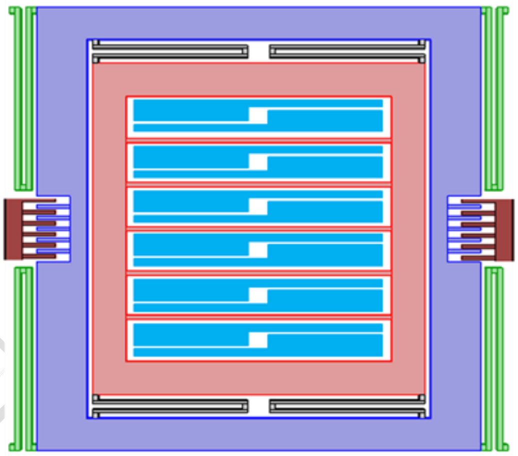
Last Name _Di Londra__ **Given Name** _Azzurra Vittoria_ **ID Number** _20210713_

Question n. 1

Draw a simple, single-mass scheme of a Coriolis-force based MEMS gyroscope, highlighting the role of the various structural elements, and motivate the reasons for using different electrostatic electrode types for the two modes.

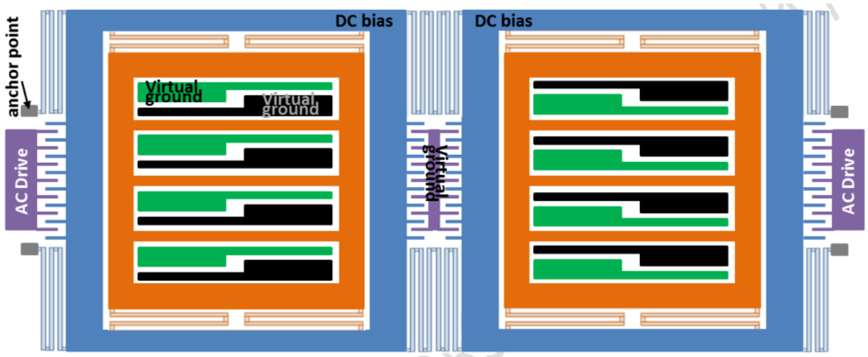
Then, draw and describe a more advanced scheme where a tuning fork architecture is adopted, and motivate the reasons towards this approach.

Finally, sketch a scheme where a double decoupling is used, and motivate the reasons towards this approach. Give guidelines on how to distribute the mass among the different forming frames.



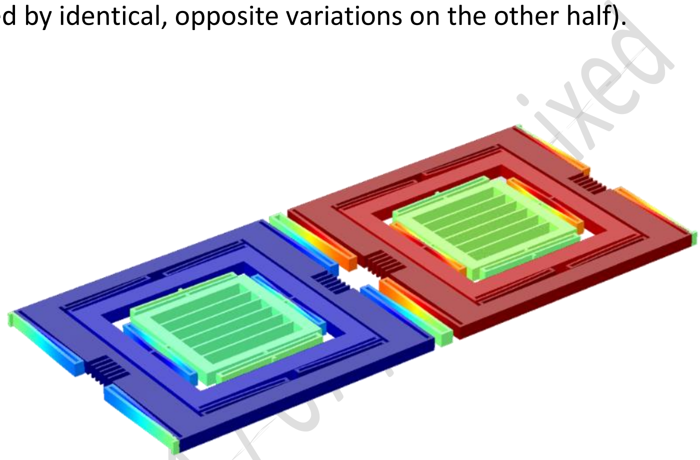
The simplest MEMS gyroscope, schematically shown aside, is formed by to main frames, the drive frame and the sense frame. The drive frame, kept suspended by suitable springs allowing motion in the x direction, is kept in stable oscillation, thus moving with a sinusoidal velocity along its axis. Its suspending springs are stiff in the orthogonal direction to avoid motion along undesired axes. The force to sustain the oscillation is provided by comb fingers, for two main reasons: (i) comb fingers allow large, linear capacitance variation and are preferred to parallel plates if large motion is desired (which is the case of MEMS gyro scopes, as the scale factor is linear with motion amplitude); (ii) comb fingers yield a lower damping contribution and are thus preferred whenever high Q factors are desirable. Nested within the drive frame, and connected to it by suitable decoupling springs, lie the sense frame. This frame is dragged by the drive frame, along the x-axis, with the same velocity, thanks to the fact that the decoupling springs are stiff in this direction. When a z-axis angular velocity is applied to the MEMS substrate, an apparent Coriolis force arises in a direction orthogonal to both the motion and the angular rate, thus in the y direction.

