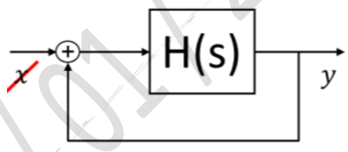
Last Name _ NONDE _ **Given Name** __ SISTO __ **ID Number** _ 20200124 _

Question n. 1

Discuss and demonstrate the Barkhausen criteria for harmonic oscillators. In the specific case of a MEMS-based oscillator, indicate what the electronic circuit needs to compensate so to cope with the Barkhausen criteria, and why a nonlinearity in the loop is needed. Finally, draw the complete scheme of one oscillator indicating the phases of the signals all along the loop nodes, writing the loop gain expression and drawing its modulus and phase diagrams.

The Barkhausen criteria state that for a generic loop characterized by a transfer function $G_{loop}(j\omega)$, in order to sustain a continuous oscillation at a desired frequency ω_0, the following conditions should hold:

$$\begin{cases} |G_{looop}(j\omega_0)| = 0\ dB \\ \measuredangle\left(G_{looop}(j\omega_0)\right) = 0° \end{cases}$$



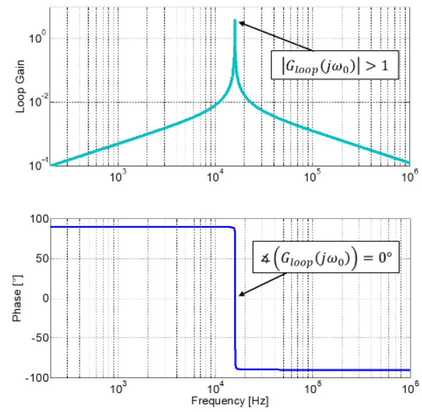
indicating that a signal should come back with the same amplitude and phase after a loop turnaround. This can be easily demonstrated by using the loop shown aside, which, in absence of any input x (i.e. in a self-sustained condition), is self-sustained only for the conditions shown above, i.e. H(s) = 1.

$$y = H(s)(\cancel{x} + y)$$

In the specific case of a MEMS-based oscillator, the element that causes energy dissipation (and thus generates the requirement for a sustaining circuit), is the damping coefficient. Its dissipation can be better quantified by evaluating its equivalent electrical model, which shows an RLC behavior. The capacitance and inductance are lossless elements, while the resistance indicates the dissipation. Its admittance is given by:

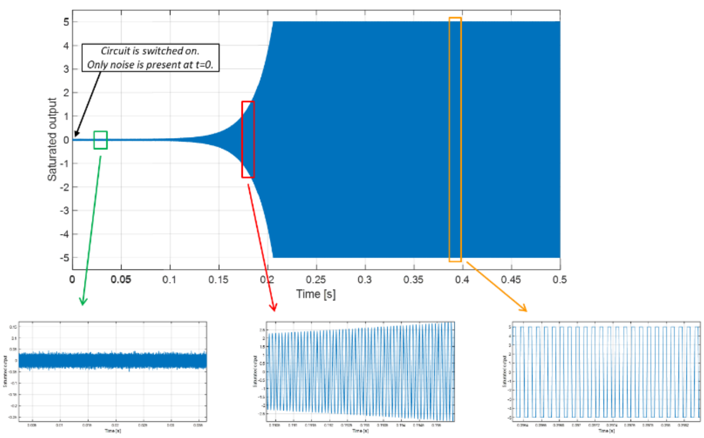
$$\frac{1}{R_{eq}} = \frac{\eta^2}{b}$$

so that ideally a sustaining circuit will need a resistive gain > R_{eq}.

