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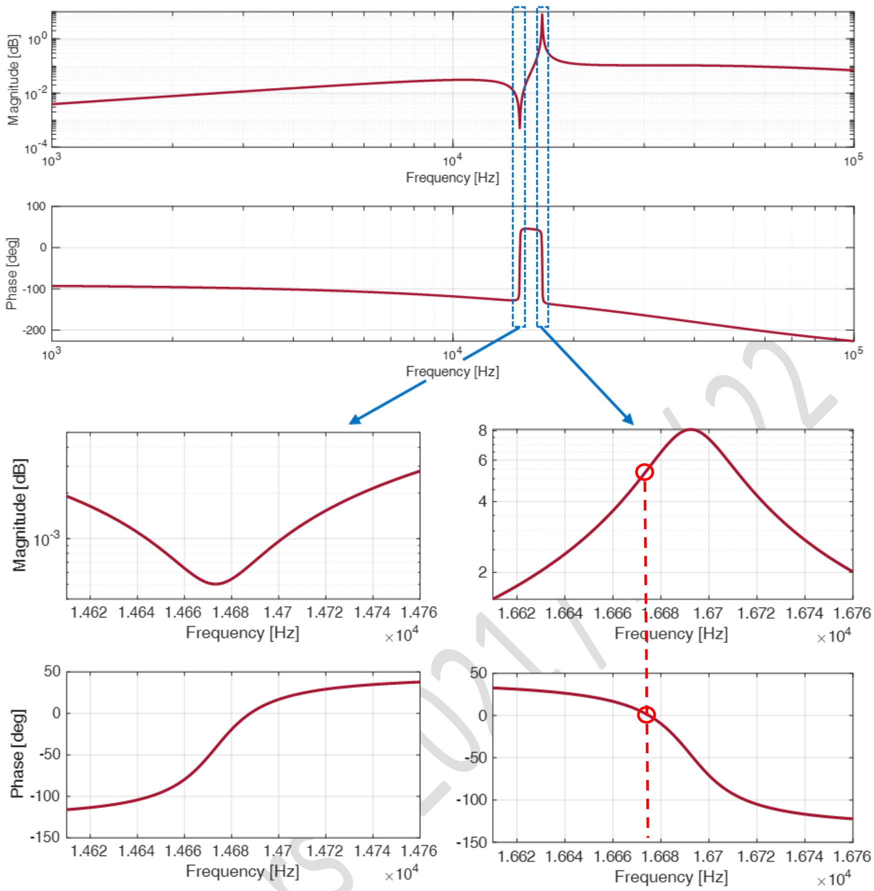
Question n. 1

The graphs aside show the Bode plots of the loop gain of a three-port MEMS-based oscillator circuit, around the resonance frequency.

Describe in detail a possible situation that can result in such a graph. Mark on the graph, and then indicate quantitatively, the oscillator resonance frequency and discuss whether the circuit can oscillate or not. How does the found oscillation frequency change with temperature?



The presented graph shows a peak, clearly due to the amplification around resonance typical of the transfer function of a 3-port resonator from applied voltage into motional current. Additionally, the fact that the graph is reported in non-dimensional units (and not in the admittance unit of Siemens) also indicates that a circuit is used to convert the motional current again into a voltage, so to close the loop and sustain the oscillation.

Nevertheless, there are some distinctive features that indicates that such a transfer function cannot be due just to a MEMS resonator and an ideal sustaining circuit that only compensates the mechanical losses. Indeed, we note:

- an anti-peak that anticipates the resonance region on the Bode modulus;
- a non-sudden decrease in the modulus after the peak region and, at the same time, a phase that returns to the same values it has before the peak;
- a slight decrease in the transfer function at higher frequencies.

