Last Name ___CASAPO___ **Given Name** ___LIA____ **ID Number** _20200611_

Question n. 1

Give a definition of spatial noise in an imaging sensor, and describe all the various sources of Fixed Pattern Noise in modern CMOS technologies. By relying on SNR and DR formulas, compare spatial noise sources with temporal noise sources as a function of integration time and area, and discuss FPN calibration procedures.

Spatial noise of an imaging sensor formed by multiple pixels represents the deterministic effect of different offset and gain (due to process spread) of the pixel-level processing chain from impinging input to digitized output. As a consequence, for a uniform input, the image sensor output is not uniform, and appears as noisy. Though the origin of this behavior is deterministic, the effect is thus equivalent to noise. As for every pixel the impinging signal is processed by a fixed pattern (formed by optical, electrical, and electronic stages), spatial noise is also called fixed-pattern noise, or FPN. In particular, signal-independent spatial noise takes the name of dark-signal nonuniformity (DSNU), while signal-dependent spatial noise takes the name of photo-response nonuniformity (PRNU).

Sources of FPN are thus all those process-spread-related nonuniformities that appear at pixel level, and can be classified into two main categories:

(i) those belonging to the physics/optics domains, in which we can include nonuniformities in the optical path (anti-reflection coatings, color filter array thickness and transmittance, micro-lenses, layer of transparent stacked materials from the filter to the silicon interface…), and nonuniformities in the silicon active area (implant defects, extension of the depletion region, interface generation/recombination centers or other defects…). Some of them affect the quantum efficiency, and thus the pixel gain (PRNU); some of them affect the dark current, and thus the pixel offset (DSNU);

(ii) those belonging to the electrical/electronics domains, in which we can include the value of the depletion region capacitance, the value of the gate capacitance of the transistor, the source follower gain, offset and threshold voltage. Again, some of them affect the PRNU, some of them affect the DSNU.

To analyze the DR and SNR formulas of e.g. a 3T imaging sensor, we can write:

$$SNR = 20 \cdot log_{10}\left[\frac{i_{ph} \cdot t_{int}}{\sqrt{q\left(i_{ph}+i_d\right)\cdot t_{int} + k_B T C + \left[\frac{C\,V_{DD}}{2^{N_{bit}}\sqrt{12}}\right]^2 + \left(i_d\, t_{int}\frac{\sigma_{DSNU,\%}}{100}\right)^2 + \left(i_{ph}\, t_{int}\frac{\sigma_{PRNU,\%}}{100}\right)^2}}\right]$$

So we can first say that at long integration times the effect of spatial noise become more visible than at short ones. This is intuitive: indeed, at long integration times temporal noise gets averaged, while this does not occur for the deterministic spatial noise sources. Apparently also in terms of area, unlike temporal noise, FPN sources see no benefits from larger areas: the formula, however, fails to show the fact that for larger areas the relative impact of process nonuniformities is lower, so that $\sigma_{DSNU,\%}$ and $\sigma_{PRNU,\%}$ will likely be lower than for ultra-small pixels.

$$DR = 20 \cdot log_{10}\left[\frac{V_{DD}\,C}{\sqrt{q\left(i_{ph}+i_d\right)\cdot t_{int} + k_B T C + \left[C\frac{V_{DD}}{2^{N_{bit}}\sqrt{12}}\right]^2 + \left(i_d\, t_{int}\frac{\sigma_{DSNU,\%}}{100}\right)^2}}\right]$$

In terms of dynamic range, increasing integration times worsen the DR even more when FPN is dominant. The advantages that one usually gets for larger areas in terms of DR are also partly reduced by FPN (though, like for SNR, larger areas will probably imply a lower percentage DSNU).

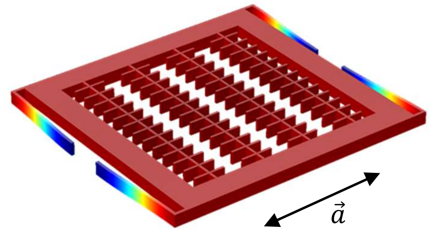
For these reasons, a FPN calibration procedure is usually applied to every imaging sensor before sending it out to the market. The calibration procedure exploits the fact that, by repeating N times the same picture, and by taking the average, pixel by pixel, of the N different images, temporal noise will get reduced by the average, while spatial noise will remain the same. Once spatial noise is measured in the described way, suitable offset and gain calibration coefficient can be stored in the digital domain (two coefficients per pixel), and used for the calibration.

