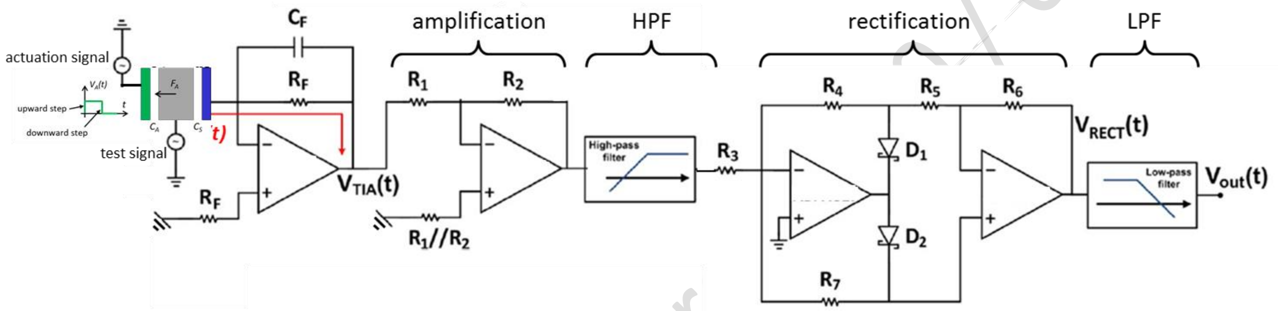
Last Name ___Dalla Mela___ **Given Name** ___Assunto__ **ID Number** ___20200220__

Question n. 1

The ring-down response curve is a widely used technique for electromechanical characterization of MEMS sensors. Assuming the sample case of electromechanical characterization of a MEMS accelerometer, describe (i) how the technique is implemented, (ii) the analytical background, and (iii) which the main parameters are that one can estimate from this measurement. How do the measurement results change as a function of temperature and package pressure?

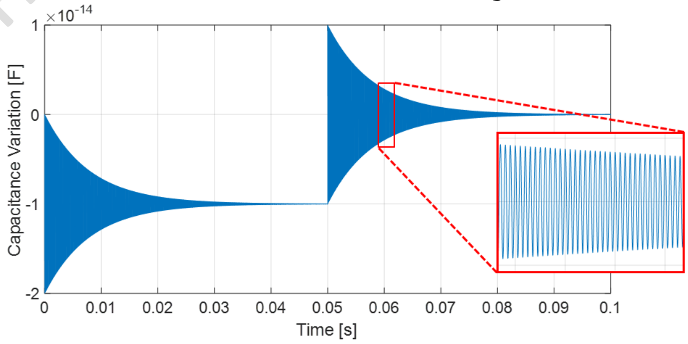
Whenever a capacitive 3-port device is available, as in the case of a MEMS accelerometer formed by the rotor and two stators, the ringdown response can be implemented by using one stator as an electrostatic actuator that delivers a pulse- or step-like input force, and the other stator as a capacitive sensor. The rotor will be kept to DC ground and biased with a small AC signal having a frequency much larger than the MEMS resonance, in order to avoid motion perturbation induced by the sense port and not applied by the drive port. The connection and a possible electronic scheme for the sense chain are represented below.



In response to a step-like actuation, the rotor (described by a second order system with complex conjugate poles if Q>0.5, or real poles if Q≤0.5) will perform a damped sinusoidal motion described by the simplified equation below, where Q is the quality factor, f_0 is the MEMS resonance frequency and $\Delta C_{S\infty}$ is the steady state capacitive change (after the ringdown end).

$$\Delta C_S(t) = \Delta C_{S\infty}\left[1 - e^{-\frac{t}{\tau}} \cdot cos(2\pi f_0 t)\right]$$

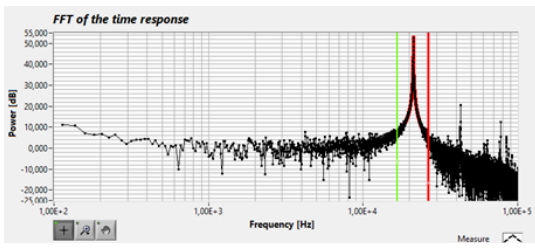
$$\tau = \frac{Q}{\pi f_0} = \frac{2Q}{\omega_0}$$



It is thus evident how the response includes information about the dynamic behavior of the MEMS, and thus both parameters (Q and f_0) can be extracted by implementing e.g. a least mean square fitting to the analytical curve (i.e. you calculate several analytical curves at variable Q and f_0, and the one that best matches the experimental one indicates your effective resonance and Q factor). As an alternative, you calculate the frequency from the average measured period and the Q factor from the expression of the time constant τ.

