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Question n. 1

Define photo response nonuniformity in a digital camera and list all the sources along the imaging pipeline that can give rise to this undesired effect.

Why is the calibration procedure for this nonideality much more complex than for dark signal nonuniformity? How is it implemented?

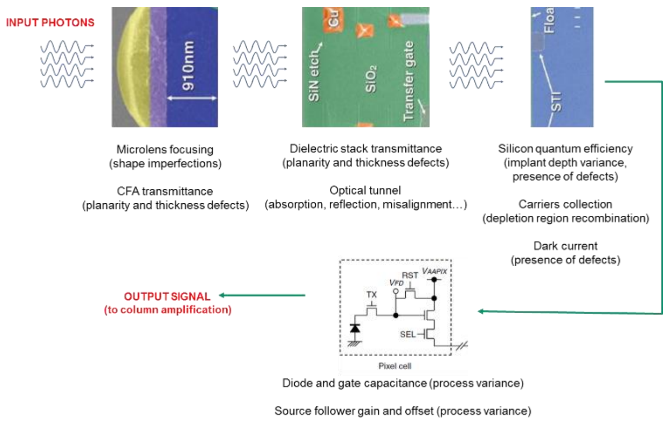
Why do errors remain even after calibration, and how do they affect the signal to noise ratio?

Photo response nonuniformity (PRNU) represents the average standard deviation found at the output of all the pixels of identical type in a digital camera, when illuminated by a uniform light source, and ideally in absence of temporal noise and DSNU. For 3-channel cameras (RGB), PRNU is thus in principle expressed separately for each R, G and B pixel type. Often, its value is expressed as a percentage of the average sensor output.

PRNU belongs thus to the category of spatial noise. Practically, its physical source is not a stochastic quantity (strictly speaking, it is not noise), but the result of several deterministic deviations discussed below. However, the result is perceived as noise and in everyday life photography it cannot be distinguished from temporal noise.

As a consequence of this definition, while DSNU is an offset related contribution, PRNU is a gain-related contribution.

Sources of PRNU can be found analyzing all the terms that form the gain from photon input to pixel output. Wherever these terms are different from pixel to pixel due to fabrication nonuniformities, PRNU will arise:



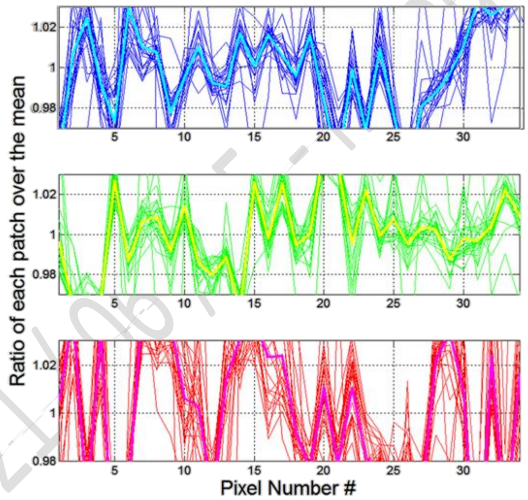
- absorption of light rays depends on the transmittance spectral density of microlenses and of color filter arrays;
- conversion from absorbed photons into photocurrent depends on the quantum efficiency, which is a function of absorption depth and thus of the wavelength. It can vary from pixel to pixel due to geometrical nonuniformities and defects in the silicon active layer;
- the photocurrent in integrated over the photodiode capacitance (or the floating diffusion capacitance in 4T structures). Geometrical nonuniformities give rise to different values of such capacitance, and thus to different gain;
- a source follower within the 3T or 4T pixel buffers the signal to the output. Small variations in the follower gain result in gain fluctuations from pixel to pixel.

The fact that PRNU is related to the spectral response of the pixel makes its calibration/compensation particularly challenging. As a clarifying example, differences between pixels occurring in the blue portion of the spectrum will affect the response to blue radiation, but not the response to red radiation. As the impinging spectrum in not known a priori, the same defects can give rise to more or less relevant effects to the output, depending on the characteristics of the input spectrum.

Practically, a universal gain correction for the pixels, which fixes PRNU for any of the infinite possible input spectral density, does not exist. This is a difference with respect to DSNU, which can be instead ideally corrected perfectly, at least at a reference temperature.

