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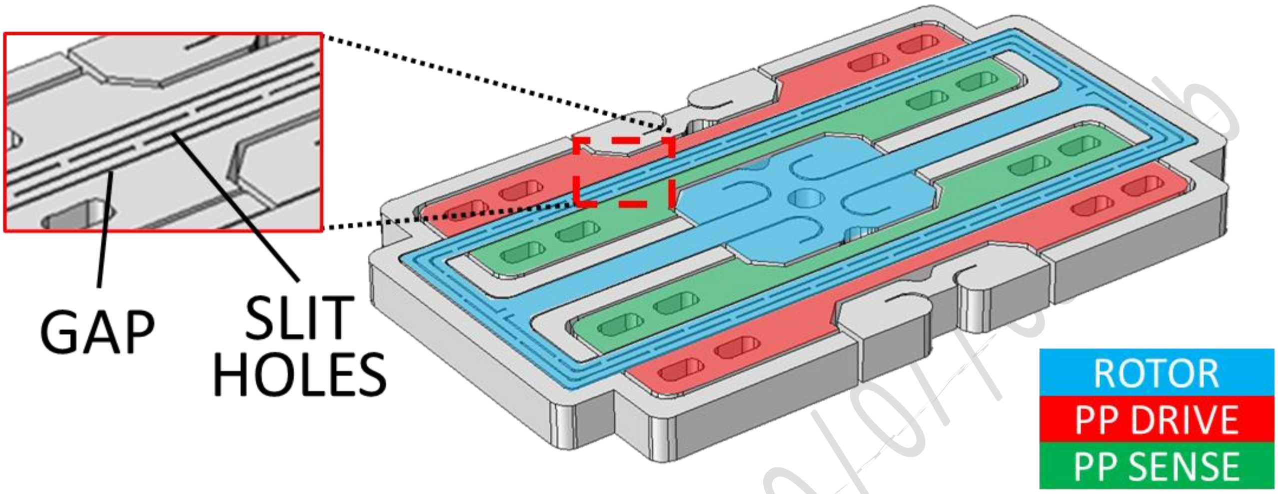
Question n. 1

A parallel-plate resonator like the one depicted below is used for MEMS-based real-time-clock applications. Derive and graph its electrical equivalent model, clearly defining the analytical expression of the model components. Describe and motivate under which conditions of pressure and motion this solution is preferable against other types of resonators. Discuss the effects of the slit holes designed on the bending structure (as highlighted in the zoom).



The shown architecture is a three-port resonator based on parallel-plate capacitive actuation and parallel-plate sensing of motion. Each of the two long, lateral suspended beams is deflected by the actuator as a consequence of the electrostatic force applied due to the voltage difference between actuator and rotor. The relationship between voltage and force can be found from:

$$F_{el}(s) = \frac{[V_{rot} - V_A(s)]^2}{2}\frac{dC_A}{dx}$$

For a small-signal approximation, where $V_A(t) = V_{A0} + \mathrm{v_a}\sin(\omega_0 t)$ and $v_a \ll V_{A0}$, and for small displacements $x \ll g$ (where g is the gap between the parallel plates), one can solve the expression above and found the transduction factor for the actuation port as:

$$F_{el}(s) = \frac{2V_{rot}v_a(s)}{2}\frac{\epsilon_0 A}{g^2} = V_{rot}v_a(s)\frac{\epsilon_0 A}{g^2} = \eta_A v_a(s) \qquad \rightarrow \qquad \eta_A = V_{rot}\frac{\epsilon_0 A}{g^2}$$

