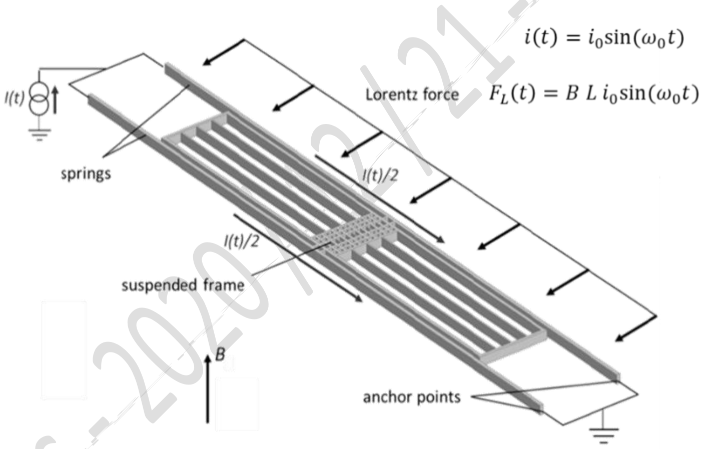
Last Name ___Suzum___ **Given Name** __Natale__ **ID Number** __20201221__

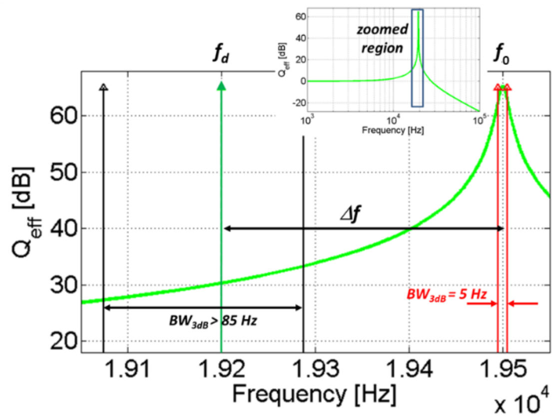
Question n. 1

You have to convince the manager of a MEMS company about the benefits of using MEMS magnetometers for a 9-axis inertial measurement unit (IMU) module: (i) discuss the general advantages (and drawbacks) of using MEMS-based solutions for magnetic field sensing; (ii) identify a solution to improve the magnetometer stability; (iii) identify a solution to improve the minimum measurable magnetic field; (iv) write the input referred magnetic field noise density and discuss the optimization of the current budget.

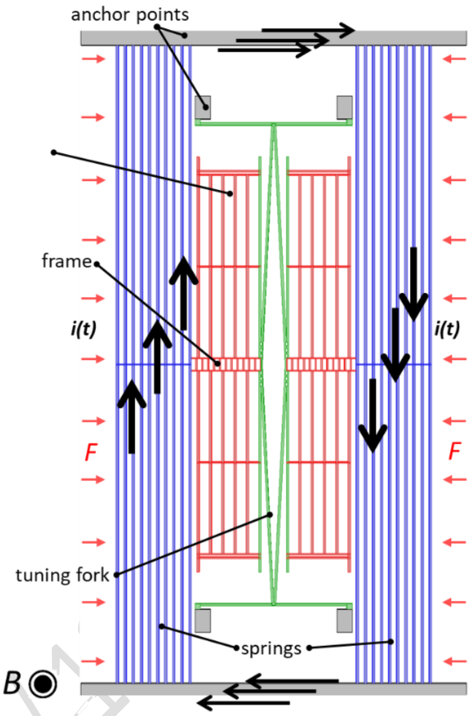
MEMS-based solutions for magnetic field sensing would enable their combination in a single-chip 9-axis inertial measurement unit, i.e. a unit formed by a 3-axis accelerometer, a 3-axis gyroscope, and a 3-axis magnetometer, where all the axes are perfectly aligned on chip, without the cost and need of assembling the magnetometer chip separately from the other MEMS sensors. Additionally, the absence of magnetic materials in a Lorentz force based MEMS magnetometer (whose schematic working principle is represented aside) enables to reach large FSR without typical constraints of such materials (nonlinearity, hysteresis ecc…). However, the high level of maturity of the existing technologies (AMR, MTJ, Hall-effect, ecc…) makes this achievement challenging, and one needs to find solutions to be competitive on all the target parameters (as usual represented by noise, bandwidth, FSR, area and consumption).



