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Question n. 1

Nowadays, the optimization of the design of sensors of various physical quantities is assisted by CAD softwares. Describe in details (i) the motivations for using a CAD software, (ii) the typical steps that one should implement in a Finite-Element-Method (FEM) simulation, and (iii) the types of simulations that can be performed along with their purpose. Finally, (iv) clarify what a "coupled multi-physics" simulation implies. Possibly, bring examples related to the topics studied in the course all along your discussion.

While a theoretical background is of paramount importance in the co-design of sensor systems (i.e. sensor + electronics), in order to understand several trade-offs and minimize their effects, at an advanced level the use of computer-aided-design software is nowadays essential.

The main motivations lie in that theoretical equations that can bring a closed-form solution have usually limitations that (a) neglect **second- or third-order effects** (e.g. mechanical or electrostatic nonlinearity in MEMS), or (b) are related to **geometrical approximations** of the system (e.g. equations referring to a point-like mass instead of a full rigid body, or equations that model 1D depletion region instead of full 3D volumes…), or finally (c) cannot easily manage **multi-physics solutions** where different sets of coupled partial differential equations should be solved (more on this point will be discussed later).

In simulating sensors, the main steps consist in (a) the definition of the **problem geometry**, which can be a full 3D description, or an approximated 2D simplification. The latter is valid whenever we have conditions of simmetricity and helps in reducing computational cost and thus the time required to get a solution. It can be however imprecise at geometry edges, where conditions of simmetricity or continuity of the problem are lost. The second step is generally (b) the **description of the set of partial differential equations** that describe our problem (e.g. the Newton's law and electrostatic laws for MEMS sensors, or the Poisson-Electron-Hole equations in semiconductor-based photodiodes…), along with **their boundary conditions** (Dirichlet or Neumann conditions). From this standpoint, modern simulators already have these sets of equations embedded, relieving the user's need for writing the entire equations code. The third step consists in (c) **generating the mesh**, i.e. the set of finite element (points and volumes) where the software will iteratively solve the set of PDE to find a converging solution. Meshes can be less or more refined, obviously leading to faster or more accurate results, and can be formed by different geometrical elements (e.g. triangles or quadrangles in 2D domains). Finally, the fourth and fifth steps consist in letting the software **solve the problem** and – if a converging solution is found – in the **analysis of the results**. The latter point is of utmost importance, as the evaluation of results from a simulator shall be always a critical steps where the obtained behavior is effectively understood.

In the specific case of MEMS sensors, the mostly adopted types of simulations are **"stationary" analysis**, e.g. to evaluate the value of an electrical capacitance in complicate electromechanical geometries, or to evaluate the stress-strain (i.e. force-displacement) relationship, finding thus the value of the elastic stiffness. In light sensors, stationary analysis can be used to evaluate the biasing conditions in a photodiode (extension of the depletion region, maximum electric field, ecc…). Additionally, the so-called **"eigenfrequency" simulations** can be performed for MEMS sensors, to evalute the freqeuncy of the modes of interest and also of undesired high-order modes.

Typically, the theoretical design will guide the first draft design of the sensor, which will be then refined through simulation steps to adjust the desired performance to the target specifications, and the robustness against possible process spread which can be verified through **parametric simulations**.

Simulators are even more important when problems to solve involve mutiple physical domains, like electrostatic, mechanics, thermal, fluidic, semiconductor… Physical quantities will interact, generating so-called **coupled multi-physics problems**: to bring a MEMS-related example, a force generated in the electrostatic domain (between the arms of a capacitor) affects the results of a mechanical simulation which, in turn, due to electrostatic softening effects (gap changes) influences again the electrostatic domain. Such coupled equations are very often hard to solve in a closed loop-form, especially where complicated geometries are under analysis. Here, the use of simulators becomes almost unavoidable.