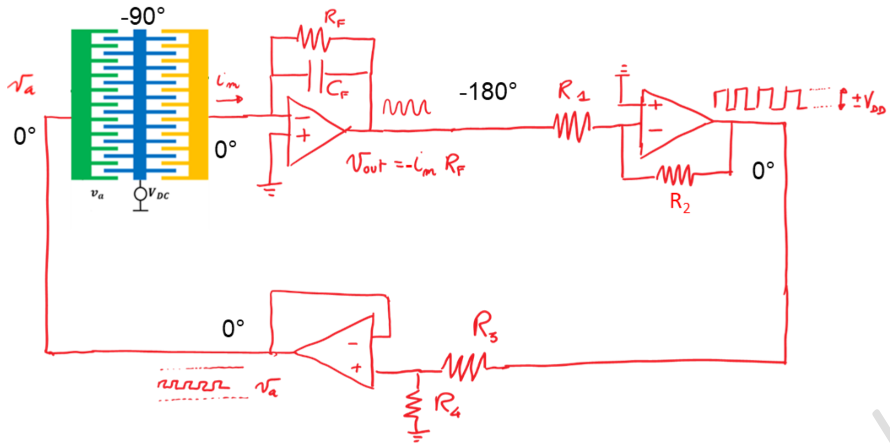


It is worthwhile to remark that it is practically impossible to implement a circuit with a resistive gain exactly equal to the one indicated above. Besides, the value of the electrical resistance changes from part to part and as a function of environmental conditions due to the spreads of b and η and to the drift of b. It is thus common to design a sustaining circuit with a resistive gain larger than R_{eq} at resonance, which includes a nonlinearity for increasing signal amplitude.

In this way, in a startup phase the loop gain will be larger than 0 dB (with a null loop phase being as stated by the Barkhausen criteria above), leading to a signal growth for the desired frequency component. As soon as saturation is reached in one electronic block, the loop gain will begin decreasing, finally stabilizing to 0 dB as required by the Barkhausen condition. An example of the ideal circuit loop gain for a MEMS-based oscillator, and of the signal time traces in the startup phase is shown below.



A real circuit will be in general formed by a more than just a single resistive gain stage. An example is given below with point-by-point justification of the role of each of the stages.

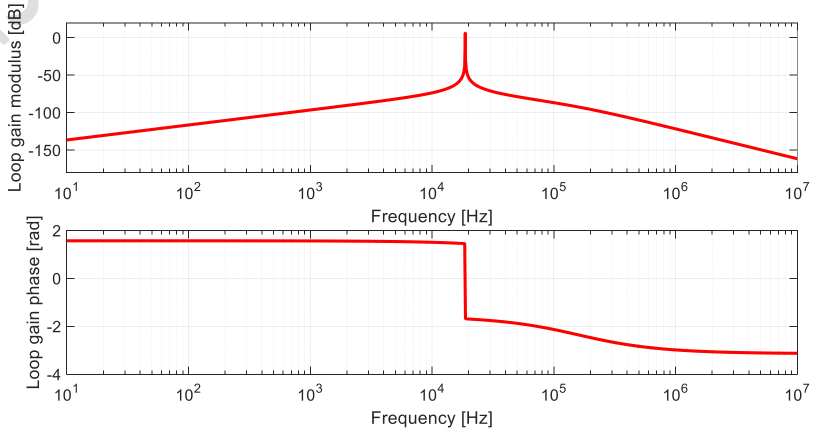


The shown circuit implements an oscillator by adopting:

- a comb-based MEMS resonator, providing a -90° phase change from input voltage to motion due to its transfer function at resonance, and +90° phase change between motion and motional current due to the capacitive effect. Overall thus, the MEMS causes no phase shift at resonance (indeed, it can be modeled just as a resistor with equivalent resistance R_{eq});
- a front-end amplifier in a transimpedance (TIA) configuration (the pole implemented by the feedback R_F/C_F is at a frequency much larger than the desired resonance). This causes an inversion of the sign between input current and output voltage. The amplifier gain is just $-R_F$, and the overall gain from input v_a to the TIA output is $-R_F/R_{eq}$;
- an inverting gain that restores the phase to 0 and provides additional gain if the feedback resistance is itself not large enough to compensate the MEMS equivalent resistance. The gain at resonance up to its output becomes $R_F/R_{eq}*R_2/R_{e1}$;
- a final stage needed to limit a too large voltage amplitude on the MEMS, to satisfy the small signal conditions needed for a single-ended MEMS resonator. The stage is formed by a voltage divider and a buffer.