An anti-peak is a feature typically associated to the presence of a feedthrough capacitance; however, a 3-port resonator with a feedthrough would show the anti-peak after (and not before) resonance. What is shown in the presented graph can be explained by the presence of a feedthrough compensation circuit that applies an overcompensation. In equations, this can be written as:

$$T(s) = \left[\frac{s\eta^2}{ms^2 + bs + k} + sC_{ft} - sC_{comp}\right] G_{eln}(s)$$

where C_{ft} indicated the feedthrough and C_{comp} indicates the equivalent compensation capacitance. If $C_{comp} > C_{ft}$, then the anti-peak appears before resonance.

Let us verify whether this hypothesis is an agreement with the phase behavior: in absence of feedthrough compensation, at low frequency we would expect the phase to have a 90° given by the feedthrough (or by the resonator low-frequency equivalent capacitance). Instead of finding this value, the graph shows a -90° value: this means that effectively, the compensation capacitance dominates on the feedthrough, leading to the shown -90°.

Another hint about the presence of compensation is that in the first region of the curve, if we have only feedthrough without compensation, the sum of sC_{eq} and sC_{ft} (both positive) cannot yield a nulling and thus an anti-peak before resonance. Only a negative term can provide such effect.

The flattening and slight decrease in the modulus at high frequencies indicates the presence of at least two electronic poles. This is also highlighted by the fact that the phase shift at resonance is about -30°, further indicating that these electronic poles are likely much less than a decade far from resonance.

Overall, the circuit can anyway oscillate as there is one point matching the Barkhausen criterion: indeed, at a frequency of approximately 16.675 kHz, as indicated in the bottom-right graph, the loop phase turns out to be 0° while the modulus is larger than 1, ensuring the oscillation start-up. No oscillation can instead arise in the region indicated in the bottom-left graph, as the loop-phase condition of 0° corresponds to a modulus lower than 1.

In presence of temperature changes, the resonator mechanical frequency drifts by -30 ppm/K, due to Young's modulus drift. Assuming that the electronic does not drift significantly, this drift will directly apply to the oscillation frequency. Minor additional drifts may be due to the change in the value of passive (resistances and capacitances) or active (transistors) electronic components with temperature.

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Question n. 2

You need to design a CMOS image sensor with a 3T technology, for scientific applications in liquid-nitrogen-cooled systems operating under a 633-nm wavelength.

The sensor requires a maximum dynamic range of 75 dB and a maximum SNR of 45 dB. The used technology has a 3.3-V supply and a specific PN junction capacitance of 0.2 fF/μm^2.

(i) considering only maximum DR and maximum SNR, suitably choose the n. of bits for the ADC, the pixel area and the lens F# number;

(ii) knowing that nitrogen remains liquid in a temperature range between -210°C and -195°C, verify whether the sizing above effectively matches the requirements; otherwise, you are given the options to (a) exploit microlenses or (b) increase the supply voltage: which option would you choose and why?

(iii) calculate the maximum percentage PRNU so that its effects do not compromise the target performance.

Physical Constants

ε_{Si} = 11.7·8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

q = 1.6 10^{-19} C;

(i)

The maximum dynamic range can be directly related to the n. of bits of an imaging sensor. Indeed, one can set quantization noise to be negligible over all other noise sources involved in the calculation of the maximum DR. These are practically represented only by kTC noise, as the maximum DR is achieved at short integration times, where dark current shot noise and DSNU are negligible.

As a consequence, we set (e.g. in terms of n. of electrons):

$$\frac{V_{DD}\,C_{dep}}{2^n}\frac{1}{\sqrt{12}}\frac{1}{q} < \frac{\sqrt{k_BTC}}{q} \;\rightarrow\; 2^n > \frac{V_{DD}\,C_{dep}}{\sqrt{k_BTC}}\frac{1}{\sqrt{12}} = \frac{DR_{MAX}}{\sqrt{12}} \;\rightarrow\; n > \log_2\left(\frac{DR_{MAX}}{\sqrt{12}}\right) = 10.66$$

We thus take 11 bits to cope with the maximum dynamic range requirement (note that the max DR in the equation above shall be turned into linear units and reads 5625). Note: assuming some margin and using e.g. 12 bits is still considered acceptable.