Looking at the general working principle of a MEMS based magnetometer, one can immediately discover that the Lorentz force, for a FSR magnetic field (e.g. few mT for the applications of interest in 9-axis units) and for maximum allowed currents of few 100 µA, is typically orders of magnitude lower than the FSR force of accelerometers or gyroscopes. While a solution to boost the effect of a tiny force is to operate at resonance (i.e. with the Lorentz current injected at the mechanical resonance frequency of the structure), known trade-offs arise, in particular (a) between noise density and bandwidth, and (b) in terms of scale-factor stability due to both frequency changes in temperature and, even more severly, to Q factor changes with temperature. One solution to improve stability against environmental changes is to adopt an off-resonance approach, similar to mode-split operation in gyroscope. This minimizes the effects of temperature changes (in particular of the Q factor), at the cost of a reduced scale factor. In turn, while the intrinsic noise density (NEMD) does not change with respect to resonant operation, effects of input-referred electronic noise will be much larger. Stability is better guaranteed if the driving frequency is itself set by another MEMS (e.g. a Tang resonator), so that drifts in frequency under temperature changes will be highly correlated between the drive frequency and the magnetometer frequency. Additionally, bandwdith can be easily extended to a consistent fraction of the distance between the drive and magnetometer frequencies (as shown in the figure).



To add a gain term to the scale factor, and thus to minimize again effects of electronic noise, a solution that consists in multiple recirculation of the same current can be adopted. At the cost of no extra current consumption, a factor of 10, or more, in sensitivity can be obtained with very limited area occupation by the spiral loop. A schematic view of such a structure, in this case for Z-axis sensing, is depicted in the figure aside. This approach is even more effective if the same current is made recirculating on all the three sensing axes. Another final step, consisting in the design of a monolithic multi-loop structure, also allows area reduction, making achievable performance competitive against other state-of-the art technologies. Typical achievable performance are a resolution in the range of 100 nT/√Hz, FSR in the order of 5 mT, bandiwdth up to 50 Hz or more, with overall current consumption in the sub-500-µA range for a device area (MEMS + Tang resonator) within 1 mm^2.



Specifically referring to noise, one can derive the expression of input-referred magnetic field density as a function of the three dominant noise terms, associated to (i) the thermomechanical contribution, (ii) to the amplifier contribution and (iii) to the feedback resistance contribution. The specific case of magnetometers is pretty interesting because we note that there are two current contributions, one that circulates in the device, to give rise to the Lorentz force, and the other one being the biasing contribution of the electronic stages.

$$\sqrt{S_{\mathrm{Bn},tot}} \propto \frac{1}{N_{loop}\, \boldsymbol{i_{MEMS}}\, L} \sqrt{\left(\sqrt{\frac{\chi_1}{\sqrt{k_n\, \boldsymbol{i_{MOS}}}}}\right)^2 + \left(\sqrt{\frac{\chi_2}{\boldsymbol{R_F}}}\right)^2 + \left(\sqrt{\chi_3\, \boldsymbol{b}}\right)^2}$$

The current budget should be allocated to (i) the sensor (i_{MEMS} above), (ii) the front-end (i_{MOS} above), (iii) the oscillator and (iv) the other stages of the sening chain.

As evident from the formula above (where χ_1, χ_2 and χ_3 include physical, geometrical and electrical effects), excluding the current contributions required for the oscillator and the other stages of the sensing chain, it is more advantageous to flow the remaining current into the sensor, as this term decreases directly all the three noise contributions – while the current in the amplifiers only reduces electronic noise and with a less than proportional law.

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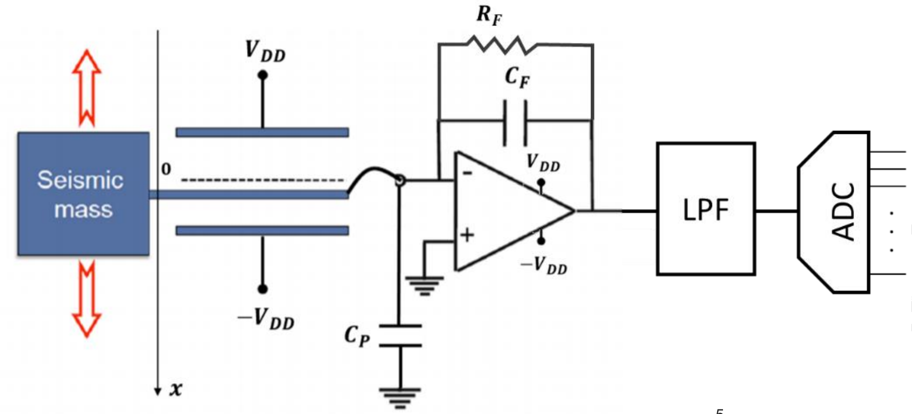
Question n. 2