Taking into account the indicated gains and at least the TIA pole, neglecting all opamp poles which are supposed to be at much larger frequencies, one obtains the following gain expression, plotted below for sample typical values.

$$G_{loop}(s) = \frac{1}{L_{eq}s + R_{eq} + \frac{1}{sC_{eq}}} \quad \frac{R_F}{1 + sR_FC_F} \quad \frac{R_2}{R_1} \quad \frac{R_4}{R_3 + R_4}$$



Note: the feedthrough capacitance has been ignored throughout the discussion. If you included it in your discussion, it will be appreciated.

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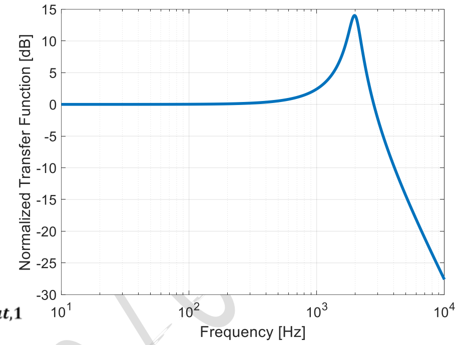
Question n. 2

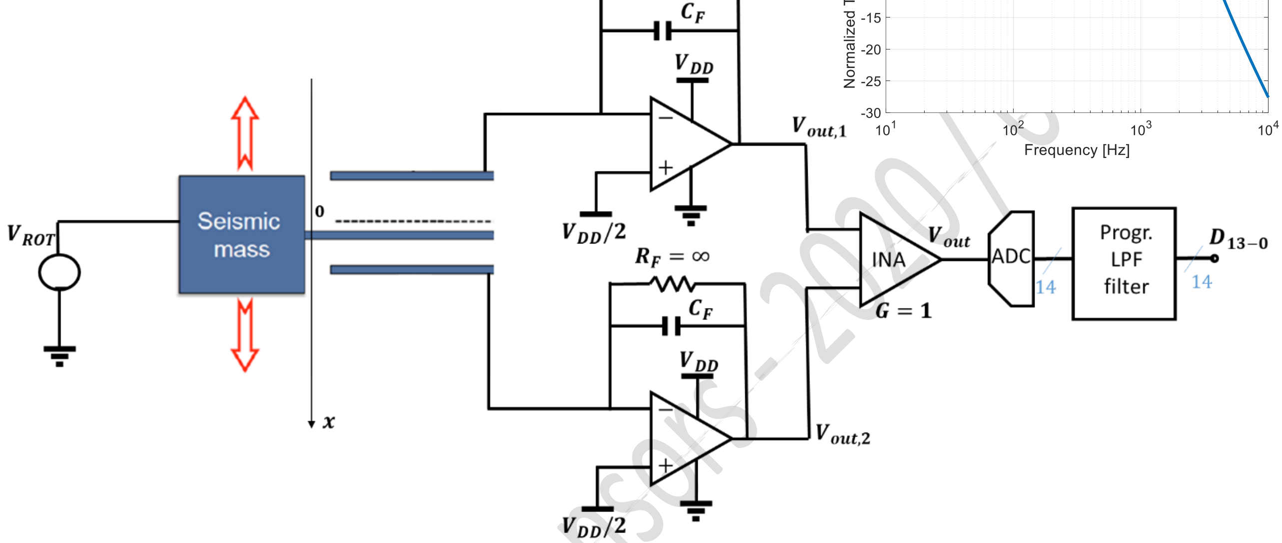
You are a student who just attended the MEMS and Microsensor course and you are looking for an accelerometer for a new IoT application.

You find the MB2020, an accelerometer with the performances you are looking for. The product datasheet specifies only a few data, which are reported in the table aside.

Supply voltage	0 - 3 V
FSR	±8 g
Output	14 bit
Consumption	150 μW
PP area	30x250 (μm)^2
PP cells	10



You are asked to do a reverse engineering on this sensor, taking advantage of other information you can extrapolate from the images.



The programmable filter at the end of the electronic chain in the figure is a second-order digital low-pass filter, characterized by real coincident poles with selectable frequency.

