to their energy). Similarly, photodetectors used in CMOS cameras rely on quantum sensors, typically based on collection of photoelectrons in PN junctions. While in the eye the generation of the electrical stimulus does not take area from the retinal surface, in CMOS sensors the electronics required to buffer the data towards the ADC and processing takes up some area, which reduces the fill factor. This is partially compensated by depositing micro lenses over the photodetector plane.

When it comes to color discrimination, CMOS sensors fully imitate the human eye during daylight vision: the presence of three different pigments on cones, yielding three different spectral functions in terms of absorption probability, is mimicked by depositing on top of the photodetectors an array of color filters (CFA). The array is usually sized such that the overall average response peaks around 555 nm, a wavelength at which also the combined response of the cones of different type peaks. To cope with this *requirement, the preferred pattern features two green filters, one blue and one red filter every four pixels. The reason why mimicking the overall brightness response of the eye is important lies in that resolution is best perceived by the brain in terms of brightness, rather than in terms of chroma. Note that, while the distribution of photoreceptors in the eye is random in terms of different pigments, in CMOS cameras it usually follows a predetermined path which gives rise to undesired aliasing effect which – combined to different color filters – may result in color fringes.*

Processing by the brain corresponds mostly to a rearrangement of color information into a different color space (brightness, saturation, and hue). This transition includes an intermediate step where color is arranged in coordinates of yellowness-blueness and greenness-redness. Additional automatic adaptation of color perception to the illuminant spectrum (color constancy) is performed at the brain level. In CMOS image sensors most of the processing corresponds to these two steps and is operated in terms of matrix transform from the camera color space into device-independent color spaces (color conversion matrix), and then an additional diagonal matrix operates a white balance step according to various possible algorithms. An additional step of color interpolation is mandatory to get three full color coordinates at each pixel location.

Overall, the performances of imaging sensors are still a bit behind those of the human eye. In terms of dynamic range, for instance, the value of 90 dB is not yet matched even using modern architectures (like 4T topologies), which approach the 75-80 dB level.

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Question n. 2

A MEMS magnetometer is based on the structure and electronics depicted aside, with:

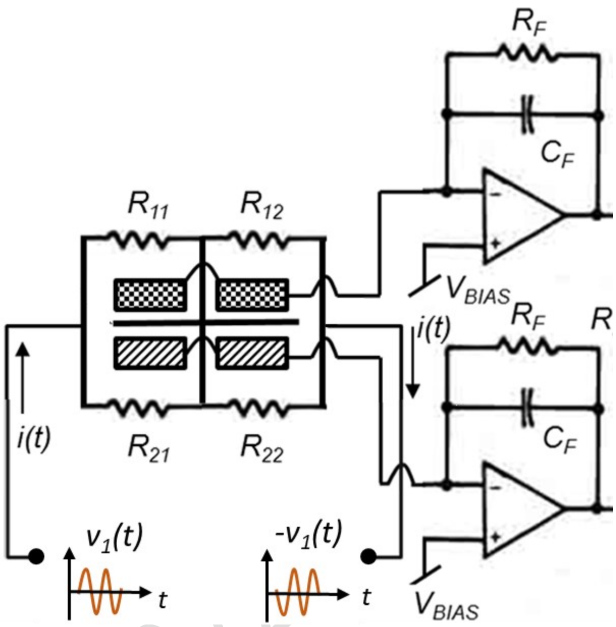
$$v(t) = V_0 \cdot \sin(2\pi\, f_d \cdot t)$$

$$R_{11} = R_{12} = R_{21} = R_{22} = 2\ k\Omega$$

Given the parameters in the table:

(i) evaluate the value of the drive resonance frequency and the optimal distribution of the current among the sensor (so the value V_0) and the front-end transistors so to cope with noise, bandwidth, linearity, and full-scale range specifications, at the same time (consider two transistors in every charge amplifier);

(ii) if the system appears to be oversized, propose to change one parameter to optimize your system design.