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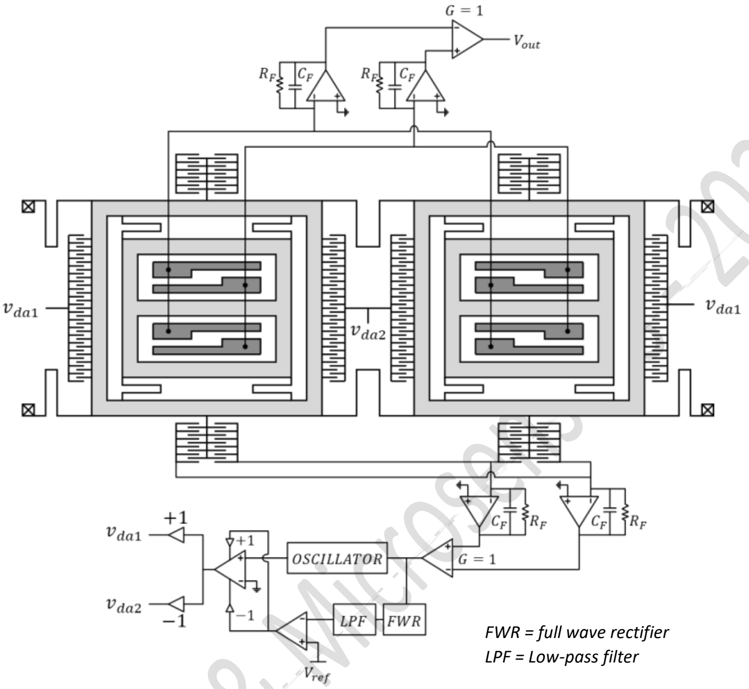
Question n. 2

You are tasked with finalizing the sizing of a dual-mass yaw gyroscope. Once fabricated, the gyro will be tested for scale-factor stability within the consumer temperature range (-40°C to 85°C) using a custom electronic setup. The relevant parameters for the device (given for the half-structure) and for the electronics are reported in the table. All parameters are given at room temperature ($T = 25°C$):

- **(i)** the maximum acceptable linearity error is 1%. Compute the required mode-split value, so to cope with the other given specifications;
- **(ii)** compute the required number of parallel plates to design a *well-balanced* system in terms of noise, adopting reasonable approximations where needed (for the sake of simplicity, assume that the Q_s factor is independent of the number of parallel plates);
- **(iii)** compute the maximum expected relative variation of the sensitivity $dV_{out}/d\Omega$, induced by the device and electronics temperature variations (for the sake of simplicity, assume that the AGC loop-gain is independent of temperature).

Physical Constants

ε_0 = 8.85 10^{-12} F/m
k_b = 1.38 10^{-23} J/K;
T = 300 K;
q = 1.6 10^{-19} C;



FWR = full wave rectifier
LPF = Low-pass filter

Table of parameters		
Full-scale range	FSR	2000 dps
Linearity error	ϵ_{lin}	1%
Drive frequency	f_d	22000
Drive mass	m_d	3 nkg
Sense mass	m_s	7 nkg
Sense mode Q factor	Q_s	500
Capacitive gap	g	1.2 µm
N. of drive fingers	N_{CF}	15
Rotor voltage	V_{rot}	10 V
Op-amp voltage noise	$\sqrt{S_{vn}}$	20 nV/$\sqrt{\text{Hz}}$
Parasitic capacitance	C_P	10 pF
Process height	h	20 µm
PP length	L_{PP}	100 µm
Feedback capacitance	C_F	200 fF
Temperature coefficient of feedback capacitance	$\frac{dC_F}{dT}$	40 ppm/K
AGC loop gain	$G_{loop_{AGC}}$	30
AGC reference	V_{ref}	1.5 V

1) The linearity error for a differential readout configuration is evaluated at the FSR to maximize the sensitivity:

$$\epsilon_{lin} = \left(\frac{y_{FSR}}{g}\right)^2 = 0.1\% \quad \Rightarrow \quad y_{FSR} = g\sqrt{\epsilon_{lin}} = 120\ nm$$

The sensitivity is evaluated at the full-scale range, but is also expressed as the ratio of drive displacement amplitude and frequency split:

$$S_y = \frac{dy}{d\Omega} = \frac{y_{FSR}}{FSR} = \frac{120\ nm}{2000\ dps} = 60\frac{pm}{dps} = 3.438\frac{nm}{rad/s}$$

$$S_y = \frac{x_{da}}{\Delta\omega_{MS}} = \frac{x_{da}}{\omega_d - \omega_s}$$

The drive displacement amplitude can be calculated by the closed loop gain of the drive loop, knowing the value of the AGC voltage reference:

$$x_{da} = \frac{V_{ref}}{\left(\frac{2N_{CF}\epsilon_0 h}{g} \cdot \frac{V_{ROT}}{C_{F,d}} \cdot \frac{2}{\pi}\right)} = 5.32\ \mu m$$