Assume initially that V_{ROT} is a DC voltage equal to V_{DD}:

(i) find the sensitivity at the ADC input (in [V/g]) and at the 14-bit digital output (in [LSB/g]);

(ii) identify a cut-off frequency for the programmable filter so to maximize the ±3 dB bandwidth. Assuming to have a well-balanced system in terms of electronic noise contributions, find the value of the feedback capacitance C_F (assume that the two front-end stages burn 70% of the total supply current, that the overdrive voltage of the front-end transistors is 0.1 V, and that the parasitic capacitance value is 3 pF);

(iii) find the electromechanical sensitivity (in [F/g]) and the gap of the parallel plates;

(iv) modify the system in order to apply a modulated voltage V_{ROT} to the rotor and draw the new electronic scheme. Which are the bounds (upper and lower) of the modulating frequency? Quote the new sensitivity. Is the presence of a mechanical DC offset bypassed?

Physical Constants

ε_0 = 8.85 10^{-12} F/m;
T = 300 K;
k_b = 1.38 10^{-23} J/K;
g = 9.81 m/s^2

(i)

Ideally we would like to accommodate within the whole voltage dynamics (0-3 V) the whole sensing range

(± 8g). This implies that the gain up to the INA output should ideally be:

$$G_{INA-IN} = \frac{3V}{16\,g} = 187.5\frac{mV}{g}$$

Similarly, as the ADC has 2^N = 16384 levels, the gain at the ADC output is:

$$G_{ADC-IN} = \frac{16384}{16\,g} = 1024\frac{LSB}{g}$$

(ii)

Without the need for an analytical solution, one can make an approximated line of reasoning. We know that a 2-coincident-real-pole filter will cause a -6 dB decrease in the response. If we set the poles frequency where the MEMS TF has a +9 dB amplitude, we will get in that point an overall -3 dB response, which will maximize[1] the bandwidth.

We thus set the programmable poles frequency at approximately 1.5 kHz, which is from now on considered our sensing bandwidth.

We can thus now equate the quantization noise contribution to the amplifier noise contribution, so to have well balanced electronic noise (we have no information about the damping coefficient and thus we assume thermo-mechanical noise negligible). The comparison is made at the ADC input:

$$\frac{V_{DD}}{2^{14}\sqrt{12}} = \sqrt{2S_{Vn}BW}\left(1+\frac{C_P}{C_F}\right)G_{INA} \approx \sqrt{2S_{Vn}\,BW}\frac{C_P}{C_F}G_{INA}$$

where the amplifier noise can be evaluated from the current, assuming saturation operation:

$$S_{Vn} = \frac{4k_BT\gamma}{g_m} = \frac{4k_BT\gamma}{2\,i_{MOS}/V_{ov}}$$

Note that each front-end transistor current i_{MOS} is equal to half of the amplifier current (two branches per amplifier), which is in turn half of the total current $i_{tot} = \frac{P}{V_{DD}} = 50\ \mu A$. i_{MOS} is thus equal to 12.5 μA.

We get in the end the feedback capacitance as:

$$C_F = \frac{\sqrt{2\frac{4k_BT\gamma}{2\,i_{MOS}/V_{ov}}BW}\,C_P\,G_{INA}}{\frac{V_{DD}}{2^{14}\sqrt{12}}} = 20\,fF$$

(iii)

At this point we can find the electromechanical sensitivity with the usual formula:

$$S_{em} = 2\frac{C_0}{g}\frac{1}{\omega_0^2}$$

where the gap can be found by equating the complete sensitivity formula as:

$$G_{INA-IN} = 2\frac{C_0}{g}\frac{V_{DD}/2}{C_F}\frac{1}{\omega_0^2}G_{INA} = 2\frac{\epsilon_0 A_{PP}N_{PP}}{g^2}\frac{V_{DD}/2}{C_F}\frac{1}{\omega_0^2}G_{INA} = 187.5\frac{mV}{g}$$

[1] The analytical solution, which involves writing the product of the MEMS transfer function and the 2-pole LPF transfer function, and equating it to +3 dB, gives a result which is very close to the approximated solution.

$$g = \sqrt{\frac{2\epsilon_0 A_{PP} N_{PP} \frac{V_{DD}/2}{C_F} \frac{1}{\omega_0^2} G_{INA}}{187.5 \frac{mV}{9.8\ m/s^2}}} = 3.8\ \mu m$$

Note that the difference between the rotor and the stators (which are kept at half the biasing voltage through the virtual ground) is equal to $V_{DD}/2$.