The so obtained force is applied to a suspended mass (a deflecting beam, in this case), which can be described by its spring-mass-damper law (with coefficients k, m and b, respectively) to find the relationship between applied force and displacement $X(s)$ in the Laplace domain:

$$X(s) = F_{el}(s)\frac{1}{ms^2 + bs + k}$$

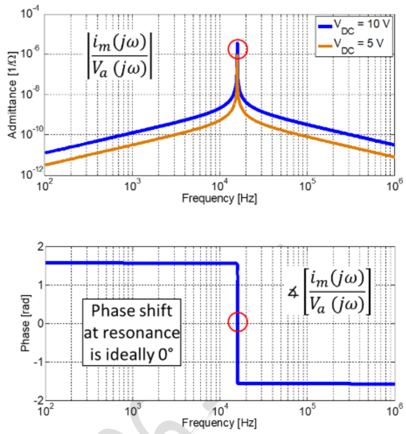
The final step is to convert this motion back into an electrical signal. This can be obtained by calculating the motional current at the output of the resonator sense port, kept to (virtual) ground:

$$i_m(t) = V_{rot}\frac{dC_S}{dt} = V_{rot}\frac{dC_S}{dx}\frac{dx}{dt} \rightarrow i_m(s) = V_{rot}\frac{dC_S}{dx}sX(s) = V_{rot}\frac{\epsilon_0 A}{g^2}sX(s) = s\eta_S X(s) \rightarrow \eta_S = V_{rot}\frac{\epsilon_0 A}{g^2}$$

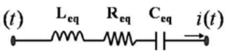
The small displacement approximation is used again. Note also how the transduction factor of the sense port, for a symmetric resonator, equals the one at the actuation port so that we can name them $\eta = \eta_A = \eta_S$.

Combining the three equations above, one can find the relationship between applied voltage and output current, which thus represents the electrical admittance of the resonator:

$$\frac{i_m(s)}{v_a(s)} = \frac{s\,\eta^2}{ms^2 + bs + k} = \frac{1}{\frac{m}{\eta^2}s + \frac{b}{\eta^2} + \frac{k}{\eta^2 s}} = \frac{1}{L_{eq}s + R_{eq} + \frac{1}{C_{eq}s}}$$

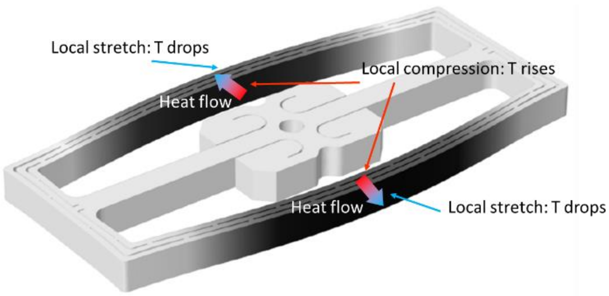


The equations show how this three-port resonator can be modeled by a two-port series of three electrical equivalent component. Note however that the third port is still present, as the voltage applied to the rotor affects the value of the transduction factor and thus of the electrical parameters. A sample graph of this admittance is reported aside. In particular, note how at the resonance frequency the model collapse into a resistance, with maximum admittance (minimum resistive losses) and no phase shift.



The model appears thus very similar to a comb-based resonator, with the difference essentially appearing only in (i) the expression of η and (ii) the need for a small displacement approximation. This already tells us that this resonator will be useful for applications where motion is small. This is not the case of drive resonators for gyroscopes, which indeed require comb-based actuation and large motion. It is instead the case of high-frequency applications of oscillators, e.g. for time keeping or synchronization. The large stiffness at such frequencies implies small displacement and, as a consequence, require parallel-plate sensing which gives a higher transduction factor. To compensate for damping introduced by the parallel plates, such applications need very low operation pressure (e.g. in the order of 100 µbar or lower).

When pressure is so low, fluid damping (i.e. damping induced by gas particles) is made negligible and other damping sources arise, like thermoelastic damping. This phenomenon, due to bending and compression of structural parts, which induce local temperature changes (heat of the compressed part, cooling of the stretched part as in the figure) induces, in turn, heat flow and dissipation (damping) inside the resonator structure. The presence of this slit holes blocks the heat flow inside the beams, thus reducing also this damping contribution and enabling very large quality factors, even with parallel-plate architectures.