The force does not cause displacements of the drive frame, which – as already discussed – is stiff along this axis. Conversely, the sense frame can displace in the y direction. This displacement is read out through parallel plates, for two main reasons: (i) the Coriolis force is usually tiny, and displacements are in the order of few 10s nm. With typical gaps in the um order, this remains a small displacement and nonlinearity out of this sensing technique remains quite limited; (ii) the sense frame does not really require a large Q factor, especially if operation occurs in mode-split.



Though in principle suitable to sense angular rates, the structure depicted above strongly suffers from effects of linear accelerations along the y-axis. Causing large displacements (up to 10 times the maximum displacement induced by the Coriolis force), though not modulated at the drive frequency, accelerations can perturb the gyroscope operation. For this reason, a tuning fork architecture, shown above, is usually adopted.

The device is essentially replicated twice, with three major tricks: (i) the two halves shall move in antiphase, with a velocity which is thus at any time equal and opposite; (ii) the two halves are connected through a tuning fork spring, which yields an additional coupling contribution for the antiphase motion, separating the antiphase mode from the in-phase mode; (iii) electrodes of the two halves ar e connected in pairs such that antiphase motion in the sense direction, excited by the Coriolis force, is effectively readout as a differential signal, while in phase motion in the y-direction, excited by accelerations, results in an overall null capacitance variation (variations on one half are compensated by identical, opposite variations on the other half).



An additional refinement in the gyroscope design consists in a double decoupling, as shown in the last figure aside. Here there are three frames per each device half. The drive frame remains the same as above. The sensing part is actually split into (i) a decoupling frame, dragged by the drive motion but without nested parallel plates, and (ii) a sense frame. The latter is connected to the substrate by springs with low stiffness in the y-direction and large stiffness along the drive axis, and to the sense frame by springs which, conversely, are stiff along the y-direction. In this way, during the drive motion the sense frame is not moving. In presence of a Coriolis force, it is dragged by the decoupling frame and yields a capacitance variation on the parallel plates which are nested inside the sense frame. This trick has the purpose of minimizing fringe field effects at the sense parallel plate edges during drive motion, and of minimizing quadrature.

In terms of area (and thus mass) distribution, it is good to maximize the area of the sense frame, which shall be subject to the Coriolis force. The drive frame does not need large mass (its purpose is just to provide the linear velocity that enab les the Coriolis action). The sense frame also does not need a large mass for two reasons: (i) it is not directly feeling the action of the Coriolis force, as it is not moving during drive motion; (ii) more mass means more effects of linear accelerations, which is undesired.

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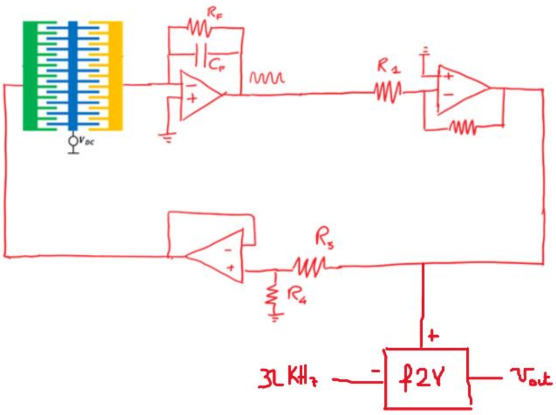
Question n. 2

The system aside is used as an ultra-linear MEMS-based temperature sensor. The block "f2V" converts a frequency difference between a square wave and a 32 kHz reference into a voltage proportional to the frequency difference.



(i) given the parameters in the table, find the required value of the resistance R_3 in order to have an output of the front-end stage of 32 mV at the reference ambient temperature;

(ii) write the linearized relationship from input temperature T to output voltage v_{out} and find the maximum nonlinearity in the sensor response in a ±50 K range around ambient temperature;

(iii) discuss the opportunity of using an amplitude-gain-control circuit in this type of system.