and thus, the frequency split is calculated inverting the formula of the sensitivity:

$$\Delta\omega_{MS} = \frac{x_{da}}{S_y} = \frac{FSR \cdot x_{da}}{y_{FSR}} = 1549\ \frac{rad}{s} \quad \Rightarrow \quad \Delta f_{MS} \approx 246\ Hz$$

$$f_s = f_d + \Delta f_{MS} \approx 22246\ Hz$$

2) To design a well-balanced system the input-referred electronic noise should be approximately equal to the intrinsic contribution of the gyroscope. The gyro force noise, referred to the NERD, is:

$$NERD_{MEMS} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{4k_B T b_s}}{2m_s x_{da}\omega_{da}} \cdot \left(\frac{180}{\pi}\right) = \frac{\sqrt{2 \cdot \frac{4k_B T \omega_s m_s}{Q_s}}}{2m_s x_{da}\omega_{da}} \cdot \left(\frac{180}{\pi}\right) = 699\,\frac{\mu\text{dps}}{\sqrt{Hz}}$$

where the factor 2 in the square root accounts for the two halves, and the factor $180/\pi$ is needed to convert to dps. The electronic noise should be thus equal to such value. The noise can be computed at the output of the sense TIAs and then referred to NERD using the full sensitivity to voltage output:

$$S_V = \frac{dV}{d\Omega} = \frac{dy}{d\Omega} \cdot \frac{dC}{dy} \cdot \frac{dV}{dC} = S_y \cdot \frac{dC}{dy} \cdot \frac{dV}{dC}$$

Where the differential parallel plate capacitance variation is:

$$\frac{dC}{dy} = 2\frac{C_0}{g} = 2\frac{N_{PP}}{g}C_{01PP}$$

$$C_{01PP} = \frac{\epsilon_0 L_{pp} h}{g} = 14.75\ fF$$

And the capacitance to voltage sensitivity accounts for the factor two due to differential readout of the two half structures:

$$\frac{dV}{dC} = 2\frac{V_{ROT}}{C_F}$$

No information is provided on the sense TIA feedback capacitance, however with reasonable assumptions the input referred op-amp voltage noise is independent on its value. Indeed:

$$NERD_{ELN} = \frac{\sqrt{2S_{vn}} \cdot \left(1 + \frac{C_P}{C_F}\right)}{S_V} \approx \frac{\sqrt{2S_{vn}} \cdot \left(\frac{C_P}{C_F}\right)}{S_y \cdot 2\frac{N_{PP}}{g}C_{01PP} \cdot 2\frac{V_{ROT}}{C_F}} = \frac{\sqrt{2S_{vn}}}{S_y \cdot 2\frac{N_{PP}}{g}C_{01PP} \cdot 2\frac{V_{ROT}}{C_P}}$$

Thus, the number of required parallel-plate cells is:

$$NERD_{ELN} \leq NERD_{MEMS} \quad \Rightarrow \quad N_{PP} \geq \frac{\sqrt{S_{vn}}}{S_y \cdot 2\frac{C_{01PP}}{g} \cdot 2\frac{V_{ROT}}{C_P} \cdot NERD_{MEMS}} = 13.69 \quad \Rightarrow \quad N_{PP} = 14$$

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3) The relative sensitivity variation is equal to the relative variation of the drive motion amplitude:

$$S_y = \frac{x_{da}}{\Delta\omega_{MS}} \Rightarrow \frac{dS_y}{S_y} = \frac{dx_{da}}{x_{da}}$$

The drive displacement amplitude depends on the variation of the drive springs stiffness, the drive quality factor, and the TIA feedback capacitance. The first two contributions are reduced by the AGC loop gain, since these disturbances are due to a variation of the actuation efficiency which is compensated by the amplitude loop. The capacitance contribution, on the other hand, affects the displacement readout and cannot be compensated by the AGC. Following these considerations, the relative variation is:

$$\frac{dx_{da}}{x_{da}} = \left(\frac{dk}{k} + \frac{dQ}{Q}\right) \cdot \frac{1}{1 + G_{loopAGC}} + \frac{dC}{dT}\Delta T = \left(-60\frac{ppm}{K}\Delta T - \frac{\Delta T}{2T}\right) \cdot \frac{1}{1 + G_{loopAGC}} + 40\frac{ppm}{K}\Delta T$$