If the MEMS accelerometer has a Q factor of lower than 0.5, the response will not be over-damped, but it will match the response of a system with two split real poles: the resonance frequency is not of much interest in this case, and the dominant pole frequency can be identified through the time constant of the transient.

As an alternative, in all cases one can apply a FFT to the captured data and look at the spectrum: from it, resonance and Q factor are immediately found in all the situations (complex conjugate, real coincident, or real split poles).

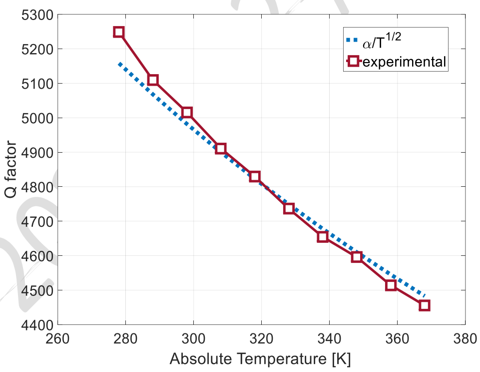


We now analyze what occurs in case of variations of the environmental parameters, i.e. the pressure and temperature at which the MEMS operates.

We assume that the MEMS is packaged, as usually occurs for MEMS accelerometers. The pressure is thus usually set at a specific value, at a reference temperature, during the device capping operation. Typical package pressures for MEMS accelerometers are in the order of e.g. 10 mbar at ambient temperature (e.g. 25°C). Around this pressure value, the damping coefficient is directly proportional to pressure, and thus the Q factor is inversely proportional to pressure.

Inner package temperature instead can change as a function of external temperature variations. What does it happen to the quality factor in this case? We can infer this from the following considerations:

- the damping coefficient $b \propto n \cdot \bar{v}$ is, in general, proportional to the density of molecules n and their mean velocity $\bar{v}$ (the number of collisions between molecules and the rotor, and their energy, determines indeed the dissipation in this case);
- from the ideal gas equation we can write that $n \propto \frac{p}{k_B T}$ and from the kinetic energy of a 1-DOF system equation we can write that $\bar{v} \propto \sqrt{k_B T}$. Additionally, as the volume is constant (the package), we know that temperature and pressure cannot change independently;
- by combining all the proportionalities above we find out that the Q factor goes with the inverse of the square root of temperature (see the graph aside).



In particular, if the temperature decreases, we will observe on our ringdown response a larger Q factor, thus a longer decay time. As a specific curious case, a system sized to have a Q factor of 0.5 at ambient temperature will become underdamped at lower temperatures (increase of Q) and overdamped at higher temperatures (decrease of Q).

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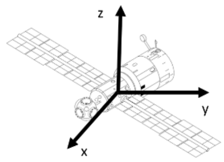
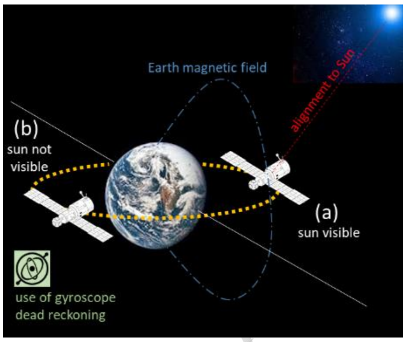
Question n. 2



Three dual-mass, tuning fork MEMS gyroscopes with comb-finger push-pull actuation and differential parallel-plate sensing, coupled to driving and sensing electronic circuits, are used on a satellite in a non-stationary orbit around the Earth to recover the angular *attitude* (i.e. the orientation with respect to Earth, or more simply the angles relative to a certain coordinate system as shown in the top figure). The satellite, indeed, can identify its attitude (see the figure aside):