You are designing a MEMS accelerometer for high-g applications, which needs to cope with the specifications in the table. The accelerometer is readout through the circuit shown below. You are asked to:

(i) define the resonance frequency in operation, and identify the pull-in voltage;

(ii) properly size the passive components R_F and C_F, the amplifier noise, and the ADC number of bits;

(iii) represent on the quoted graph below (a) the sum of the elastic force and the linearized electrostatic force, and (b) the inertial force for a FSR input, and then identify the rotor position.



Physical Constants

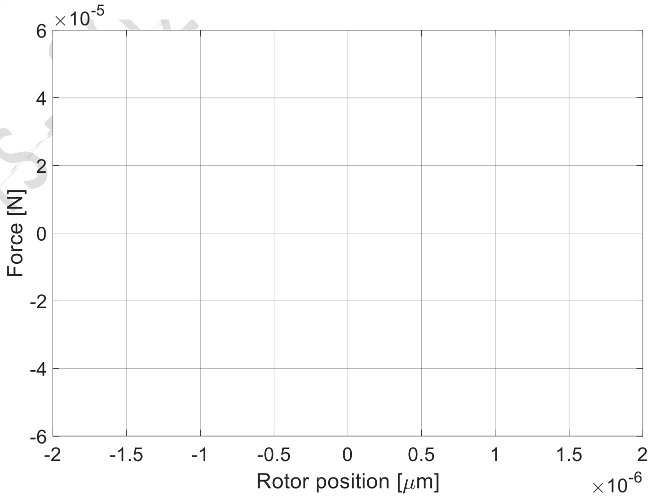
ε_0 = 8.85 10^{-12} F/m

k_b = 1.38 10^{-23} J/K;

T = 300 K;

q = 1.6 10^{-19} C;

Parameter [unit]	Value
FSR	±64 g
Linearity error	2%
Flat sensing Bandwidth	30 Hz - 300 Hz
Resolution	100 µg/√Hz
Supply voltage	±3 V
Gap between parallel plates	1.6 µm
Electromechanical sensitivity	3 fF/g
Mass	12 nkg
Quality factor	5
Parasitic capacitance	10 pF



(i)

It is known that the differential parallel-plate configuration suffers from nonlinearity arising from the inverse proportionality between the capacitance and the gap. This relationship can be linearized for small displacements, resulting in an error which is maximum for the largest displacements, i.e. at the FSR acceleration. We thus first of all set the maximum displacement as the motion corresponding to a FSR acceleration, according to the two following equations:

$$x_{max} = \sqrt{\epsilon_{lin}} \cdot g = \sqrt{\frac{2}{100}} \cdot g = 226\ nm$$

$$x_{max} = \frac{a_{FSR}}{\omega_0^2} \rightarrow f_0 = \frac{\omega_0}{2\,\pi} = \frac{\sqrt{\frac{a_{FSR}}{x_{max}}}}{2\,\pi} = 8.38\ kHz$$

We note how the resonance frequency is a bit larger than usual, due to the required large FSR. This is, by the way, the value of the frequency in operation, which is set by the natural frequency combined with electrostatic softening.

The knowledge of the electrostatic softening requires the calculation of the MEMS capacitance rest value. Defining the native mechanical stiffness k_m and the equivalent electrostatic stiffness k_{el}, we get:

$$S_{em} = 2\frac{C_0}{g}\frac{1}{\omega_0^2} \rightarrow C_0 = \frac{S_{em}\, g\, \omega_0^2}{2} = 678\, fF$$

$$\omega_0 = \sqrt{\frac{k_m + k_{el}}{m}} = \sqrt{\frac{k_m - \frac{2C_0}{g^2}V_{DD}^2}{m}} \rightarrow k_m = \omega_0^2 \cdot m + \frac{2C_0}{g^2}V_{DD}^2 = 33.3\frac{N}{m} + 4.8\frac{N}{m} = 38.1\frac{N}{m}$$

Which allows to calculate, by definition, the pull-in voltage of our differential parallel-plate configuration:

$$V_{PI} = \sqrt{\frac{k_m}{\frac{2C_0}{g^2}}} = 8.47\, V$$

We note how the biasing value of 3 V is safely far from the instability condition.