Parameter [unit]	Value
Natural resonance frequency	32 kHz
Process thickness	32 μm
Comb-finger cells (per port)	32
Comb-finger gap	3.2 μm
Rotor voltage	32 V
Supply Voltage	±3.2 V
Damping coefficient at 300 K	3.2 10^{-7} kg/s
Amplifier feedback capacitance	3.2 pF
Amplifier feedback resistance	320 kΩ
Resistance R_4	3.2 kΩ
Frequency to voltage gain f2V	32 mV/Hz
Young's modulus T coefficient	-60 ppm/K

Physical Constants

ε_0 = 8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

T = 300 K;

(i)

The system implements a MEMS-based oscillator. We first verify whether the front-end operation acts as a trans-resistance or trans-capacitance amplifier. Given the 32 kHz operation frequency, and given that the first stage pole lies at:

$$f_p = \frac{1}{2\pi R_F C_F} = 155\ kHz$$

we can conclude that operation occurs in trans-resistance mode as the pole falls beyond the operating frequency. In this situation, the relationship between the drive voltage and the front-end output is:

$$v_{out,1} = -i_m R_F$$

For resonant operation, the equivalent electrical model of the MEMS is just its equivalent resistance, thus, neglecting the – sign and considering just the modulus, we get:

$$v_{out,1} = v_d \frac{4}{\pi} \frac{R_F}{R_{eq}} = v_d \frac{4}{\pi} \frac{R_F}{b} \eta^2 = v_d \frac{4}{\pi} \frac{R_F}{b} \left(V_{DC} \frac{dC}{dx} \right)^2$$

where v_d is the drive square wave amplitude and the factor $4/\pi$ accounts for the first harmonic of the square wave. The equivalent resistance yields a value of 9.7 MΩ.

As the drive wave v_d is just the supply voltage reduced by the resistive voltage divider, we get:

$$v_{out,1} = V_{DD} \frac{R_4}{R_4 + R_3} \frac{4}{\pi} \frac{R_F}{R_{eq}}$$

Setting the output to 32 mV and solving for R_3 we find a resistance value of 10.2 kΩ.

(ii)

The Young's modulus E is known to drift in temperature with the given -60 ppm/K = 2α coefficient. Correspondingly, the same temperature variation occurs on the stiffness, which is linear with E. When linearizing, one usually assumes that the frequency, proportional to the stiffness square root, goes with ½ of that coefficient, so α = -30 ppm/K.

In this situation, the relationship between temperature and output voltage is simply:

$$v_{out} = G_{f2V} \cdot [f_0(1 + \alpha\Delta T) - 32\ kHz] = G_{f2V} \cdot [f_0(1 + \alpha\Delta T) - 32\ kHz]$$

Assuming that the resonator frequency f_0 at the reference temperature T_0 matches the reference value at 32 kHz, one gets:

$$v_{out} = G_{f2V} \cdot \Delta f_{lin}(T) = G_{f2V} \cdot f_0 \cdot \alpha \cdot \Delta T \quad \rightarrow \quad \frac{v_{out}}{\Delta T} = G_{f2V} \cdot f_0 \cdot \alpha = -30.7\frac{mV}{K}$$

When assuming the full nonlinear relationship between temperature and frequency, one can write:

$$\Delta f(T) = \frac{1}{2\pi}\sqrt{\frac{k(T)}{m}} - \frac{1}{2\pi}\sqrt{\frac{k(T_0)}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_0(1+2\alpha\Delta T)}{m}} - \frac{1}{2\pi}\sqrt{\frac{k_0}{m}} = f_0\left(\sqrt{(1+2\alpha\Delta T)} - 1\right)$$

where k_0 represents the stiffness at the reference temperature. The nonlinearity, assumed at the percentage deviation between the linearized and nonlinear expression, normalized to the maximum temperature change, can be calculated as:

$$\epsilon_{lin,\%} = \frac{\Delta f(T) - \Delta f_{lin}(T)}{\Delta f_{FSR}} \cdot 100 = \frac{\cancel{f_0}\left(\sqrt{(1+2\alpha\Delta T)} - 1\right) - \cancel{f_0}\alpha\Delta T}{\cancel{f_0}\left(\sqrt{(1+2\alpha\Delta T_{max})} - 1\right)} \cdot 100$$

For the maximum temperature, + 50°C, the nonlinearity turns out to be only:

$$\epsilon_{lin,\%} = \frac{\left(\sqrt{(1+2\alpha\Delta T_{max})} - 1\right) - \alpha\Delta T_{max}}{\left(\sqrt{(1+2\alpha\Delta T_{max})} - 1\right)} = \left(1 - \frac{\alpha\Delta T_{max}}{\left(\sqrt{(1+2\alpha\Delta T_{max})} - 1\right)}\right) = 0.08\ \%$$

confirming that the sensor is quite linear.