At the same time, we know that the maximum SNR, ignoring by now the PRNU, is related only to photon shot noise. Indeed, at maximum signals, where we expect the maximum SNR, we also expect that its associated shot noise dominates over other noise contributions:

$$SNR_{MAX} = 20\log_{10}\frac{i_{ph,MAX}t_{int}}{\sqrt{k_BTC + q\left(i_d + i_{ph,MAX}\right)t_{int}}} \approx 20\log_{10}\frac{i_{ph,MAX}t_{int}}{\sqrt{q i_{ph,MAX}t_{int}}} = 20\log_{10}\sqrt{\frac{Q_{MAX}}{q}}$$

$$= 20\log_{10}\sqrt{N_{el}} \;\rightarrow\; N_{el} = SNR^2_{MAX,lin} = 31623$$

We thus know that the pixel shall host this number of electrons in order to guarantee the required maximum SNR. We can directly calculate the pixel area, as the supply voltage is known:

$$\frac{C'_{pn} \cdot A_{pix} \cdot V_{DD}}{q} = N_{el} \;\rightarrow\; l_{pix} = \sqrt{\frac{N_{el} q}{C'_{pn} \cdot V_{DD}}} = 2.77\ \mu m$$

Assuming to design a balanced system, we set the diffraction limit such that it matches the achieved pixel side. We thus obtain:

$$d_{Airy} = 2.44 \frac{\lambda}{D} f = 2.44\,\lambda\, F_{\#} = l_{pix} \;\rightarrow\; F_{\#} = \frac{l_{pix}}{2.44\,\lambda} = 1.8$$

Alternatively, one can choose an F number such that the optics blurs the image by a dimension which is twice the pixel size, so that no aliasing occurs during spatial sampling. This is obtained for an F number of 3.6.

(ii)

Let us now verify whether kTC noise enables achieving the required DR. After converting temperature in Kelvin, and choosing the highest temperature as a worst case, we find that:

$$DR_{MAX} = 20 \log_{10} \frac{N_{el}}{\frac{\sqrt{k_B T C}}{q}} = 71.9\ dB$$

This result does not match the requirements. Using microlenses cannot bring an improvement in the maximum DR, as the technique neither increases the maximum number of electrons in the photodiode well, nor decreases kTC noise. Conversely, increasing the maximum voltage also increases the maximum number of electrons in the well, enabling to improve the DR.

Assuming not to change the number of bits and pixel area, and thus taking into account also quantization noise, we get that the expression below:

$$DR_{MAX} = 20 \log_{10} \frac{V_{DD,new} C'_{pn} A_{pix}}{\sqrt{k_B T C'_{pn} A_{pix} + \left(\frac{V_{DD,new}\ C'_{pn} A_{pix}}{2^n \sqrt{12}} \right)^2}} = 75\ dB$$

Brings a DR larger than 75 dB for a supply voltage of 7.7 V at 78.2 K.

(iii)

We know that PRNU appears for large signals, overcoming eventually photon shot noise. In order to limit PRNU effects, we thus force that its effect, even at the largest signal, remains comparable (or negligible) with respect to photon shot noise.

In formulas this can be expressed as:

$$\%_{prnu} i_{ph,MAX} t_{int} \le \sqrt{q i_{ph,MAX} t_{int}} \;\rightarrow\; \%_{prnu} \le \frac{\sqrt{q i_{ph,MAX} t_{int}}}{i_{ph,MAX} t_{int}} = \sqrt{\frac{q}{Q_{MAX}}} = \frac{1}{\sqrt{N_{el}}}$$

Very interestingly, we note that this condition depends only on the maximum charge inside the well. Numerically, the maximum acceptable PRNU turns out to be lower than 0.006, or 0.6 %. We note that this value is hardly achievable without PRNU calibration, that will be thus likely required.