(ii)

The first consideration we can draw is about the required flat bandwidth. A lower bound of 30 Hz requires to place a pole at least one decade before, i.e. at 3 Hz

At the same time, an optimization of the front-end gain requires to fill the voltage dynamics for accelerations corresponding to the FSR. Assuming to work in a charge amplifier configuration, we know that the overall gain from input acceleration to output voltage becomes:

$$\frac{V_{out}}{a} = 2\frac{C_0}{g}\frac{V_{DD}}{C_F}\frac{1}{\omega_0^2}$$

By forcing that a FSR acceleration gives an output corresponding to the supply, we size the feedback capacitance:

$$C_F = 2\frac{C_0}{g}\cancel{V_{DD}}\frac{1}{\omega_0^2}\frac{a_{FSR}}{\cancel{V_{DD}}} = 2\frac{C_0}{g}\frac{1}{\omega_0^2}a_{FSR} = 2C_0\frac{x_{FSR}}{g} = 192\, fF$$

Interestingly, we note how the sizing is independent of the supply voltage, and is just equal in these conditions to a fraction of twice the MEMS capacitance, where the fraction corresponds to the ratio of the maximum displacement over the gap.

At this point we easily size the resistor:

$$R_F = \frac{1}{2\pi\, C_F\, f_p} = \frac{1}{2\pi\, 192\, fF\, 3\, Hz} = 276\, T\Omega$$

Noise generated by this resistor can be calculated at an intermediate value in the flat portion of the required bandwidth, e.g. around 165 Hz:

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$$\sqrt{S_{R_F,a}\,(s)} = \frac{\sqrt{\frac{4k_BT}{R_F}\frac{1}{s\,C_F}}}{\frac{V_{DD}}{a_{FSR}}} \rightarrow \sqrt{S_{R_F,a}\,(165\,Hz)} = \frac{\sqrt{\frac{4k_BT}{R_F}\frac{1}{2\pi\,\backslash 65\,Hz\,C_F}}}{\frac{V_{DD}}{a_{FSR}}} = 26\frac{\mu g}{\sqrt{Hz}}$$

Continuing with noise sizing, before setting the value of the amplifier noise we calculate the intrinsic thermomechanical contribution:

$$\sqrt{S_{tm,a}} = NEAD = \sqrt{\frac{4k_BT\,b}{m^2}} = \sqrt{\frac{4k_BT\,\omega_0}{m\,Q}} = 12\frac{\mu g}{\sqrt{Hz}}$$

We can finally size the required amplifier noise to fulfill the specification on noise (and then we will size the ADC to make quantization noise negligible).

$$\sqrt{S_{amp,a}} = \frac{\sqrt{S_{V,in}\left(1+\frac{C_P}{C_F}\right)^2}}{\frac{V_{DD}}{a_{FSR}}} = \sqrt{\left(100\frac{\mu g}{\sqrt{Hz}}\right)^2 - \left(26\frac{\mu g}{\sqrt{Hz}}\right)^2 - \left(12\frac{\mu g}{\sqrt{Hz}}\right)^2} = \sqrt{\left(96\frac{\mu g}{\sqrt{Hz}}\right)^2} \approx 100\frac{\mu g}{\sqrt{Hz}}$$

Which yields an amplifier noise of $88\frac{nV}{\sqrt{Hz}}$.

For the calculation of quantization noise, we know that the required number of bits is set through the dynamic range as:

$$n_{levels} = \frac{2\cdot a_{FSR}}{a_{min}} = \frac{2\cdot a_{FSR}}{100\frac{\mu g}{\sqrt{Hz}}\sqrt{300\,Hz - 30\,Hz}} = 77900 \rightarrow n_{bit} = \log_2(n_{levels}) = 17$$

The associated quantization noise is:

$$\sqrt{S_{quant,a}} = \frac{2\cdot a_{FSR}}{2^{17}\sqrt{12}\sqrt{270\,Hz}} = 17\frac{\mu g}{\sqrt{Hz}}$$

Which remains negligible in the quadratic noise sum.