The optimal calibration usually occurs through the following steps:

- a set of N known representative spectral reflectances (e.g. a color checker) is used under a known illuminant (e.g. a sunlight illuminant, like the D65);
- for each of the N-th spectral reflectances, several images are captured and averaged to eliminate temporal noise contributions. DSNU should be also eliminated from all pixels through DSNU calibration;
- the procedure is repeated for all the N spectra. Afterwards, 3xN graphs plotting, for each R, G and B type channels, the normalized output (i.e. the output with respect to the average output) of all the sensor pixel can be drawn. An example is shown in the figure for a simplified case of 24 spectra and 35 pixels only.
- as expected, for a given color channel, the deviation of each pixel from the average will not be the same for the N different spectra. However, there will typically be a correlation, and the average (bold curves in the graph) can be taken as the optimal curve to apply gain correcting coefficient to compensate PRNU.

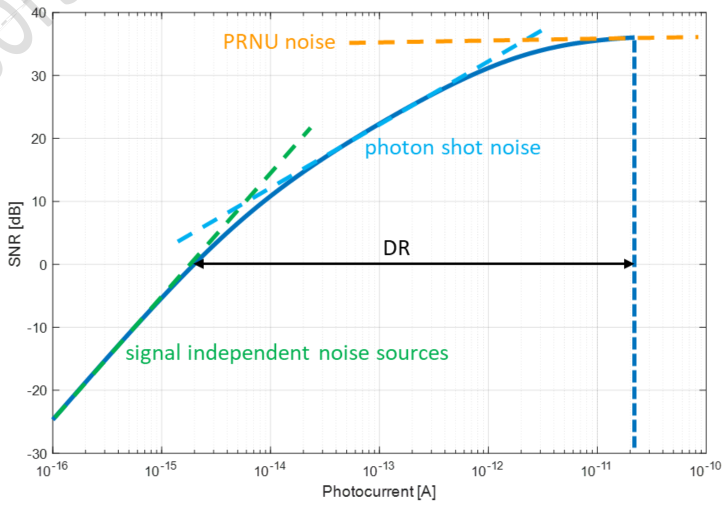


The result is that PRNU will be reduced, typically from values around 10% before calibration to values in the order of 1% after calibration.

The residual PRNU can be easily evaluated as the product of the % PRNU multiplied by the signal value. As a consequence, we expect the PRNU to:

- be the dominant noise contribution at large signal values (large photocurrents);
- cause the SNR to flatten at large signals, as it is itself proportional to S.

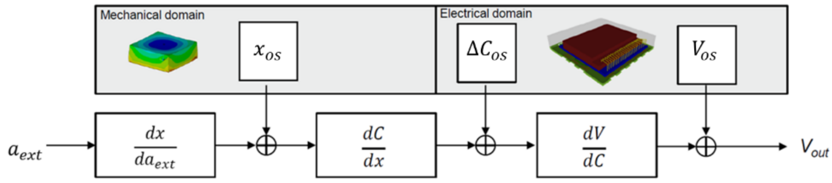
An example of PRNU effects on the SNR is shown in the graph.



Last Name __ZONA___ **Given Name** ___BIANCA__ **ID Number** __20210625_

Question n. 2

The figure aside is the acquisition chain of a parallel-plate MEMS accelerometer affected by various offset sources. Other parameters of the sensor are indicated in the table.



(i) compute the expression and the numerical value of the gain blocks of the figure ($\frac{dx}{da_{ext}}, \frac{dC}{dx}, \frac{dV}{dC}$), and, neglecting by now the offset sources, the accelerometer full-scale range in gravity units;

(ii) compute the expression and the numerical value of the three input-referred zero-g-offset contributions (in terms of gravity units);

(iii) assume that you can change one parameter to improve the zero-g-offset performance down to less than 1 g. Which one would you choose? Which trade-offs do you expect?

Parameter [unit]	**Value**
Displacement offset x_{os}	84 nm
Differential capacitive offset ΔC_{os}	7 fF
Output voltage offset	9 mV
Natural resonance frequency	3 kHz
Process thickness	40 μm
Parallel-plate sensing cells	5
Parallel-plate length	120 μm
Parallel-plate gap	1.6 μm
Rotor voltage and supply voltage	2 V
Proof mass	15 nkg
Amplifier feedback capacitance	160 fF

Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m
$k_b = 1.38\ 10^{-23}$ J/K;
T = 300 K;
$q = 1.6\ 10^{-19}$ C;

(i)

The three gain block are those characteristic of a parallel-plate MEMS accelerometer. We thus have :

$$\frac{dx}{da_{ext}} = \frac{1}{\omega_0^2} = \sqrt{\frac{k_{tot}}{m}} = \sqrt{\frac{k_m + k_{el}}{m}}$$

where the electrostatic stiffness can be evaluated as :

$$k_{el} = -2\frac{C_0}{g^2}V_{DD}^2 = -0.41\ N/m$$

The resulting gain turns thus to be $\frac{dx}{da_{EXT}} = 3\frac{nm}{m/s^2} = 0.03\frac{\mu m}{\hat{g}}$.