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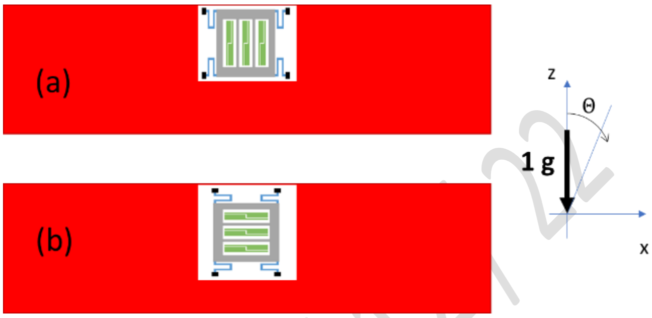
Question n. 3

You work in a company developing toolkits, and you want to upgrade your manual alignment tool shown aside, substituting the fluidic bubble with a high-precision sensor. You thus look for a MEMS inclinometer, which is a sensor based on a single-axis MEMS accelerometer and gravity.

(i) choose the best mounting direction of the accelerometer, according to the figure aside: (a) or (b). Then, write the relationship between inclination angle θ and input acceleration, for small inclination angles. Assuming that your target precision in measuring inclination is 0.02° rms, find the required accelerometer resolution (g rms);



(ii) choose a suitable triplet of values for the accelerometer parameters (k, m, Q) assuming that the application has a target -6 db bandwidth of 500 Hz;

(iii) draw and comment with motivations a suitable electronic readout chain (from applied voltage to digital output) to cope with the described application.

Physical Constants

ε_0 = 8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

T = 300 K;

(i)

When quasi-statically positioned on the surface whose inclination needs to be measured, the accelerometer feels an acceleration in the sensing direction, as a function of the inclination angle θ that, looking at the pictures aside can be written as:

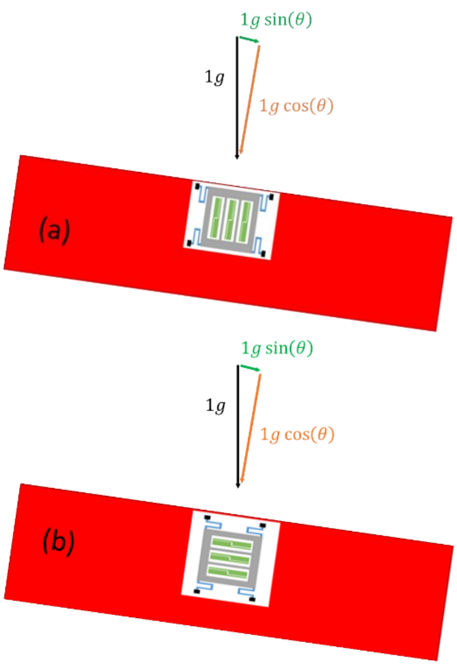
$$a = 1g \cdot \sin(\theta) \qquad for\ case\ (a)$$

$$a = 1g \cdot \cos(\theta) \qquad for\ case\ (b)$$

At small inclination angles, the variation in the acceleration in the sensing direction per unit angles can be thus written as:

$$sin\theta \approx \theta \rightarrow \frac{da}{d\theta} \approx 1g \qquad for\ case\ (a)$$

$$cos\theta \approx 1 \rightarrow \frac{da}{d\theta} \approx 0 \qquad for\ case\ (b)$$



We thus choose a mounting of type (a), as this guarantees the highest sensitivity and, besides, avoids a huge offset at the input.

Given the target resolution of 0.02°, converting it into radians and exploiting the sensitivity found above, the requirements in terms of minimum acceleration to measure turns out to be:

$$\sigma_a = \sigma_\theta \cdot \frac{da}{d\theta} = 0.02° \cdot \frac{\pi\ rad}{180°} \cdot \frac{1\ g}{rad} = 349\ \mu g_{rms}$$

(ii)

A few considerations can be drawn to easily cope with the given requirements:

- as an accelerometer works on the flat portion of the 2-pole transfer function, it is better to avoid overdamped responses, so to avoid a quality factor larger than 0.5;
- at the same time, there is no use in lowering the Q factor more than 0.5, which would cause a pole splitting and limit the bandwidth to values lower than resonance;
- for a quality factor of exactly 0.5, the poles of the electromechanical transfer function are real coincident and we thus get -6dB at 500 Hz if we set the resonance frequency exactly at 500 Hz;
- the third condition that we exploit to size the system is related to noise. We know that thermomechanical noise density (NEAD) shall reach a maximum value given by the integrated resolution found above, divided by the square root of the 500-Hz bandwidth.