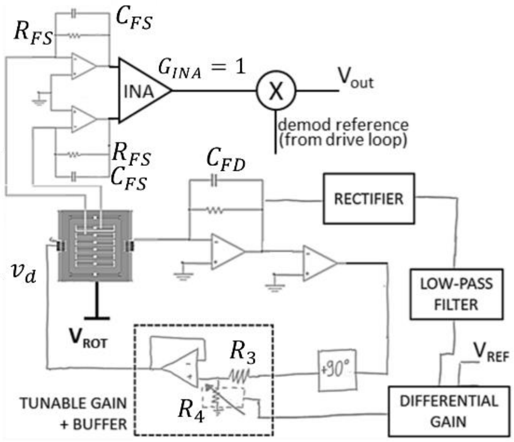
- in the half-orbit (a) that looks towards the Sun, using CMOS radiation detectors and the direction of the Earth magnetic field (in this phase, as the satellite attitude is known through the mentioned sensors, the gyroscope is not used but its offset can be self-calibrated and nulled);
- as the satellite cannot rely on the Sun position during the other half (b) of the orbit around the Earth (when eclipsed by the Earth itself), the satellite needs to rely on dead reckoning of the orientation through the gyroscopes (in this half-period temperature is kept between 5°C and 20°C by a control system).

The satellite overall orbit around the Earth lasts exactly 90 minutes. Consider just the z-axis gyroscope, whose parameters are shown in the Table: you need to evaluate whether the gyroscope can cope with this application by answering the following points.

Drive frequency [Hz]	20000
Mode split [Hz]	400
Sense mass [kg]	$2.5\ 10^{-9}$
Drive mass [kg]	$1\ 10^{-9}$
Full-scale range [dps]	±2000
Linearity error [%]	0.5
Process gap [μm]	1.5
Process height [μm]	20
Drive Q factor	5000
Sense Q factor	500
Number of comb	40
Rotor voltage [V]	16
S.E. sense capacitance [fF]	320
Circuit supply [V]	±3
Amplifier noise [nV/√Hz]	10
R_3 [kΩ]	100
Parasitic capacitance [pF]	3
INA gain	1

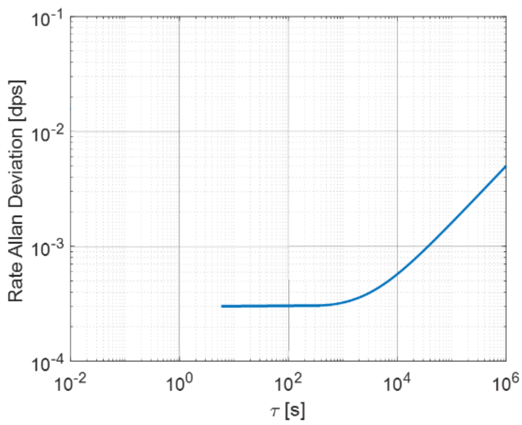
(i) find the nominal drive displacement amplitude, so to cope with linearity requirements. Additionally, compute the sensitivity in terms of displacement per unit rate (in [m/dps]);

(ii) compute the driving voltage v_d (squarewave push-pull drive) so to obtain the target displacement in nominal conditions and size the resistors R_4 of the voltage divider in the figure aside.



(iii) compute the intrinsic Noise Equivalent Rate Density NERD. Find the values of the feedback resistors and capacitors R_{FS} and C_{FS} of the shown sense chain to optimize the scale factor and to have the feedback pole two decades far from the operating frequency. Then compute the electronic noise density.

(iv) consider the Allan deviation graph partly represented aside. Complete it by adding the missing portion of the curve at low observation intervals. Comment on the possibility to use this sensor for the proposed satellite inertial navigation with a maximum attitude target error of 1°. Suggest solutions to improve the performance.



Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K;

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m;

(i)

To cope with the given linearity requirement, the displacement of the proof masses along the sense axis is constrained by the parallel-plate linearity limits:

$$\epsilon = \left(\frac{\Delta y}{g}\right)^2 = 0.005 \Rightarrow \Delta y < g\sqrt{\epsilon} = 106\ \text{nm}$$

Given the Full-Scale Range, it is then possible to find the corresponding drive displacement x_d through the displacement sensitivity S_y:

$$S_y = \frac{\Delta y_{max}}{\Omega_{FSR}} = \frac{x_d}{\omega_{split}} \cdot \frac{\pi}{180} \quad \Rightarrow \quad x_d = \frac{\Delta y_{max}\omega_{split}}{\Omega_{FSR}} \cdot \frac{180}{\pi} = 7.64\ \mu\text{m}$$

(ii)

Considering a push-pull square-wave driving of amplitude $\pm V_d$, the total force F_d exerted on the half-mass is:

$$F_d = V_d \frac{4}{\pi}\, 2\,\eta = \frac{8}{\pi}\, V_d\, V_{ROT} \left(\frac{2\epsilon_0 h}{g} N_{CF}\right)$$

where the factor $4/\pi$ converts the squarewave amplitude to the amplitude of its first harmonic, the additional factor 2 is due to the force being applied with the push-pull technique on half the mass and η is the transduction factor of a single actuation port.

The drive displacement is then given by the force multiplied by the factor Q_d/k_d (Hooke's law at the resonant frequency), where the drive-mode stiffness can be evaluated as $k_d = \omega_d^2(m_s + m_d)$. Combining all these considerations:

$$x_d = F_d \frac{Q_d}{k_d} \quad \Rightarrow \quad V_d = \frac{\pi\, x_d\, g\, \omega_d^2(m_s + m_d)}{16\, V_{ROT}\, \epsilon_0\, N_{CF}\, Q_d} = \pm 0.22\ \text{V}$$

This implies that the resistor R_2 should be sized to partition the square-wave clamped to supply voltage down to the correct value. This will be done by the AGC and it turns out that R_2 will be set to $4\ k\Omega$.

Note: deviations by a factor 2 or ½ due to misinterpretation of the data (number of comb single-ended or differential ecc…) were not considered errors in the correction.

(iii)

The intrinsic Noise Equivalent Rate Density $NERD_i$ can be evaluated as:

$$NERD_i = \frac{1}{\sqrt{2}} \frac{\sqrt{4k_B T b_s}}{2 m_s x_d \omega_d} \frac{180}{\pi} = \sqrt{\frac{k_B T}{2 Q_s m_s \omega_d}} \frac{1}{x_d} \frac{180}{\pi} = 840 \frac{\mu\text{dps}}{\sqrt{\text{Hz}}}$$

(note that factor $1/\sqrt{2}$ that accounts for the two halves of the device).

To optimize the Scale Factor, one can match the voltage variation at the INA output to the ± 3 V supply voltage. One can compute the maximum (single-ended) sense-capacitance variation ΔC_{max} as:

$$\Delta C_{max} = \frac{\partial C}{\partial y} \cdot \Delta y_{max} = \frac{C_S}{g} \cdot \Delta y_{max} = 22\ \text{fF}$$

Then, the INA voltage output is:

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$$V_{INA} = \Delta C_{max} \frac{V_{ROT}}{C_{FS}} 2 = V_{DD} \quad \Rightarrow \quad C_{FS} = 2 \frac{V_{ROT}}{V_{DD}} \Delta C_{max} = 241 \text{ fF}$$

One can now size the feedback resistor R_{FS} in order to have the pole f_p of the charge amplifier two decades below the operation frequency as requested:

$$f_p = \frac{1}{2\pi R_{FS} C_{FS}} = \frac{f_d}{100} \quad \Rightarrow \quad R_{FS} = \frac{100}{2\pi f_d C_{FS}} = 3.3 \text{ G}\Omega$$