Assume now that the resistance of the springs R_{11} and R_{21} is 1% larger than R_{12} and R_{22}:

(iii) using the data of point (i), in absence of any magnetic field, calculate the resulting peak voltage on the rotor and at the output of one of the charge amplifiers. Does this term appear as an offset at the output?

Full-scale range	5 mT
Current budget (MEMS + front-end)	2 mA
Overall spring length	1.5 mm
Parallel-plate gap	1 µm
Maximum linearity error	0.1 %
Resonance frequency	10 kHz
Quality factor	500
Effective mass	1.7 nkg
Single ended overall PP capacitance	250 fF
MOSFET coefficient k_n	0.05 mA/V²
Positive input bias (V_{BIAS})	1.2 V
Parasitic capacitance	10 pF
Bandwidth	50 Hz
Resolution target	100 nT/√Hz
Feedback capacitance	400 fF

Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$k_b = 1.38\ 10^{-23}$ J/K;

T = 300 K;

(i)

Among the several constraints that we are given, the easiest to size are the split value and maximum displacement so to cope with bandwidth and linearity, respectively:

$$\Delta f = 3 \cdot BW$$

$$x_{max} = \sqrt{\epsilon_{lin}} \cdot g = 31.6\ nm$$

With this sizing, it is possible to find the maximum current that, flowing through the springs, generates the maximum displacement for a field corresponding to the FSR:

$$I_{MEMS,peak} = x_{max} * \frac{2 * k}{FSR * L * Qeff} = 1.7\ mA \rightarrow I_{MEMS,rms} = \frac{I_{MEMS,peak}}{\sqrt{2}} = 1.2\ mA$$

This leaves 0.8 mA available for the electronics. We have to check whether this budget is enough to cope with noise requirements together with Brownian noise:

$$\sigma_B = \sqrt{\left(\frac{4\sqrt{k_B T b}}{I_{MEMS,peak} L}\right)^2 + 2 \cdot 2 \cdot \frac{4 k_B T \gamma}{2\sqrt{k_n \cdot I_{MOS}}}\left(\frac{1+\frac{C_P}{C_F}}{SENS}\right)^2}$$

Where I_{MOS} is 200 μA (1/4 of the budget), b can be derived from the parameters and the sensitivity is given by:

$$SENS = 2\frac{I_{MEMS,peak}\, L\, Q_{eff}}{2 \cdot k}\frac{V_{BIAS}}{C_F}\frac{C_0}{g} = 9.5\ \frac{V}{T}$$

With these values, the input referred noise density becomes 62 nT/√Hz, thus coping with the requirements. The corresponding voltage required to provide the current is just:

$$V_0 = I_{MEMS,peak} * \frac{R}{2} = 1.7\ V$$

NOTE: alternative reasonable sizing approaches were considered positively in the correction.

(ii)

With the proposed approach, the system appears to be oversized in that we can relax some parameters while still coping with noise requirements. An idea could be to reduce the current. An alternative could be to widen the split value, to make the design more robust against possible variation in the split value between the magnetometer mode and the frequency used to drive the Lorentz current into the device.

(iii)

The symmetricity of the resistance of the driving springs is lost in this situation. As a consequence, the rotor is no longer at the ground potential but oscillates with a spurious voltage at the drive frequency given by:

$$|V_{spur}| = \left|V_0\left(\frac{R}{2.01 \cdot R} - \frac{1.01 \cdot R}{2.01 \cdot R}\right)\right| = 8.4\ mV$$

The corresponding current and voltage through a branch of the charge amplifiers pair becomes:

$$|I_{spur}| = |V_{spur} \cdot (2\pi f_d C_0)| = 0.13\ nA$$

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$$\left|V_{CA,spur}\right| = \frac{\left|I_{spur}\right|}{2\pi f_d C_F} = 5.3\ mV$$

However, this effect is identical on both branches, and is then suppressed at the output by the differential sensing. It thus not implies an output offset.