To calibrate DSNU, multiple images will be taken in dark, under no signal. To calibrate PRNU, multiple images will be taken under various "typical" light sources, e.g. exploiting a calibrated source and the reflectance (or transmittance) by a standard color chart, whose spectra reproduce typical reflectance spectra that we encounter in everyday lifw.

Such a calibration will not be perfect, leaving residual DSNU and PRNU in the order of 1% or few %, which will become visible, typically, only at very long integration times.

Last Name ___CASAPO___ **Given Name** ___LIA____ **ID Number** _20200611_

Question n. 2



A MEMS capacitive accelerometer, to be installed aboard the International Space Station (ISS) for physics experiments, should be designed. The centrifugal acceleration generated by the satellite rotation perfectly balances Earth gravity acceleration, so that micro-gravity experiments can be performed.

The main target parameters for the accelerometer are: an overall area of the Silicon device of (3x3) mm^2 (70% of which is taken by the rotor, as a rough assumption), a FSR of 5 m*g* (*g* = 9.81 m/s^2), and an input-referred resolution of 1 n*g*/√Hz over a 30 Hz bandwidth. The differential capacitive electrodes are based on comb-finger sensing cells (see the figure, the maximum displacement is limited by fringe effects). Other parameters are given in the Table aside.

Parameter [unit]	**Value**
Process height [μm]	100
Process gap [μm]	1.24
Operating temperature [K]	298
Maximum displacement [nm]	125
Amplifier noise [nV/√Hz]	2
Parasitic capacitance [pF]	3
Feedback capacitance [pF]	5
Rotor voltage [$V_{ac,rot}$]	2
Target area [mm^2]	3x3
Target noise density [ng/√Hz]	1
Target bandwidth [Hz]	30
Target full-scale range [g]	0.005

(i) calculate the minimum quality factor Q to cope with noise specifications (assume that noise should be evenly distributed among mechanical and electronic sources);

(ii) the accelerometer is readout through a modulated AC voltage applied at the rotor, with the stators to the virtual ground of differential charge amplifiers. Calculate the required number of comb fingers to cope with noise due to electronic sources. Why is the accelerometer using comb-finger detection?

(iii) choose the number of bits of the ADC to cope with the required dynamic range specifications, and evaluate its impact on noise.

Physical Constants
k_b = 1.38 10^{-23} J/K;
ε_0 = 8.85 10^{-12} F/m;
ρ = 2350 kg/m^3;

(i)

We begin by writing the noise formula for thermomechanical contribution, and by equating it (in terms of power) to half of the available noise budget (so that, with the two contributions being equal, we will match the noise specifications).

$$\sqrt{S_{n,tm}} = \sqrt{\frac{4 \cdot k_B \cdot T \cdot b}{m^2}} = \sqrt{\frac{4 \cdot k_B \cdot T \cdot \omega_0}{m \cdot Q}} = \frac{1}{\sqrt{2}} \frac{ng}{\sqrt{Hz}} \cdot 9.8 \frac{\frac{m}{s^2}}{g} = 6.9 \cdot 10^{-9} \, \frac{m}{s^2}$$

To complete the calculation we need to calculate the mass, which is readily done, and to calculate the resonance frequency, which is set by the target maximum displacement.

$$m = \rho \cdot (0.7 \cdot 9 \cdot 10^{-6} m^2) \cdot 100 \ \mu m = 1.48 \ \mu kg$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{m \cdot a_{FSR}/x_{max}}{m}} = \sqrt{\frac{a_{FSR}}{x_{max}}} = 626 \frac{rad}{s} \qquad \rightarrow \quad f_0 = 100 \ Hz$$

Note how the mass is relatively large, compared to e.g. consumer applications (usually in the ten nkg range), and the resonance is low (usually in the few kHz range), in both cases due to the ultra low-noise requirements.

The Q factor can be thus calculated as:

$$Q = \frac{4 \cdot k_B \cdot T \cdot \omega_0}{m \cdot \left(6.9 \cdot 10^{-9} \ \frac{m}{s^2}\right)^2} = 144000$$

We find a very large value, again determined by the tight noise requirements.