The TCA sensitivity is also affected by the capacitance variation, with opposite sign, thus cancelling out in the rate-to-voltage sensitivity:

$$\frac{dS_V}{S_V} = \left(\frac{dk}{k} + \frac{dQ}{Q}\right) \cdot \frac{1}{1 + G_{loopAGC}}$$

The variation evaluated at both ends of the temperature range results in:

$$\left.\frac{dS_y}{S_y}\right|_{-40°C} = +0.34\%$$

$$\left.\frac{dS_y}{S_y}\right|_{85°C} = -0.31\%$$

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Question n. 3

A camera for high dynamic range acquisitions needs to be optimized when capturing pictures at 0.5 ms integration time, for scientific applications.

The camera is characterized by a 4T topology. Before implementing correlated double sampling (CDS) and before spatial noise (FPN) calibration, the camera shows the Photon Transfer Curve (PTC) in the figure.

Table of parameters		
Operating temperature	T	$280\ K$
Integration time	t_{int}	$0.5\ ms$
Dark current	i_d	$0.05\ fA$
Floating diffusion capacitance	C_{int}	$0.5\ fF$
Circuit biasing voltage	V_{DD}	$20\ V$
ADC number of bits	N_{bit}	14

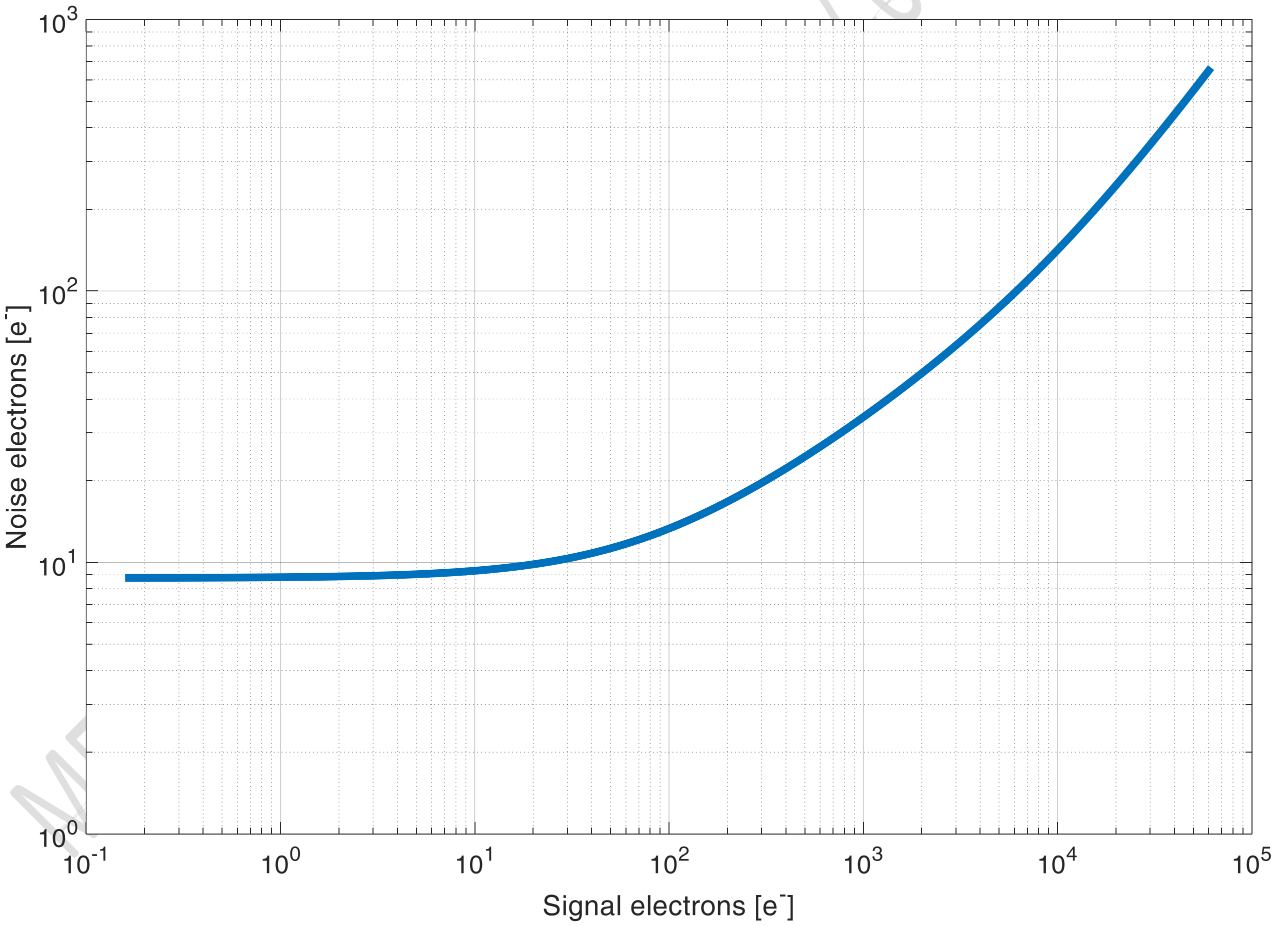
- **(i)** properly size the kTC noise reduction factor that needs to be achieved through the CDS operation;
- **(ii)** properly size the PRNU compensation factor that needs to be reached through the FPN calibration;
- **(iii)** on the same (or redrawn) quoted graph, plot the new PTC, and calculate the initial and final dynamic range and the initial and final maximum SNR.