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Question n. 3

You need to finalize the design of an oscillator for a symmetric MEMS resonator with a 100 kHz resonance.

(i) Size the feedback capacitance C_f of the charge amplifier for a 100-nm target displacement. Size the 2nd stage to satisfy the phase criteria for oscillation, with unity gain at resonance.

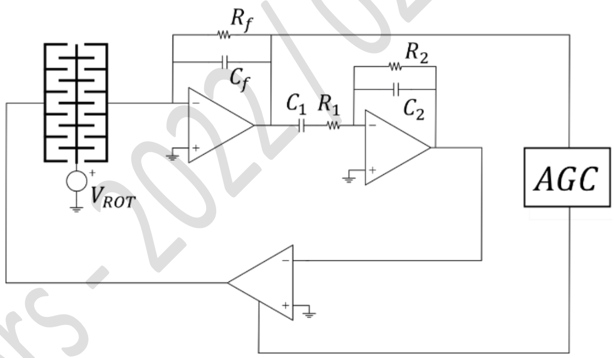
(ii) Draw and size a feedthrough compensation circuit for a 50-fF feedthrough capacitance, using resistances < 100 kΩ.

(iii) For high Q-factors, the frequency vs phase relationship around resonance is given by:

$$\Delta f = \frac{f_0}{2Q}\Delta\phi$$

Using the formula, calculate the deviation in the operating frequency from resonance at room temperature, due to the phase-shift introduced by the circuit poles. Then, repeat the calculation at the maximum operating temperature, considering the resonance frequency and quality factor dependence on temperature.

AGC reference voltage	V_{REF}	$5\,V$
MEMS resonance frequency	f_0	$100\,kHz$
Target displacement	x_d	$100\,nm$
Transduction coefficient	η	$80\,\mu A/(m/s)$
Mass	m	$3\,nkg$
1st stage feedback resistance	R_F	$16\,G\Omega$
2nd stage feedback capacitance	C_2	$100\,pF$
Feedthrough compensation capacitance	C_{comp}	$1\,pF$
Quality factor (room temp.)	Q_0	1000
Max operating Temperature	T	$125\,°C$



Physical Constants

ε_0 = 8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

T = 300 K;

i)

Assuming the AGC being composed by rectifier + LPF + comparator with Vref = 5 V:

$$V_{REF} = x_d \cdot \frac{1}{C_F} \cdot \frac{2}{\pi} \cdot \eta_{dd} \quad \rightarrow \quad C_F = \frac{x_d}{V_{REF}} \cdot \frac{2}{\pi} \cdot \eta_{dd} = 1.0186\,pF$$

The second stage is sized in order to satisfy the phase criteria on phase and to have unity gain at resonance. Since the comparator is inverting (-180° phase shift) the second stage must operate as an integrator.

Therefore, I choose to set the two poles two decades before the resonance frequency and set the gain at the resonance frequency equal to 1:

$$T_{90D}(s) = \frac{sC_1R_2}{(1+sC_1R_1)(1+sC_2R_2)}$$

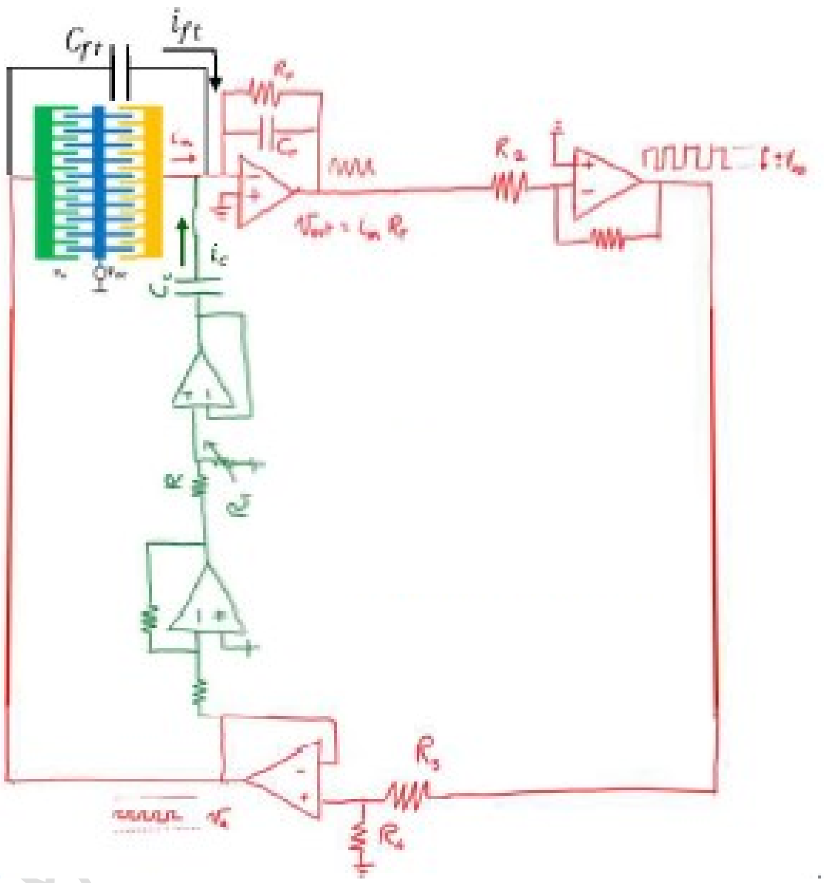
1. $f_p = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_2 C_2} = 1\,kHz$
2. $|T_{90D}(j\omega_0)| = \frac{1}{2\pi f_0 C_2 R_1} = 1$

From eq.2 → $R_1 = \frac{1}{2\pi f_0 C_2} = 15.9\ k\Omega$

From eq.1 → $C_1 = \frac{1}{2\pi f_p R_1} = 10\ nF$ *and* $R_2 = \frac{1}{2\pi f_p C_2} = 1.59\ M\Omega$

ii)

The compensation circuit is something similar to the one in green in the following picture



Assume to size the resistances of the first inverting stage to be of the same value, the voltage divider must be sized in order to satisfy the following:

$$V_a \cdot \frac{R_1}{R_1 + R} \cdot 2\pi f_0 \cdot C_{comp} = V_a \cdot 2\pi f_0 \cdot C_{ft}$$

Choosing $R = 100\ k\Omega$ → $R_1 = \frac{R \cdot C_{ft}}{(C_{comp} - C_{ft})} = 5.26\ k\Omega$

iii)

The phase shift introduced by circuit poles at room temperature is:

$$\Delta\phi = 2 \cdot \mathrm{atan}\left(\frac{f_p}{f_0}\right) = 1.1459°$$

Therefore $\Delta f = \frac{f_0}{2Q} \Delta\phi \cdot \frac{\pi}{180} = 1\ Hz$

Then going to the max operating frequency and remembering the relationships between temperature, Q and f:

$$Q(T_{max}) = Q_0 \cdot \frac{\sqrt{300\ K}}{\sqrt{398\ K}} = 868.2$$

$f_0(T_{\max}) = f_0 \cdot \left(1 + TC_f \cdot \Delta T\right) = 99.7\ kHz$ *with* $TC_f = -30 \frac{ppm}{K}$

and as before:

$$\Delta\phi = 2 \cdot \mathrm{atan}\left(\frac{f_p}{f_0(T_{max})}\right) = 1.1493°$$

$\Delta f = \frac{f_0(T_{\max})}{2Q(T_{max})} \Delta\phi \cdot \frac{\pi}{180} = 1.15\ Hz$

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