(ii)

Concerning electronic noise, the dominant contribution is assumed to be due to the amplifier voltage noise (no other contributions are mentioned in the text). We can write input referred noise as the output noise amplified by the parasitic-induced gain.

$$\sqrt{S_{n,eln}} = \frac{\sqrt{S_{Vn}\left(1 + \frac{C_P}{C_F}\right)^2}}{SF} = \frac{\sqrt{S_{Vn}\left(1 + \frac{C_P}{C_F}\right)^2}}{2 \frac{C_0}{L_{ov}} \frac{V_{DD}}{C_F} \frac{1}{\omega_0^2}} = 6.9 \cdot 10^{-9} \ \frac{m}{s^2}$$

Note that the capacitive gain $\frac{C_0}{L_{ov}}$ is for the comb finger configuration and not for the capacitive one. This gain can be further expressed in order to highlight the dependency on the comb number:

$$\frac{C_0}{L_{ov}} = \frac{2\epsilon_0 N_{CF} h L_{ov}/g}{L_{ov}} = \frac{2\epsilon_0 N_{CF} h}{g}$$

In this way by combining the last equations we find:

$$N_{CF} = \frac{\sqrt{S_{Vn}\left(1 + \frac{C_P}{C_F}\right)^2}}{2 \frac{2\epsilon_0 h}{g} \frac{V_{DD}}{C_F} \frac{1}{\omega_0^2} \cdot 6.9 \cdot 10^{-9} \ \frac{m}{s^2}} = 158.5 \quad \rightarrow \quad N_{CF} = 159$$

The use of comb fingers in this situation is motivated by at least two major needs: the first one is to achieve large quality factors to lower thermomechanical noise. Using comb fingers avoids squeezed-film damping and thus enables high-Q operation. The second is the need to avoid electrostatic softening which, at such low resonance frequency, would easily induce pull-in effects.

(iii)

The dynamic range is the ratio of the maximum to the minimum measurable signal. The minimum measurable signal will be found through noise integrated over the bandwidth. We can do this calculation in any unit. For the sake of simplicity, we do this in units of acceleration:

$$DR = 20 \cdot \log_{10} \frac{2 \cdot FSR}{\sigma_{n,acc}} = 20 \cdot \log_{10} \frac{10 \ mg}{1 \frac{ng}{\sqrt{Hz}} \sqrt{30 \ Hz}} = 20 \cdot \log_{10}(1.8 \cdot 10^6) = 125 \ dB$$

Note that the factor 2 at the numerator accounts for positive and negative accelerations. To cope with such a large dynamic range, the required number of bits becomes:

$$N_{bit} = \log_2(1.8 \cdot 10^6) = 20.8 \quad \rightarrow \quad 21 \ bit$$

The corresponding LSB in terms of acceleration, and quantization noise, can be calculated as:

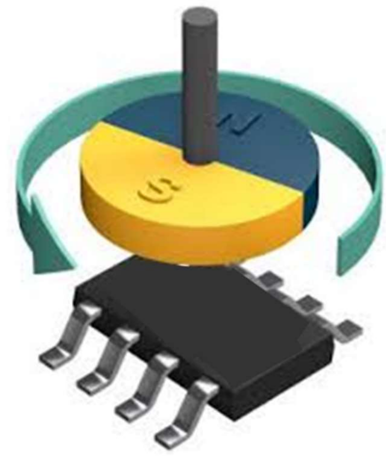
$$LSB = \frac{FSR}{2^{21}} = 4.77\ ng$$

$$\sigma_{quant} = \frac{LSB}{\sqrt{12}} = 1.37\ ng$$

Which is effectively negligible if compared to the summed value of thermomechanical end electronic noise (5.5 n*g*), especially taking into account the fact that noise contributions sum up quadratically.

Last Name ___CASAPO___ **Given Name** ___LIA____ **ID Number** _20200611_

Question n. 3



A 2-axis in-plane Lorentz-force MEMS magnetometer, coupled to a permanent magnet, is used for precise control of a steering wheel system of a Formula 1 car. As schematically shown in the picture, the rotation of the wheel magnet, relative to the fixed magnetometer, generates a sinusoidal response as a function of the rotation angle, i.e. of the relative orientation of the wheel.

The magnetic field modulus, measured at the distance where the magnetometers are positioned, is 50 mT.

The electronics is formed by a capacitive front-end interface, followed by a 40x-gain differential voltage amplifier and a demodulator with a 2x-gain to compensate demodulation losses.

(i) calculate the target sensitivity (in terms of output voltage change per unit field), and, after calculating the corresponding capacitive sensitivity [in fF/μT], draw the differential capacitive change as a function of the rotation angle for the two sensing axes;

(ii) calculate the required Lorentz current to match the above calculated capacitive sensitivity;

(iii) calculate the overall input-referred magnetic field noise density and calculate the error in the angle estimation of the steering wheel (in $°_{rms}$) when the rotation is 45° and when is 90° (hint: the derivative of the arctan function is: $\frac{d(\arctan(x))}{dx} = \frac{1}{1+x^2}$).