Physical Constants

ε_{Si} = 8.85 10^{-12}·11.7 F/m

k_b = 1.38 10^{-23} J/K;

q = 1.6 10^{-19} C;



(i)

The implementation of the CDS operation always leave a residual effect of kTC noise, due ti process spreads. It is important that this residual noise is brought down to the level of other noise sources (or even to lower levels), in such a way that it does not worsen the overall noise performance.

In this case, we observe that at low signal values (flat region of the PTC) the dark current contributions and quantization contributions are given by:

$$\sigma_{i_d} = \frac{\sqrt{q \cdot i_d \cdot t_{int}}}{q} = 0.4\ e^-_{rms}$$

$$\sigma_{quant} = \frac{V_{DD}}{\sqrt{12}\ 2^{N_{bit}}} \frac{C_{int}}{q} = 1.1\ e^-_{rms}$$

While effectively kTC noise remains the dominant contribution:

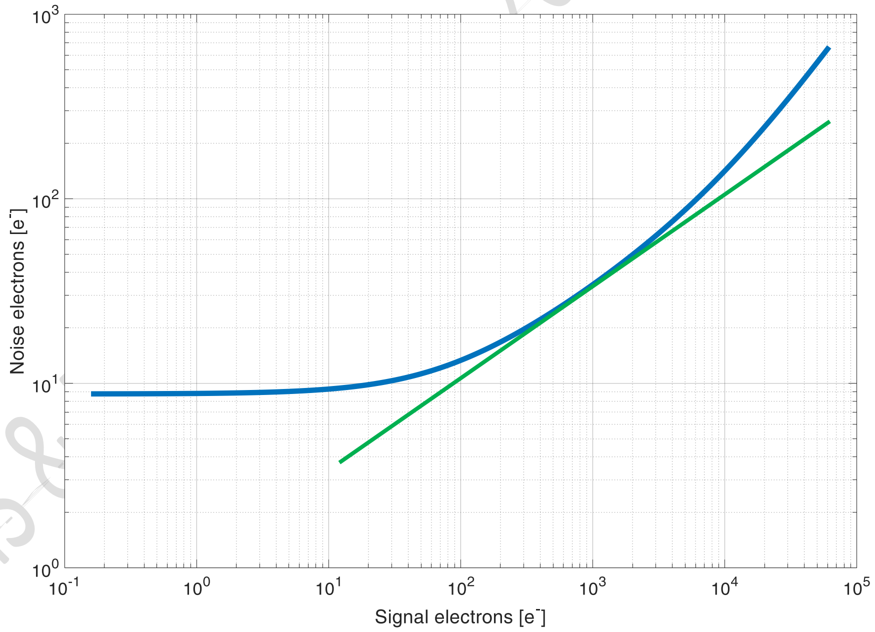
$$\sigma_{kTC} = \frac{\sqrt{kTC_{int}}}{q} = 8.7\ e^-_{rms}$$

which corresponds to the value which we read on the graph on the flat region. A well dimensioned system will require kTC noise to equal (or a bit below) the above-mentioned sources. Setting it to too large values would imply that the kTC noise reduction factor is excessively overstressed.
We thus easily find:

$$R_{kTC} = \frac{8.7\ e^-_{rms}}{\sqrt{(0.4\ e^-_{rms})^2 + (1.1\ e^-_{rms})^2}} = 7.4$$

Note that we assumed DSNU to be negligible as we had no information on this contribution.