Finally, the NERD due to the electronics $NERD_e$ can be computed by transferring all the electronic noise sources (the voltage noise of the sense operational amplifier and the noise of R_{FS}) to the INA output and then refer it back to the angular rate input:

$$NERD_e = \sqrt{2} \sqrt{S_v \left(1 + \frac{C_P}{C_{FS}}\right)^2 + \frac{4k_B T}{R_{FS}} \cdot \left(\frac{1}{\omega_d C_{FS}}\right)^2} \left(\frac{\Omega_{FSR}}{V_{DD}}\right) = 144 \frac{\text{μdps}}{\sqrt{\text{Hz}}}$$

Note the factor $\sqrt{2}$ that takes into account the two (uncorrelated) operational amplifiers and feedback resistors. Note also that by setting the value of C_{FS}, the sensitivity at the INA output was implicitly set to V_{DD}/Ω_{FSR}.

Note: deviations by a factor 2, ½ or 1/√2 due to misinterpretation of the data (number of PP single-ended or differential, NERD of single mass or two masses ecc…) were not considered errors in the correction.

(iv)

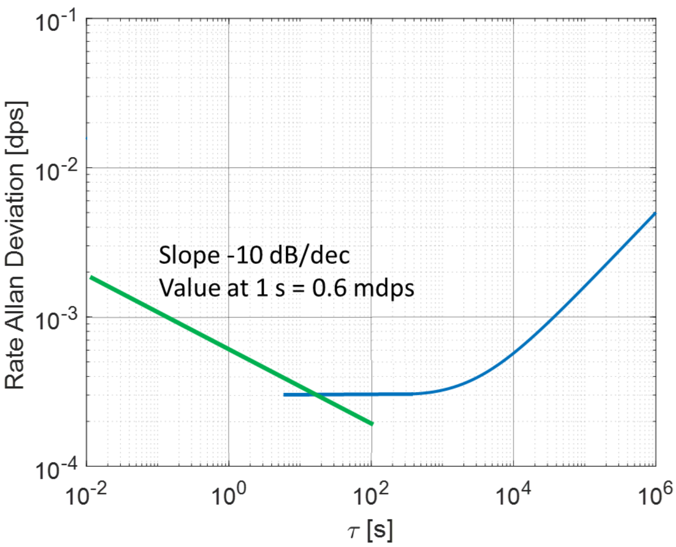
We know that the Allan deviation plot for large integration times is dominated by long-term drift, flicker noise and bias instability that give rise to straight lines with slope 0 and +1/2. At shorter integration times, the white noise dominates and is characterized by a line with slope -1/2. With a known slope, we just need one point belonging to this line to complete the Allan plot. The most convenient way to answer is recalling that (in presence of white noise only) the Allan deviation at integration time $\tau = 1$ s is equal to $\sigma_{AV} = \frac{NERD}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1\text{ s}}} \sqrt{NERD_i^2 + NERD_e^2} = 615$ μdps. Overall, the missing portion of the graph is described by:

$$\sigma_{AV}^{(white)} = \frac{600 \text{ μdps}}{\sqrt{\tau}}$$

As for the use of this sensor on the described satellite, we need to understand whether the gyroscope can cope with a 1° whole angle drift over a half orbit, i.e. 45 minutes or 2700 seconds. From the graph, we observe an Allan deviation of the *angular rate* of about 400 μdps at 2700 s. But we are actually interested in the drift of the *attitude angle* of the satellite. This derives from the integration of the rate drift. Therefore, we can compute the worst-case angle Allan deviation $\sigma_{AV,\theta}$ by multiplying the rate Allan deviation $\sigma_{AV,\Omega}$ with the integration time. In formulas:

$$\sigma_{AV,\theta}(2700 \text{ s}) = \sigma_{AV,\Omega}(2700 \text{ s}) \cdot 2700 \text{ s} = 0.6°$$

We can therefore conclude that this sensor is able to cope with a requirement of 1° angle drift over 45 minutes.



What if one still wanted to improve the long-term performance of this gyroscope? One observation we can make is that this gyro operates in an environment with temperature controlled between 5°C and 20°C. We can safely assume that if this temperature control improved, the Allan deviation section with slope +1/2 would improve as well.