Quantity [unit]	Value
Circuit supply voltage [V]	±5
Process thickness [μm]	12
Process gap [μm]	1
Resonance frequency [kHz]	45
Stiffness [N/m]	220
Quality factor [-]	1000
Lorentz drive frequency [kHz]	44.5
Single-ended sense capacitance [fF]	800
N. of recirculation loops	15
Loop length [μm]	800
Rotor-stator voltage difference [V]	5
Low-pass filter bandwidth [Hz]	25
Amplifier feedback capacitance [pF]	1
Amplifier input noise [nV/√Hz]	12
Parasitic capacitance [pF]	5

Physical Constants

$\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m

$k_B = 1.38 \cdot 10^{-23}$ J/K

T = 300 K

(i)

The FSR for each axis is obtained when the permanent magnet fiel dis perfectly aligned with the sensing axis. It is thus ±50 mT. As the circuit supply is ±5 V, we immediately infer that the target sensitivity should be 0.1 V/mT.

The capacitive sensitivity in a Lorentz force magnetometer with recirculation loop can be calculated as :

$$\frac{\Delta C}{B} = 2\frac{C_0}{g}\frac{i \cdot N_{loop} \cdot L\, Q_{eff}}{2\, k_{1/2}} = \frac{C_0}{g}\frac{i \cdot N_{loop} \cdot L\, Q_{eff}}{k_{1/2}}$$

And should equal in this case the following value:

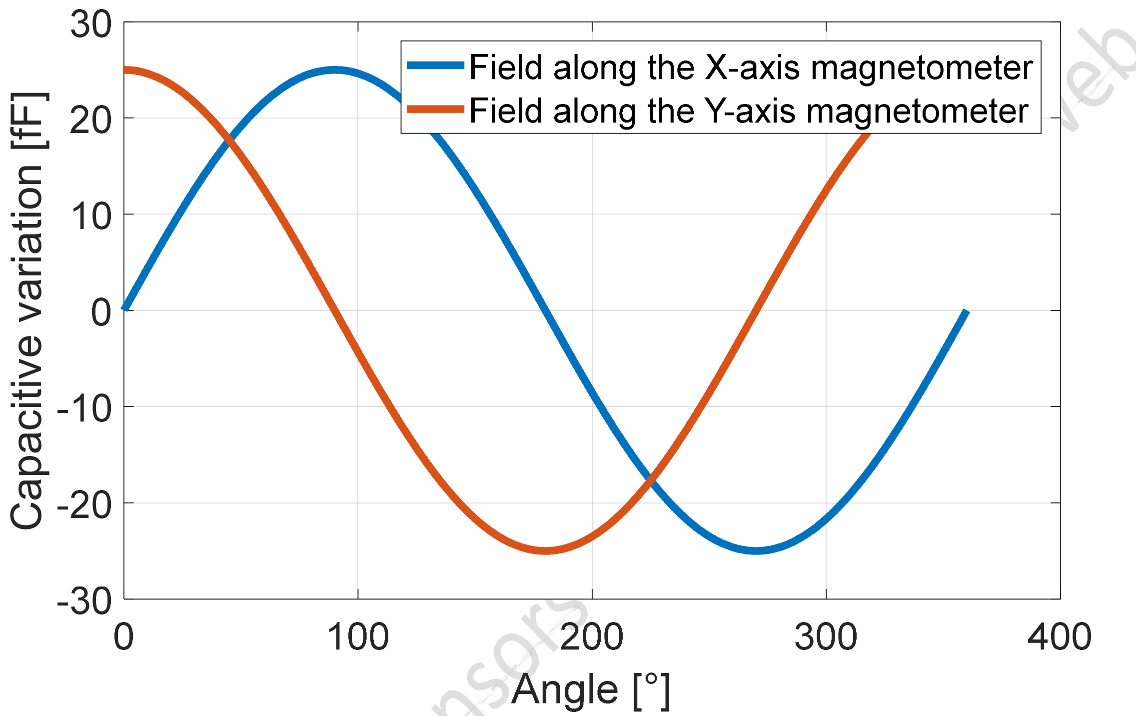
$$\frac{\Delta C}{B} = \frac{\frac{\Delta V_{out}}{B}}{\frac{V_{rot}}{C_F}\, G_{INA}} = 5 \cdot 10^{-13}\frac{F}{T} = 0.5\frac{fF}{mT}$$

In the formula above, we have taken into account the INA gain but not the additional 2x gain of the demodulation stage, because a factor 2 is also lost in synchronous demodulation.