(ii)
Similarly, in the calibration operation of fixed pattern noise, the aim will be to minimize PRNU effects down to a value that equals photon shot noise for the maximum input signal. From the graph, we can draw the curve with a slope of ½ that corresponds to photon shot noise:



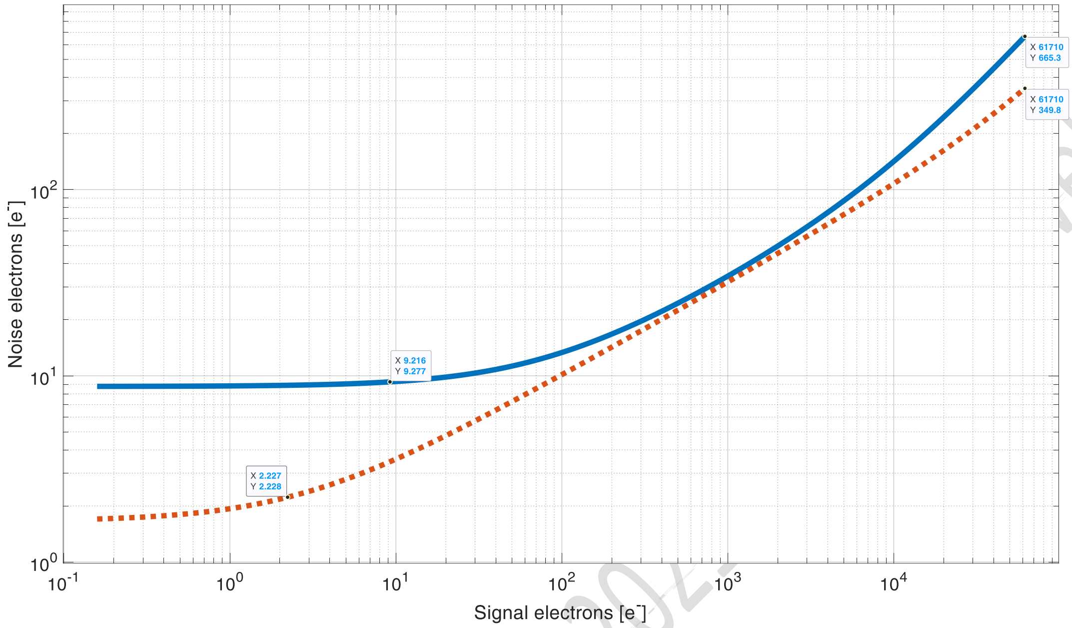
We note that at the maximum signal, photon shot noise corresponds to about 250 electrons, while PRNU noise equals about 650 electrons.
The required compensation factor is thus:

$$Comp_{PRNU} = \frac{650\ e^-_{rms}}{250\ e^-_{rms}} = 2.5$$

(iii)
We can now draw the new graph without even doing calculations, but just relying on what we sized so far. In particular, we know that at low signal levels we will have roughly a factor √2 the value of the quadratic

sum of dark current shot noise and quantization noise (indeed, we sized kTC noise reduction factor so to have kTC noise equal to that quadratic sum). This yields 1.6 electrons rms.

Conversely, at the highest bound of the graph we will have √2 the value of photon shot noise, which corresponds to about 350 electrons rms. The graph appears thus as below.



Through the markers, we have highlighted the values of noise for the points corresponding to the maximum signal ($\frac{V_{DD}C_{int}}{q} \approx 62000$ electrons). Additionally, we have highlighted the point in the graph where signal equals noise (SNR = 1).

We can thus easily find the DR and maximum SNR for the two conditions:

$$SNR_{initial_{\max}} = 20\log_{10}\frac{62000}{650} = 39.5\ dB$$

$$SNR_{final_{\max}} = 20\log_{10}\frac{62000}{350} = 44.9\ dB$$

$$DR_{initial} = 20\log_{10}\frac{62000}{9.3} = 76.5\ dB$$

$$DR_{final} = 20\log_{10}\frac{62000}{2.2} = 88.9\ dB$$