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Question n. 2

An imaging sensor based on a pinned photodiode process, without correlated double sampling, is characterized by the photon transfer curves (PTC) shown aside. In particular, the blue solid curve is captured at a temperature T_1 = 250 K, and the orange dashed line is captured at T_2 = 350 K.

Parameter [unit]	**Value**
Total number of active pixels [-]	3000 · 4000
Active pixel sensor size [mm^2]	3 · 4
Process dark current density [$fA/\mu m^2$]	0.1
Sensor supply voltage [V]	3

Considering the other parameters given in the table:

(i) calculate the sensor quantum efficiency and calculate the % PRNU;

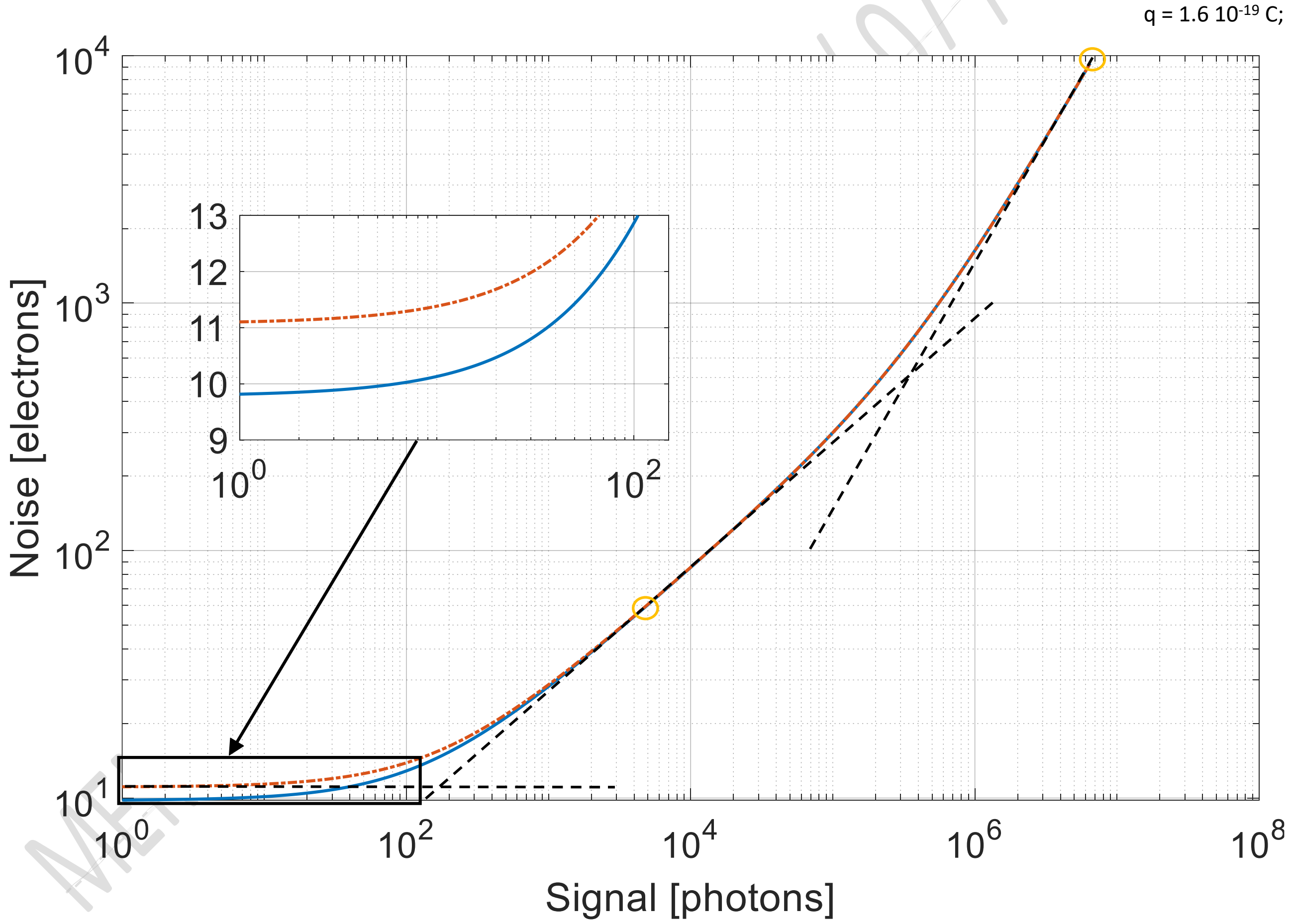
(ii) estimate the value of the capacitance at the integration node;