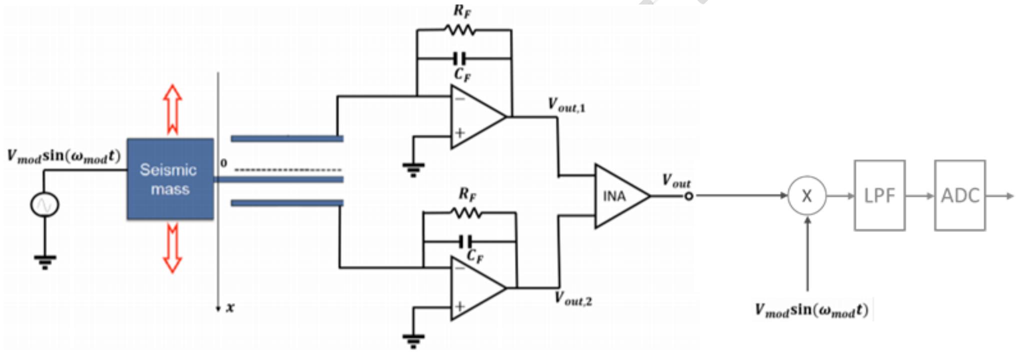
The resulting electromechanical sensitivity turns out to be $0.26\ fF/(m/s^2)$ or $2.5\ fF/g$.

(iv)

A modulation of the rotor voltage is used to correctly readout signals close to DC, without the unrealistic approximation of an infinite feedback resistance.

The technique is also useful to bypass offset voltages of the amplifier, but it does not bypass the mechanical offset of the sensor (indeed, a mechanical offset is read out exactly like a DC acceleration, i.e. a constant rotor displacement).

A scheme of the so-implemented solution is shown below, which yields the same sensitivity as calculated above if the INA gain is 2 (which compensates the factor 2 lost by the modulation/demodulation approach). One will need an additional demodulation stage (i.e. a multiplication of the INA output by the modulating signal), followed by an analog LPF and ADC.



With this type of sensing scheme, the modulating frequency should be much higher than the MEMS resonance frequency (we can e.g. select 100 kHz). However, the higher the frequency, the higher the bandwidth required by the amplifiers, and in turn their consumption. So, for portable applications, going to larger value makes no sense.

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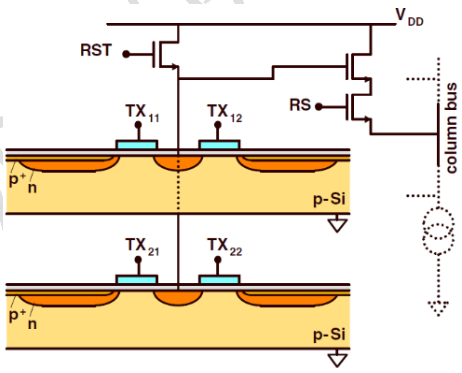
Question n. 3

You are designing a CMOS imaging system for the next-generation mobile phones of a famous company. The camera module is based on two separate sub-systems with different lenses and sensors. The first is a wide-angle camera, the second is a tele-objective camera. The specifications require their field of view (FOV) to be of 110° and 35°, respectively. Another design specification requires for both the sensors to feature 12 Mpixels, with a square sensor and square pixel geometry (the same number of pixels indeed helps in merging the two images in software post-processing).