(iii)

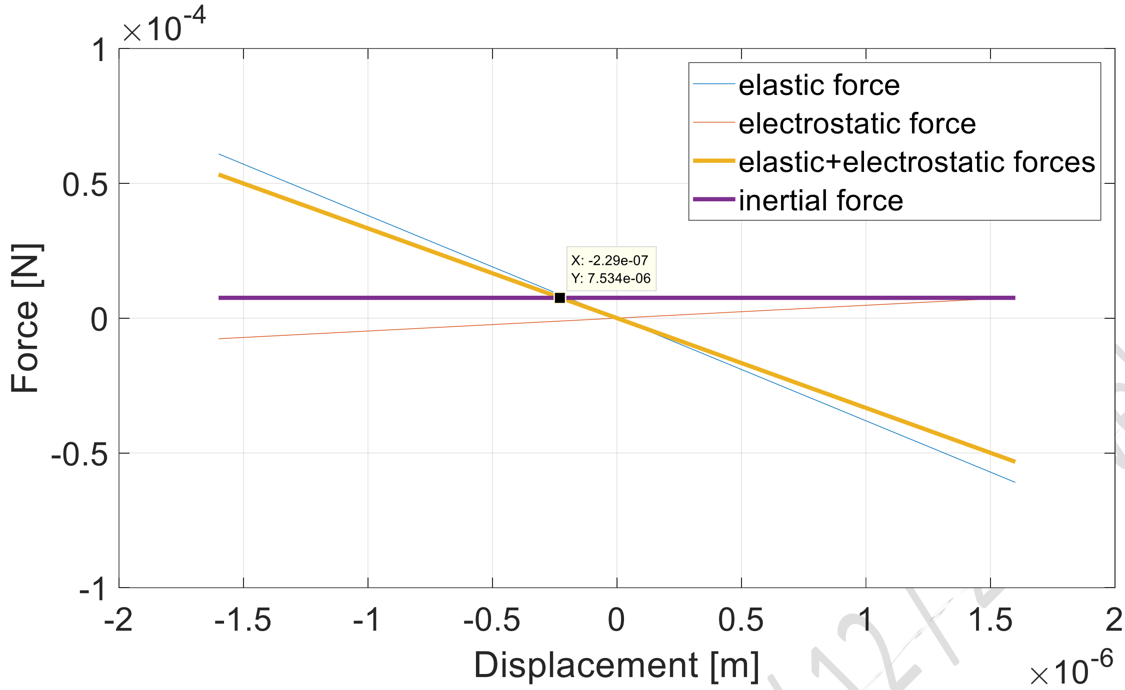
The inertial force is a constant and is obviously independent of the gap. The elastic force always tries to oppose to displacements, while the linearized electrostatic force has an opposite sign.

Their expressions are respectively given as:

$$F_{in} = m\cdot a_{FSR}$$

$$F_k - F_{elec} = -k_m\cdot x + \frac{2C_0}{g^2}V_{DD}^2 x$$

As shown in the graph, their intersection correctly lies at a value of the x coordinate corresponding to 226 nm, which was indeed set by design as the displacement corresponding to an input acceleration at the FSR.



×10^-4
1
0.5
0
-0.5
-1
Force [N]
elastic force
electrostatic force
elastic+electrostatic forces
inertial force
X: -2.29e-07
Y: 7.534e-06
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Displacement [m]
×10^-6

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Question n. 3

You take a picture of a green painting that occupies an area of 2 m^2, located at a distance of 5 m. Your camera features a 12-Mpixel APS based on the 3T topology without micro-lenses. Due to the crowd observing the painting, you are looking at the scene tilted by 30° with respect to the direction perpendicular to the surface. The total optical power that illuminates the painting is 2.4 W. The surface acts as a Lambertian reflector with 20% surface reflectance, whose intensity, measured in units of [W/sr], obeys Lambert's cosine law: $I_{Lamb}(\beta) = I_{Lamb,0}\cos(\beta)$.

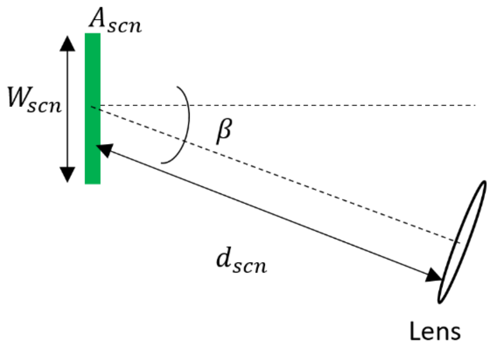
β is the observation angle and $I_{Lamb,0}$ is the peak intensity along the direction perpendicular to the surface. The table provides additional information on the scene and the camera.