Next, the differential capacitive gain under the small displacement approximation is :

$$\frac{dC}{dx} = 2\frac{C_0}{g} = 166\frac{fF}{\mu m}$$

Finally, the gain of a charge amplifier can be written as :

$$\frac{dV}{dC} = \frac{V_{DD}}{C_F} = 12.5\frac{mV}{fF}$$

Overall, the acceleometer scale-factor becomes

$$S = 2\frac{C_0}{g}\frac{V_{DD}}{C_F}\frac{1}{\omega_0^2} = 62.1\frac{mV}{\hat{g}}$$

where $\hat{g}$ represents the gravity unit. The saturation limit, neglecting offset contributions, can be evaluated by assuming that the maximum acceleration corresponds to the full electronic supply :

$$FSR = \frac{\pm V_{DD}}{S} = 32.3\ \hat{g}$$

(ii)

The various offset sources can be brought back as input-referred terms by dividing them by the correspondingly required transfer functions :

$$a_{os_{x_{os}}} = \frac{x_{os}}{dx/da_{ext}} = x_{os}\ \omega_0^2 = 2.8\ \hat{g}$$

$$a_{os_{\Delta C_{os}}} = \frac{\Delta C_{os}}{dC/dx \cdot dx/da_{ext}} = \frac{\Delta C_{os}\ \omega_0^2\ g}{2\ C_0} = 1.4\ \hat{g}$$

$$a_{os_{V_{os}}} = \frac{V_{os}}{dV/dC \cdot dC/dx \cdot dx/da_{ext}} = \frac{V_{os}\ \omega_0^2\ g\ C_F}{2\ C_0\ V_{DD}} = 0.15\ \hat{g}$$

(iii)

The dominant offset terms appear to bet hose related to native displacement imbalance and to native capacitive imbalance. The parameter that helps in mitigating this offset contribution is the resonance frequency, which shall be lowered.

However, trade-offs arise with (i) the maximum sensing bandwidth, which will be reduced accordingly and (ii) the full-scale range, which also decreases as the scale factor increases given the drop in ω_0.

In conclusions, parallel-plate capacitive accelerometer appear to be critical in terms of trade-off between input-referred offset and full-scale range.

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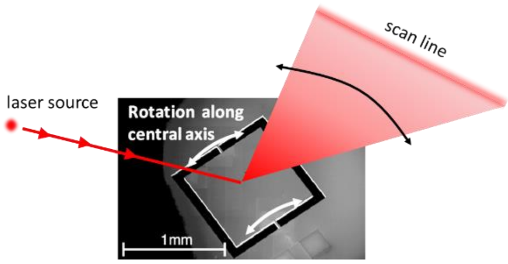
Question n. 3

A MEMS suspended square plate is used as a beam reflecting, tilting mirror (a.k.a. MEMS micromirror) to project a scan line on a screen. It is formed by a silicon mass, suspended by a pair of torsional springs as shown in the figure.



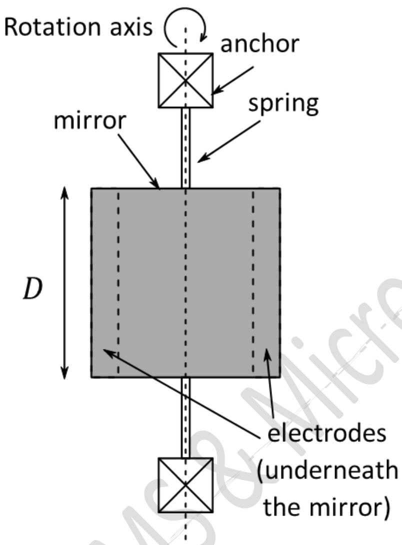
Process height	h	20 µm
Laser wavelength	λ	680 nm
Parallel-plate Gap	g	100 µm
Mirror side	D	1 mm
Quality factor	Q	500
Parallel-plate length	L_{pp}	1 mm
Parallel-plate width	W_{pp}	200 µm
Silicon density	ρ	2330 kg·m^{-3}
Target tilt angle	θ	±1°
DC actuation voltage	V_{DC}	50 V
AC actuation voltage	v_{ac}	50 V

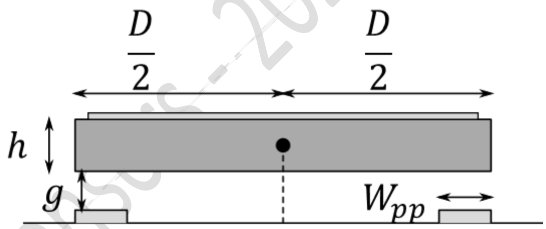
The device is actuated at resonance by electrostatic parallel plates designed underneath the mirror (see the additional schematic figures below). The plates are driven by anti-phase AC signals, centered on the same DC value, to generate the device rotation. The rotor is biased to ground. You are asked to:

(i) choose the resonance frequency to match the target tilt angle, neglecting non-linear effects;
(ii) compute the intrinsic rms noise (units of degrees) that affects the position of the projected spot;
(iii) estimate the spot-size (units of degrees) of each projected point along the scan line, due to diffraction effects. How many "pixels" can fit into one scan line, using this device? Briefly comment the result.