It is important to note that the Allan deviation plot of a sensor at large integration strides is strongly dependent on the environmental condition. At large integration times, the performance of a sensor is a combination of the environmental conditions during the measurement and the ability of the device itself to reject such conditions.

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Question n. 3

A Silicon epitaxial layer of 3-µm thickness and with a surface impurity depth of (10 ± 3) nm is used for an imaging sensor of blue radiation in medical applications using a 405-nm laser. Overlapping on top of the whole sensor lies a blue filter. The readout circuit is based on a 3-transistor topology. No micro-lenses are available. Other sensor and circuit parameters, along with their maximum spread, are reported in the table.

Quantity (@ 405 nm)	Value	±3σ dev
Absorption coefficient [m^{-1}]	10^7	-
Active Si thickness [µm]	3	-
Impurity depth [nm]	15	±3
Input wavelength [nm]	405	-
Pixel area [μm^2]	2x2	-
Minimum transistor area [μm^2]	0.35x0.7	
Depletion region [µm]	0.6	-
Dark current density [nA/cm^2]	1	±0.15
Gate capacitance [fF]	0.3	±0.01
Input photon flux [$ph/s/m^2$]	10^{15}	-
Integration time [ms]	30	-
Filter transmittance	0.6	±0.02
ADC number of bits	10	-
Supply voltage [V]	1.8	-

(i) find the quantum efficiency and the measured number of electrons when a controlled photon flux of 10^{15} $ph/s/m^2$ is impinging on the pixel surface;

(ii) assuming the mean value for all the given parameters, calculate the various temporal noise contributions in terms of rms number of electrons;

(iii) assuming that process spreads between the different quantities are uncorrelated, calculate the different spatial noise contributions in terms of rms number of electrons;

(iv) assuming that you are able to implement an offset calibration, quantify the improvement in terms of dynamic range.

Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$\varepsilon_{Si} = 11.7$

$q = 1.6\ 10^{-19}$ C

$k_B = 1.38\ 10^{-23}$ J/K

T = 300 K

1) The quantum efficiency is defined as the ratio between the collected electrons over the incident number of photons. It can be found knowing the absorption coefficient (α), the implant/impurity depth (x_1) and the active thickness (x_2):

$$\eta \simeq (e^{-\alpha x_1} - e^{-\alpha x_2})T_{blue} = 0.52.$$

Notice that also the filter transmittance coefficient must be included in the computation (product of the silicon quantum efficiency itself and the filter efficiency).

The generated photocurrent can be computed as:

$$i_{ph} = \Phi \cdot A_{pix} \cdot FF \cdot \eta \cdot q$$

The only unknown is the fill factor (FF), which is the ratio of the total pixel area over the active area. Given the minimum transistor area, for a three-transistor topology, the fill factor is:

$$FF = \frac{A_{pix} - A_{eln}}{A_{pix}} = \frac{A_{pix} - A_T * 3}{A_{pix}} = 0.82.$$

The photocurrent and the resulting measured number of electrons are:

$$i_{ph} = 0.27\ fA.$$

$$\#el = \frac{i_{ph} \cdot t_{int}}{q} = 50.58.$$

2) First of all let us compute the integrating capacitance value, which is the sum of the gate capacitance and the diode capacitance.

$$C_{int} = C_g + C_{dep} = 0.86\, fF.$$

The different types of temporal noise contribution we should take care of are:

a. Reset Noise:

$$\sigma_{res,el} = \sqrt{\frac{kTC_{int}}{q^2}} = 11.8\, el,$$

b. Shot noise:

$$\sigma_{shot,el} = \sqrt{\frac{(i_d + i_{ph})t_{int}}{q}} = 7.6\, el,$$

c. ADC quantization noise:

$$\sigma_{quant,el} = \sqrt{\left(\frac{Vref}{2^{N_{bit}}\sqrt{12}} C_{int}\right)^2} \,/q = 2.7\, el.$$

The total temporal noise value in terms of rms number of electrons is thus:

$$\sigma_{temp,el} = \sqrt{\sigma_{res,el}^2 + \sigma_{shot,el}^2 + \sigma_{quant,el}^2} = 14.3\, el.$$

3) There are two main sources of FPN: "dark signal non-uniformity" (DSNU) and "photo response non-uniformity" (PRNU):

a. DSNU is caused in this case by the dark signal (whose variations represent offset variations in absence of any signal) and the gate capacitance variations (whose variations cause output offset variations even for the same value of the dark current).

b. PRNU is caused in this case by the variation of the filter transmittance, gate capacitance, and impurity depth variations (all affect the gain from photons to output).