Table of parameters		
Power illuminating the scene	P_{scn}	$2.4\ W$
Scene reflectance	R_{scn}	20%
Scene width	W_{scn}	$1\ m$
Scene height	H_{scn}	$2\ m$
Scene-camera distance	d_{scn}	$5\ m$
Sensor area	A_{sensor}	$8.8 \cdot 6.6\ mm^2$
F-number	$F_\#$	4
Number of pixels	$N_h \cdot N_v$	4000×3000
Fill factor	FF	45%
Depth of n+ implant	x_n	$200\ nm$
Depleted layer thickness	x_{dep}	$1.2\ \mu m$
Epitaxial layer diffusion length	x_{diff}	$40\ \mu m$
Epitaxial layer thickness	x_{epi}	$10\ \mu m$
Red pixel transmittance	T_{red}	55%
Green pixel transmittance	T_{green}	85%
Blue pixel transmittance	T_{blue}	48%
Specific gate capacitance	C'_{ox}	$1\ fF/\mu m^2$
Dark current	i_d	$0.2\ fA$

(i) considering a $500\ nm$ wavelength, calculate the quantum efficiency η of the sensor pixels;

(ii) calculate the maximum photocurrent (choose a reasonable value of the quantum efficiency if you were not able to solve the previous question). The lens aperture is set to a F-number equal to 4;

(iii) calculate the corresponding SNR considering an integration time of $8\ ms$, making reasonable assumptions if needed.



Physical Constants

ε_{Si} = 8.85 10^{-12}·11.7 F/m

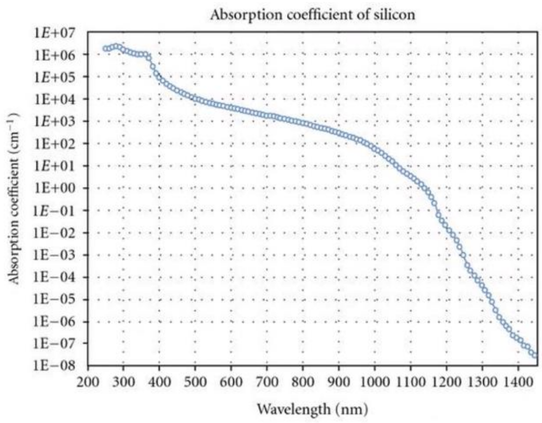
k_b = 1.38 10^{-23} J/K;

h = 6.62 10^{-34} J·s;

c = 3 10^8 m/s;

T = 300 K;

q = 1.6 10^{-19} C;



(i)

Given that the diffusion length is much larger than the epitaxial layer thickness, all the carriers generated within the epitaxial layer must be considered in the calculation of the quantum efficiency. From the plot, the absorption coefficient α at a 500 nm wavelength is equal to $10^4\ \mathrm{cm}^{-1}$, thus the absorption efficiency is:

$$\eta_{abs} = e^{-\alpha x_n} - e^{-\alpha x_{epi}} = 81.9\%$$

The quantum efficiency for each pixel is then readily obtained considering the transmittance of each color filter:

$$\eta_{red} = \eta_{abs} T_{red} = 45\%$$

$$\eta_{green} = \eta_{abs} T_{green} = 69.5\%$$

$$\eta_{blue} = \eta_{abs} T_{blue} = 39.3\%$$

(ii)

The sensor is best exploited by fitting the height of the scene into the width of the sensor. Thus the magnification factor is:

$$m = \frac{W_{sensor}}{H_{scn}} = 0.0044$$

The distance from the object is very large with respect to the distance between the lens and the sensor (reasonable assumption), thus the focal length is calculated through the magnification factor:

$$f = m \cdot d_{scn} = 22\ \mathrm{mm}$$

The total power reflected by the scene is:

$$P_r = P_{scn} R_{scn} = 0.48\ \mathrm{W}$$

and the intensity along the direction of the camera, for a Lambertian reflector, is:

$$I_{Lamb}(30°) = \frac{P_r}{\pi} \cos(30°) = 19.1\ \mathrm{nW/sr}$$

The diameter of the lens is calculated using the focal length and the F-number:

$$D_{lens} = 5.5\ \mathrm{mm}$$

The solid angle subtended by the lens and centered on the scene is:

$$\Omega_{lens} = \frac{\pi \left(\frac{D_{lens}}{2}\right)^2}{d_{scn}^2} = 950.3 \cdot 10^{-9}\ \mathrm{sr}$$