Physical Constants

ε_{Si} = 8.85 10^{-12}·11.7 F/m
k_b = 1.38 10^{-23} J/K;
T = 300 K;





(i)

The effective electrostatic torque applied to the mirror is the given by the difference of the forces generated by each parallel plate times the lever arm:

$$M = \left(\frac{D}{2} - \frac{W_{pp}}{2}\right) \cdot (F_{el1} - F_{el2}) = \frac{D}{2}\left(\frac{(V_{DC} + v_{ac})^2}{2}\frac{\partial C}{\partial z} - \frac{(V_{DC} - v_{ac})^2}{2}\frac{\partial C}{\partial z}\right)$$
$$= \left(\frac{D}{2} - \frac{W_{pp}}{2}\right) \cdot \frac{4V_{DC}v_{ac}}{2} \cdot \frac{\partial C}{\partial z} = (D - W_{pp})V_{DC}v_{ac} \cdot \frac{\partial C}{\partial z}$$

where the capacitance variation is assumed in the linear range for small displacements:

$$\frac{\partial C}{\partial z} = \frac{C_0}{g} = \frac{\epsilon_0 W_{pp} L_{pp}}{g^2}$$

with $C_0 = 17.7$ fF. The z coordinate is related to the tilt angle ϑ by the usual relation:

$$z = \left(\frac{D}{2} - \frac{W_{pp}}{2}\right)\tan(\vartheta) \approx \left(\frac{D}{2} - \frac{W_{pp}}{2}\right)\vartheta$$

The transfer function from input AC voltage to tilt angle is thus:

$$\frac{\vartheta}{v_{ac}} = (D - W_{pp})V_{DC}\frac{\epsilon_0 W_{pp} L_{pp}}{g^2} \cdot \frac{Q}{k_\vartheta}$$

The required torsional stiffness is:

$$k_\vartheta = (D - W_{pp})V_{DC}\frac{\epsilon_0 W_{pp} L_{pp}}{g^2} \cdot Q \cdot \frac{v_{ac,max}}{\vartheta_{max}} = 1.014 \cdot 10^{-5}\ \text{Nm}$$

the moment of inertia of the structure is:

$$I = \frac{\left(\frac{D}{2}\right)^2 \cdot (m_1 + m_2)}{3} = \frac{1}{3}\left(\frac{D}{2}\right)^2 \cdot D^2 h\rho$$

so the resonance frequency results:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_\vartheta}{I}} = 8.15\ \text{kHz}$$

(ii)

The torque-to-angle transfer function is a second order low-pass filter with a quite large quality factor. We can calculate the effect of thermomechanical noise by multiplying it times the DC value of the transfer function:

$$S_\vartheta = \frac{\sqrt{4k_bTb_\vartheta}}{k_\vartheta} = \frac{\sqrt{\frac{4k_bT\omega_0 I}{Q}}}{k_\vartheta}$$

then integrating this value over the equivalent bandwidth:

$$BW_{eq} = \frac{\omega_0 Q}{4} \quad \Rightarrow \quad \sigma_{\vartheta,deg} = \frac{\sqrt{\frac{4k_bT\omega_0 I}{Q}}}{k_\vartheta} \cdot \sqrt{\frac{\omega_0 Q}{4}} \cdot \left(\frac{180}{\pi}\right) = 33.2\ \text{m}°$$

Last Name __ZONA___ **Given Name** ___BIANCA__ **ID Number** __20210625_

(iii)

Approximating the mirror as a circular aperture with diameter D, the diffraction limit in terms of angle can be calculated as:

$$\Delta\vartheta_{diff} = 2.44\frac{\lambda}{D} = 95.1\ \mathrm{m}^\circ$$

The total field-of-view is four times the tilt angle, by Snell reflection laws. The total number of different pixels that can be distinguished is thus the ratio of the total field-of-view over the diffraction limit:

$N_{pix} = \frac{4\vartheta}{\Delta\vartheta} \approx 42$