(iii) estimate the number of bits used by the imaging sensor ADC.

Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K;

$q = 1.6\ 10^{-19}$ C;



(i)

The calculation of the quantum efficiency can be done by considering the central part of the PTC (slope ½), where photon shot noise dominates. Indeed, in this region we know that the following relationship gives the value of noise σ_{el} in rms electrons:

$$\sigma_{el} = \frac{\sqrt{q i_{ph} t_{int}}}{q}$$

As the photocurrent is just equal to the number of signal electrons N_{el}, multiplied by the elementary charge, and captured per unit integration time t_{int}, we can as well write that:

$$\sigma_{el} = \frac{\sqrt{q \frac{qN_{el}}{t_{int}} t_{int}}}{q}$$

Finally, as the number of captured electrons per unit time is related, by definition, to the number of input photons per unit time through the quantum efficiency η, we can write:

$$\sigma_{el} = \frac{\sqrt{q \frac{qN_{ph}\eta}{t_{int}} t_{int}}}{q} = \frac{q\sqrt{N_{ph}\eta}}{q} = \sqrt{N_{ph}\eta} \quad \rightarrow \quad \eta = \frac{\sigma_{el}^2}{N_{ph}}$$

By choosing a suitable point in the curve, in the central part of the ½ slope region, we give a good estimate of η: with $\sigma_{el} = 60$ and $N_{ph} = 5000$ we find a quantum efficiency of 0.72. Similar values are obtained also by choosing other pairs of $(\sigma_{el},\ N_{ph})$ in this region.

The PRNU is instead determined by the portion of the curve with slope 1. In this region we know that:

$$\sigma_{el} = \frac{PRNU_{\%} i_{ph} t_{int}}{q} = \frac{PRNU_{\%} \frac{qN_{ph}\eta}{t_{int}} t_{int}}{q} = PRNU_{\%} N_{ph}\eta \rightarrow \quad PRNU_{\%} = \frac{\sigma_{el}}{N_{ph}\eta}$$

By choosing the saturation point of the curve, with $\sigma_{el} = 10^4$ and $N_{ph} = 7 \cdot 10^6$ we find $PRNU_{\%} = 0.2\ \%$.

(ii)

We observe the presence of two curves at different temperatures, and we know that kTC noise depends on both temperature and the capacitance at the integration node. This suggests us that the following system can be written, looking at the 0-slope portion of the PTC (the zoomed part of the image):

$$\begin{cases} \frac{k_B T_2 C}{q^2} + \left(\sigma_{quant}^2 + \sigma_{dark}^2 + \sigma_{DSNU}^2\right) = \sigma_{el,T_2}^2 \\ \frac{k_B T_1 C}{q^2} + \left(\sigma_{quant}^2 + \sigma_{dark}^2 + \sigma_{DSNU}^2\right) = \sigma_{el,T_1}^2 \end{cases} \qquad \begin{cases} \alpha T_2 + \beta = 11.1^2 \\ \alpha T_1 + \beta = 9.9^2 \end{cases}$$

where the simplified version has assumed $\alpha = \frac{k_B C}{q^2}$ and β as the sum of all signal-independent noise sources except for kTC noise. The solution of this simple system yields $\alpha = 0.25$ and $\beta = 35.5$. By solving for the value of the capacitance at the integration node, we find:

$$\alpha = \frac{k_B C}{q^2} = 0.25 \quad \rightarrow \quad C = 0.25 \cdot \frac{q^2}{k_B} = 0.46\ fF$$

with the corresponding kTC noise being in the order of 7-9 electrons depending on the temperature value.