Overall we can write :

$$\begin{cases} Q = 0.5 \\ f_0 = 500\ Hz \\ NEAD = \dfrac{\sqrt{4k_BTb}}{m} = \sqrt{\dfrac{4k_BT\omega_0}{m \cdot Q}} = \sqrt{\dfrac{8\pi k_BTf_0}{m \cdot Q}} = \dfrac{349\ \mu g_{rms}}{\sqrt{500\ Hz}} = 15.6\dfrac{\mu g}{\sqrt{Hz}} \end{cases}$$

From which we find a mass value of 3.5 nkg.

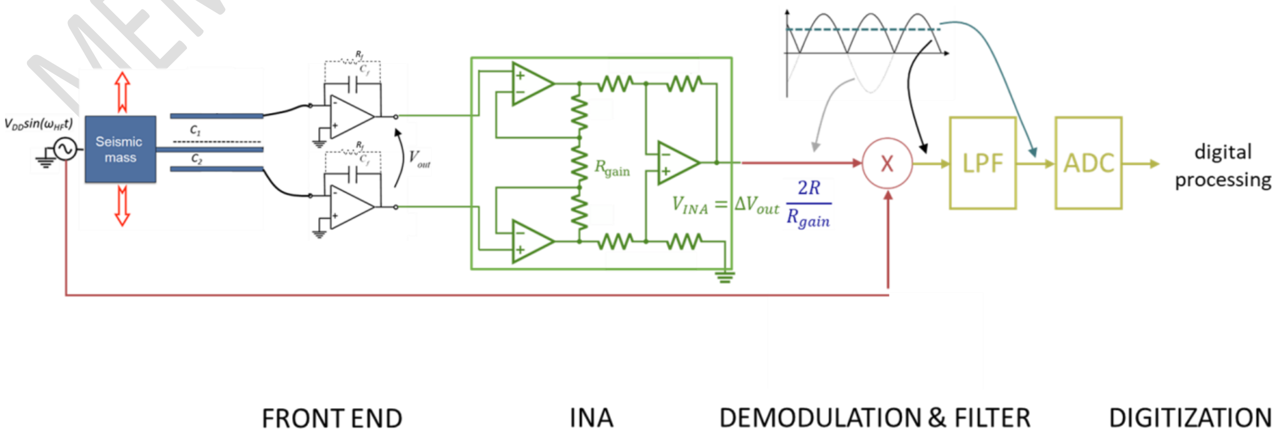
The resulting stiffness turns out to be:

$$k = (2\pi f_0)^2 m = 0.035\frac{N}{m}$$

Note: other solutions using a higher resonance frequency with filtering and/or slightly different Q factors can be accepted, but are sub-optimal (e.g. a higher resonance lowers the scale factor and thus makes the design of the electronics more challenging). Whatever the choice of f_0, *Q* and *m*, a control that the NEAD is matched shall be always done.

(iii)

As the sensor needs to read static values of acceleration, when lying on the object whose angle is to be measured, we need to use a modulated sensing chain. The rotor will be applied a certain AC voltage, while the two stators will be differentially readout by a pair of charge amplifiers with their positive inputs tight to the ground potential.



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Though we have not enough information to accurately size the entire chain (we do not know the MEMS transduction coefficient), we can anyway make some considerations. The modulation frequency shall be much larger than 500 Hz, e.g. 50 kHz. The feedback capacitance shall be sized to maximize the amplifier output, without causing saturation. At the same time, the resistance and the amplifier noise sources shall not worsen the resolution. A stage will be required to convert the differential output into a single-ended information, followed by a demodulation stage (multiplier and low pass filter). The ADC at the end of the sensing chain, after the demodulation step, shall accommodate a number of bit that matches with the required dynamic range.