(iii)

AGC stages are widely used in MEMS-based oscillator, whenever one needs to get a controlled motion amplitude. It is, e.g., the case of MEMS gyroscopes, where the scale factor is proportional to the drive amplitude. There, the AGC is used to keep the scale factor stable against variations of the Q factor.

Also in the situation considered in this exercise the Q factor can change with temperature. However, as nothing in the system discussed here is related to the drive motion amplitude, there is actually no need for an amplitude control (AGC) stage.

Besides, an AGC would prefer a trans-capacitance front-end stage, rather than a trans-resistance amplifier, in order to have a signal at the front-end output proportional to the displacement and not to the velocity.

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Question n. 3

You are developing a 3T CMOS imaging system for automotive applications, based on an infrared laser at 830 nm. The target specifications and process parameters are shown aside.

- **(i)** choose a pixel size that exactly matches optical limits, and calculate the fill factor and the dark current;
- **(ii)** evaluate the maximum SNR and verify whether it matches the specifications;
- **(iii)** evaluate the DR and verify whether it matches the specifications.

Dark current density	10 aA/(μm)^2
Area of 3 MOS transistors	2 μm x 3 μm
Lens diameter	10 mm
Focal length	40 mm
Depletion capacitance/area	0.1 fF/(μm)^2
Gate capacitance	0.2 fF
Supply Voltage	3 V
Target maximum SNR	50 dB
Maximum pixel size	10 μm
Target DR at 1 ms	75 dB

Physical Constants

ε_{Si} = 8.85 10^{-12}·11.7 F/m
k_b = 1.38 10^{-23} J/K;
T = 300 K;
q = 1.6 10^{-19} C;

(i)

At such long wavelength in the near infrared range, diffraction yields an airy disk spot as large as:

$$d_{Airy} = 2.44\frac{\lambda}{D}\cdot f = 2.44\,\lambda\,F_{\#} = 8.1\,\mu m$$

And we thus choose this value as our pixel side, $d_{pix} = d_{Airy}$, in order to have a balanced system as required in the text.

The fill factor is immediately calculated as:

$$FF = \frac{A_{active}}{A_{pixel}} = \frac{d_{pix}^2 - A_{3MOS}}{d_{pix}^2} = 0.91 = 91\%$$

For the dark current calculation, we need to take care about the fact that is gathered only underneath the sensor area. We thus get:

$$i_d = J_d \cdot d_{pix}^2 \cdot FF = \ 0.6\,fA$$

(ii)

The maximum SNR in a 3T topology depends only on the maximum number of electrons that can be collected in the potential well. In particular, as photo-generation is a Poisson process, neglecting spatial noise we get:

$$SNR_{max} = 20\cdot\log_{10}\sqrt{N_{el}} = 20\cdot\log_{10}\sqrt{\frac{Q_{max}}{q}} = 20\cdot\log_{10}\sqrt{\frac{C_{int}V_{DD}}{q}}$$

The integration capacitance is the sum of the gate capacitance and the depletion capacitance per unit area multiplied by the active area:

$$C_{int} = C_g + C'_{dep}\cdot d_{pix}^2\cdot FF = 6.2\,fF$$

We thus get a maximum SNR of 50.6 dB, which matches the specifications.

(iii)

The well-known DR formula, neglecting all noise contributions for which we have no data (spatial noise, quantization noise…) is:

$$DR = 20 \cdot \log_{10} \frac{C_{int} V_{DD}}{\sqrt{q \cdot i_d \cdot t_{int} + k_B \cdot T \cdot C_{int}}}$$

The formula yields 71 dB. As we have neglected all other noise sources (quantization, PRNU, DSNU…), it is certain that the final system will actually be even worse and will not match the target value.

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