The total power impinging on one pixel can be calculated considering that all the light collected by the lens is shared by a fraction of the pixels on the sensor. Such fraction is given by:

$$n_{pix} = \frac{A_{scn} \cos(30°) \cdot m^2}{A_{sensor}} N_h N_v = \frac{A_{scn} \cos(30°) \cdot m^2}{A_{pix}}$$

where the area of the scene, projected along the observing direction as $A_{scn} \cos(30°)$, is magnified by the lens and evaluated as a fraction of the total sensor area.

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Note that the sensor has square pixels, as easily calculated by the given data:

$$L_{pix} = \frac{W_{sensor}}{N_h} = \frac{H_{sensor}}{N_v} = 2.2\ \mu\text{m} \qquad \Rightarrow \qquad A_{pix} = (2.2\ \mu\text{m})^2$$

Finally, accounting also for the fill-factor as the sensor does not mount any microlenses, the total optical power impinging on a single pixel is calculated as:

$$P_{pix} = \frac{I_{Lamb}(30°)\,\Omega_{lens}}{n_{pix}} FF = \frac{\frac{P_r}{\pi}\cos(30°)\,\Omega_{lens}}{A_{scn}\cos(30°)\cdot m^2} A_{pix} FF$$

The same expression can be derived by first considering the reflected power per unit area of the scene:

$$\frac{\frac{P_r}{\pi}\cos(30°)\,\Omega_{lens}}{A_{scn}\cos(30°)\cdot m^2}$$

And multiplying it by the pixel area and the fill factor.

The number of photoelectrons impinging on one pixel is:

$$N_{ph} = \frac{P_{pix}}{\frac{hc}{\lambda}} = 20563\ \text{photons}$$

and the photocurrent generated in a green pixel (so, the maximum one) is:

$$i_{ph} = qN_{ph}\eta_{green} = 2.29\ \text{fA}$$

(iii)

The integration capacitance C_{int} is obtained as the sum of the photodiode capacitance C_{dep} and the parasitic capacitance C_p. This contribution can be roughly estimated by considering that each of the three transistors will occupy approximately 1/3 of the available area (i.e. the pixel area minus the photodiode area):

$$C_{dep} = \frac{\epsilon_0 \epsilon_r A_{pix} FF}{x_{dep}} = 0.188\ \text{fF}$$

$$C_p \approx C'_{ox} \frac{A_{pix}(1-FF)}{3} = 0.887\ \text{fF}$$

$$C_{int} = C_{dep} + C_p = 1.08\ \text{fF}$$

The noise contributions we can consider are the photocurrent shot noise, the dark current shot noise, and the reset noise. Since we have no information on the ADC, we will assume that its design is such that quantization noise is negligible. These contributions, in number of electrons, are:

$$\sigma_{shot,ph,N} = \frac{\sqrt{2qi_{ph}\cdot\frac{1}{2t_{int}}\cdot t_{int}^2}}{q} = \frac{\sqrt{qi_{ph}t_{int}}}{q} = 10.7\ \text{e}_{\text{rms}}$$

$$\sigma_{shot,dark,N} = \frac{\sqrt{2qi_d\cdot\frac{1}{2t_{int}}\cdot t_{int}^2}}{q} = \frac{\sqrt{qi_d t_{int}}}{q} = 3.16\ \text{e}_{\text{rms}}$$

$$\sigma_{reset,N} = \frac{\sqrt{\frac{k_b T}{C_{int}} \cdot C_{int}^2}}{q} = \frac{\sqrt{k_b T C_{int}}}{q} = 13.19\ \mathrm{e_{rms}}$$

The SNR is thus:

$$\mathrm{SNR} = 20\log_{10}\left(\frac{\frac{i_{ph} t_{int}}{q}}{\sqrt{\sigma_{shot,ph,N}^2 + \sigma_{shot,dark,N}^2 + \sigma_{reset,N}^2}}\right) = 20\log_{10}\left(\frac{114.47}{17.27}\right) = 16.42\ \mathrm{dB}$$