(iii)

We now also know the summed value of signal-independent noise sources (excluding kTC noise), which is:

$$\beta = \left(\sigma_{quant}^2 + \sigma_{dark}^2 + \sigma_{DSNU}^2\right) = 35.5 \ \rightarrow \ \sqrt{\left(\sigma_{quant}^2 + \sigma_{dark}^2 + \sigma_{DSNU}^2\right)} = 6\ e_{rms}^-$$

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As the process is based on pinned photodiodes, we assume that the dark current is low enough (and the DSNU as well) to make its noise contribution negligible. We will later verify this hypothesis. In this situation, (almost) all the 6 electrons will be given by quantization noise, which allows to easily find the number of bits of the sensor ADC as:

$$\sigma_{el} = \frac{\frac{V_{DD}}{2^{n_{bit}}\sqrt{12}}C}{q} \quad \rightarrow \quad n_{bit} = \log_2\left(\frac{V_{DD}C}{q\sqrt{12}\,\sigma_{el}}\right) = 8.69 \quad \rightarrow \quad n_{bit} = 9$$

For the verification of the hypothesis, we can roughly estimate the pixel area from the total number of pixels and the overall sensor area as:

$$A_{pix} = \frac{A_{sensor}}{N_{pixel}} = (1\ \mu m)^2$$

We have no information on microlenses, fill factor etc… but we just take the worst case for which the dark current density is collected over the entire pixel area, giving thus $i_d = 0.1\ fA$. For a typical integration time in the range of 0.5 ms to 5 ms, the associated shor noise contribution remains in the range of 0.55 to 1.5 electrons rms, confirming that shot noise (and reasonably also DSNU) are effectively negligible.

We conclude with a comment: quantization noise is well dimensioned. Indeed, its value is comparable or even slightly lower than the dominant noise source, which is in this case the reset noise. This agrees with the fact that CDS is not implemented for this sensor, as stated in the exercise text.

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Question n. 3

A z-axis gyroscope, with differential parallel plate sensing, designed for motion tracking application, features the parameters reported in the table. You are asked to:

(i) find the sensitivity (in [m/dps]) and the linearity error at the full-scale-range;

(ii) find the thermomechanical and the electronic (of amplifiers and feedback resistors) contributions to the input-referred noise density, in [dps/√Hz] (you are supposed to properly size the feedback resistor of the amplifiers);

Parameter [unit]	**Value**
Process height [μm]	25
Process gap [μm]	1.5
Drive displacement [μm]	6
Amplifier input noise [nV/√Hz]	20
Parasitic capacitance [pF]	4
Rotor voltage [$V_{DC,rot}$]	15
Frequency split [Hz]	500
Drive frequency [kHz]	25
Sense mass (half structure) [nKg]	4
Target full-scale range [dps]	±4500
Sense Q-factor	800
Sense capacitance (all device, single-ended) [fF]	200
Supply voltage [V]	0 – 3
N. of quadrature compensation cells	14
Drive mode angle α vs nominal direction [rad]	$10^{-4} \pm 10^{-6}$

(iii) for a given production lot, the statistic angle error α between the drive frame motion and the nominal drive motion direction is given by a mean value of 10^{-4} rad and a statistical deviation (3σ) of 10^{-6} rad. Accurately draw the compensation technique (at electromechanical level) for the quadrature error. Compute the required voltage difference for the tuning plates in order to compensate for the mean quadrature error. Compute the residual maximum offset (after calibration) in dps.