During a steering wheel rotation, the magnetic field sensed along the two axis will change as a sinusoidal function described by:

$$B_x = B_{FSR} \cdot \sin(\theta) \qquad B_y = B_{FSR} \cdot \cos(\theta)$$

The maximum capacitance change along each axis is equal to $\frac{\Delta C}{B} \cdot B_{max} = 25\ fF$, and thus the graph is the one represented below:



(ii)

The required Lorentz current is found from the first reported formula, and can be evaluated as:

$$\frac{C_0}{g} \frac{i \cdot N_{loop} \cdot L\, Q_{eff}}{k_{1/2}} = 0.5 \frac{fF}{mT} \quad \rightarrow \quad i = \frac{0.5 \frac{fF}{mT}}{\frac{C_0}{g} \cdot \frac{N_{loop} \cdot L\, Q_{eff}}{k_{1/2}}}$$

With an effective quality factor $Q_{eff} = \frac{f_0}{2\Delta f} = 45$, we obtain a required current of 254 µA.

(iii)

Given the value of the damping coefficient:

$$b = \frac{k_{\frac{1}{2}}}{\omega_0 Q} = 7.8 \cdot 10^{-7}\ kg/s$$

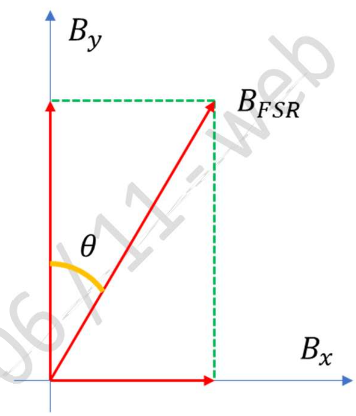
It is possible to estimate the overall noise contribution as the sum of thermomechanical and electronic noise. This yields:

Last Name ___CASAPO___ **Given Name** ___LIA____ **ID Number** _20200611_

$$\sqrt{S_{n,B}} = \sqrt{S_{n,B,tm} + S_{n,B,eln}} = \sqrt{\left(\frac{4}{N_{loop}\ i\ L}\sqrt{k_B T b}\right)^2 + \left(\frac{\sqrt{2S_{Vn}}\left(1+\frac{C_P}{C_F}\right)G_{INA}}{\Delta V_{out}/B}\right)^2} =$$

$$= \sqrt{\left(74\frac{nT}{\sqrt{Hz}}\right)^2 + \left(40\frac{nT}{\sqrt{Hz}}\right)^2} = 85\frac{nT}{\sqrt{Hz}}$$

How does this noise propagate to angle noise? We have to consider the relationship between angle and magnetic field, which we can re-write looking at the picture aside:



$$\theta = \arctan\left(\frac{B_x}{B_y}\right)$$

To linearize the relationship and write an angle noise expression, we make use of the suggested derivative and take the quadratic sum of noises:

$$\frac{d\theta}{dB_x} = \frac{1}{B_y}\frac{1}{1+\left(\frac{B_x}{B_y}\right)^2} \qquad \frac{d\theta}{dB_y} = -\frac{B_x}{B_y^2}\frac{1}{1+\left(\frac{B_x}{B_y}\right)^2}$$

$$\sigma_\theta^2 = \sigma_{B_x}^2\left(\frac{d\theta}{dB_x}\right)^2 + \sigma_{B_y}^2\left(\frac{d\theta}{dB_y}\right)^2 = \sigma_B^2\left[\left(\frac{d\theta}{dB_x}\right)^2 + \left(\frac{d\theta}{dB_y}\right)^2\right] = S_{n,B}\cdot BW\left[\left(\frac{d\theta}{dB_x}\right)^2 + \left(\frac{d\theta}{dB_y}\right)^2\right]$$

We can now write noise in the angle estimation starting for the two considered angles. At 45°, the two magnetic fields components are both equal to $B_{max}/\sqrt{2}$, thus:

$$\sigma_{\theta=45^\circ} = \sqrt{S_{n,B}\cdot BW\left[\left(\frac{1}{2B_y}\right)^2 + \left(\frac{-1}{2B_y}\right)^2\right]}\frac{180^\circ}{\pi} = \sqrt{S_{n,B}\cdot BW\frac{1}{B_{max}^2}}\frac{180^\circ}{\pi} = 0.49\cdot 10^{-3}\ {}^\circ{}_{rms}$$

At 90°, the x-axis field component is null, while the y-axis one equals B_{max}, thus:

$$\sigma_{\theta=9\ \circ} = \sqrt{S_{n,B}\cdot BW\left[\left(\frac{1}{B_y}\right)^2\right]}\frac{180^\circ}{\pi} = \sqrt{S_{n,B}\cdot BW\frac{1}{B_{max}^2}}\frac{180^\circ}{\pi} = 0.49\cdot 10^{-3}\ {}^\circ{}_{rms}$$

Noise is thus equal in the two situations.