Since all the contributions are assumed to be uncorrelated, we should sum up quadratically the standard deviations. One can calculate the percentage values as the ratio between the ±3-sigma data and the mean data. We thus get:

$$\sigma_{DSNU,\%} = \sqrt{\sigma_{dark,\%}^2 + \sigma_{cap,\%}^2} =$$

$$= \sqrt{\left(100 \cdot \frac{2 \cdot 3\sigma_{id}}{i_d}\right)^2 + \left(100 \cdot \frac{2 \cdot 3\sigma_{Cg}}{C_g}\right)^2} =$$

$$= \sqrt{\left(100 \cdot \frac{2 \cdot 0.15}{1}\right)^2 + \left(100 \cdot \frac{2 \cdot 0.01}{0.3}\right)^2} = 30.7\%$$

Notice that a factor 2 has been considered in order to take into account the $\pm 3\sigma$ spread. The added noise due to dark signal non uniformities, expressed in terms of electrons, is therefore:

$$\sigma_{DSNU,el} = \frac{i_d t_{int} \sigma_{DSNU,\%}}{q \cdot 100} = 1.88\, el$$

For the PRNU computations, variations on the impurity depth and the filter transmittance must be considered as well. Photon to electrons gain is not linear with the former contribution and thus we take the full calculation in terms of the exponential law variations.

$$\sigma_{\eta,\%} = \frac{\left(e^{-\alpha\,(x_{imp}-3\sigma_{imp})} - e^{-\alpha\,(x_{imp}+3\sigma_{imp})}\right)}{e^{-\alpha\,(x_{imp})}} \cdot 100$$

$$\sigma_{Tblu,\%} = 100 \cdot \frac{2 \cdot 3\sigma_{Tblu}}{T_{blu}}$$

Thus we obtain the %PRNU and the PRNU in terms of number of electrons as:

$$\sigma_{PRNU,\%}= \sqrt{\sigma_{\eta,\%}^2 + \sigma_{Tblu,\%}^2 + \sigma_{cap,\%}^2} = 10.7\%$$

$$\sigma_{PRNU,el} = \frac{i_{ph} t_{int} \sigma_{PRNU,\%}}{q \cdot 100} = 5.4\ el$$

4) The dynamic range without offset calibration can be computed as:

$$DR_{no\_calib} = 20\log_{10}\left(\frac{\frac{V_{DD}C_{int}}{q}}{\sqrt{\frac{kTC_{int}}{q^2} + \left(\frac{Vref}{2^{N_{bit}}\sqrt{12}}C_{int}\right)^2 + \frac{i_d t_{int}}{q} + \sigma_{PRNU,el}^2 + \sigma_{DSNU,el}^2}}\right) = 57\text{dB}$$

With respect to all the calculations above, note the absence of the noise term related to the photon shot noise.

If pixel-by-pixel offset calibration of nonuniformities (in particular of dark current values) can be performed, the DSNU contributions will be eliminated. The DR will become:

$$DR_{offset_{calib}} = 20\log_{10}\left(\frac{\frac{V_{DD}C_{int}}{q}}{\sqrt{\frac{kTC_{int}}{q^2} + \left(\frac{Vref}{2^{N_{bit}}\sqrt{12}}C_{int}\right)^2 + \frac{i_d t_{int}}{q} + \sigma_{PRNU,el}^2}}\right) = 57.8dB$$

The DN range improvement is very moderate because DSNU is not the dominant noise source:

$$\Delta DR = 0.8dB.$$

The dominant noise source is indeed kTC noise. Note however that the application of correlated double sampling (CDS) only does not remove all offset contributions (e.g. dark current fluctuations are not eliminated by CDS!).