Physical Constants

ε_0 = 8.85 10^{-12} F/m

k_B = 1.38 10^{-23} J/K

T = 300 K

(i)

The sensitivity in terms of displacement per unit angular rate [m/dps] is computed through its definition:

$$S_y = \frac{x_d}{\Delta\omega} \cdot \frac{\pi}{180} = 33.3\ \frac{\text{pm}}{\text{dps}}$$

We can thus compute the maximum displacement of the sense frame, i.e. at the FSR:

$$y_{max} = S_y \cdot FSR = 150\ \text{nm}$$

Using the obtained value, it is possible to compute the maximum linearity error for the differential parallel plate topology, given by the formula:

$$\epsilon_{\%} = \left(\frac{y_{max}}{g}\right)^2 \cdot 100 = 1\%.$$

(ii)

Let us compute the intrinsic Noise Equivalent Rate Density $NERD_i$ (the factor 2 at the denominator accounts for the two-mass structure: not an error if you missed it):

$$NERD_i = \sqrt{\frac{k_B T\, b_s}{2}}\frac{1}{x_d \omega_d m_s}\frac{180}{\pi} = \sqrt{\frac{k_B T}{2 Q_s m_s \omega_d}}\frac{1}{x_d}\frac{180}{\pi} = 3.2\ \frac{mdps}{\sqrt{Hz}}.$$

The Noise Equivalent Rate Density due to electronics, NERD$_e$, is computed as follows:

$$NERD_e = \sqrt{2}\frac{\sqrt{S_v\left(1+\frac{C_p}{C_f}\right)^2 + \frac{4k_BT}{Rf}\left(\frac{1}{\omega_d C_f}\right)^2}}{\left(\frac{V_{dd}}{2\,FSR}\right)}$$

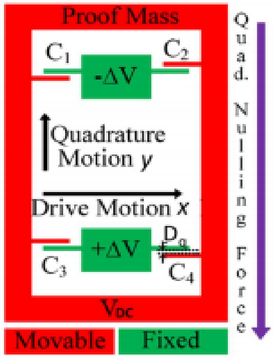
Where the factor $\sqrt{2}$ takes into account the two uncorrelated OPAMP and feedback resistor noises, C_p is the parasitic capacitance, and the factor 2 at the denominator in the last term is due to the single rail of the supply which goes from 0 V to 3 V. The feedback capacitance C_f should be dimensioned in order to maximize the sensitivity, achieving a rail to rail signal at the FSR[1].

$$C_f = 2 \cdot \frac{V_{DC,rot}}{\frac{V_{dd}}{2}} \cdot \frac{C_0}{g} \cdot y_{max} = 400\, fF$$

where C_0 represents the sense capacitance. With C_f, we can compute the NERD$_e$, getting 1.3 mdps/√Hz.

(iii)

The electromechanical compensation strategy is the adoption of a set of four parallel plates arranged and biased as shown in the figure.



The resulting electrostatic force in the y direction is given by the formula:

$$|F_{QC}| = \frac{4\epsilon_0 h}{g^2}\, V_{DC,rot}\, \Delta V\, x_d.$$

First of all we should compute the sense stiffness projected along the drive direction:

$$k_{ds} \simeq \bar{\alpha}\, k_s = \bar{\alpha}\, m_s \left(2\pi(f_d + f_{split})\right)^2 = 0.001\frac{N}{m}.$$

$\bar{\alpha}$ is the mean value of the angle error between the sense and the drive frame.

From that, we can compute the voltage difference to be applied to the compensation electrodes in order to null the quadrature error:

$$\Delta V = \frac{k_{ds} x_d\, g^2}{4\epsilon_0 h\, V_{DC,rot}\, x_d\, N_{cc}} = 1.24\, V$$

The residual maximum quadrature error can be computed as:

$$B_q = \frac{k_{ds,3\sigma}}{2\, m_s \omega_d}\frac{180}{\pi} = \frac{\alpha_{3\sigma} m_s [2\pi(f_d + f_{split})]^2}{2\, m_s \omega_d}\frac{180}{\pi} = 46.8\, dps$$

[1] We are neglecting the presence of the quadrature signal which in principle might saturate the output of the charge amplifier.

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