Max lens to sensor distance d_{max}	5.5 mm
Min lens to sensor distance d_{min}	2.5 mm
Teleobjective lens field of view FOV_{to}	35°
Wide-angle lens field of view FOV_{wa}	110°
N of pixels per sensor N_{px}	12 M
Area of the shared electronics A_{eln}	(1.1 µm)^2
Lens aperture D_{lens}	1.5 mm
Average wavelength λ	500 nm
Dark current density J_d	10^{-5} A/m^2
Sense node capacitance C_{sense}	0.15 fF
Quantum efficiency η	0.5
Integration time t_{int}	8 ms
Scene distance d_{scene}	2 m
Photon flux ϕ_{ph}	10^{10} ph/s/m^2

Constraints on the mobile phone thickness and lens manufacturing require a minimum/maximum distance between the lens and the sensor of approximately 2.5 mm and 5.5 mm respectively. Finally, the used CMOS technology allows the fabrication of pinned photodiodes sharing part of the electronics among 4 pixels (the so-called 1.75T topology shown aside), with an overall area occupation A_{eln} by the four transfer gates and the other shared transistors of (1.1 µm)^2. No pixel-level microlenses are used.



You are required to design the remaining system parameters and evaluate the performance:

(i) choose the focal length of the two lenses and the required size of the two sensors so to meet the specifications on the FOV and to minimize the difference in the two sensor sizes. Evaluate then the available area A_{PPD} for the pinned photodiode in each of the two sensors pixels;

(ii) assuming an average wavelength of 500 nm, evaluate whether the two systems are diffraction limited or sensor limited. Is there any other effect that we are neglecting? How could we make it effectively negligible?

(iii) evaluate the number of photoelectrons accumulated in the two pixels when capturing a uniform scene with an emitted photon flux of 10^{10} ph/s/m^2 originating at a 2 m distance, already integrated over the mobile phone lenses solid angle, at an 8-ms integration time;

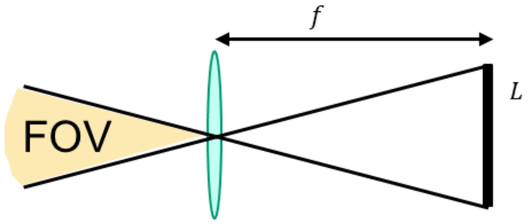
(iv) evaluate the SNR for the pixels of the two sensors. Propose solutions to improve the system.

Physical Constants

$q = 1.6\ 10^{-19}$ C;
$T = 300$ K;
$k_b = 1.38\ 10^{-23}$ J/K;
$\varepsilon_0 = 8.85\ 10^{-12}$ F/m;
$\varepsilon_{Si} = 11.7^*\ \varepsilon_0$;

(i)

Given the limited space in a mobile phone, it is usually difficult to implement tele-objective lenses, while it is, on the contrary, relatively easy to implement wide-angle lenses. As a consequence, for the tele-objective it is obvious to choose the maximum available distance (note that as the object distance s_1 will be much, much larger than the image distance s_2, we can assume very reasonably that $f = s_2$). The focal length is thus:



$$f_{to} = d_{max} = 5.5\, mm$$

The square-sensor side is thus geometrically found (with the FOV expressed in rad) considering the schematic figure above as:

$$L_{to} = 2 \cdot \tan\left(\frac{FOV_{to}}{2}\right) f_{to} = 3.5\, mm$$

The distance of the wide angle lens can be chosen by looking at the same formula:

$$L_{wa} = 2 \cdot \tan\left(\frac{FOV_{wa}}{2}\right) f_{wa}$$

As the wide angle FOV is much larger than the tele-objective FOV, in order to keep the smallest difference in sensor size we have to choose the minimum focal length and thus the minimum distance between the WA sensor and its lens:

$$f_{wa} = d_{min} = 2.5\, mm$$

which yields $L_{wa} = 7.1\, mm$ (any other choice of focal length would have given a larger sensor size).

The overall side of each pixel is chosen by dividing the sensor area by the pixels number.

$$L_{pix-to} = \sqrt{\frac{L_{to}^2}{N_{pix}}} = 1\, \mu m \qquad L_{pix-wa} = \sqrt{\frac{L_{wa}^2}{N_{pix}}} = 2\, \mu m$$

Note thus how the area for the wide-angle pixels is larger. We can thus already expect better performance from this sensor. A group of four pixels is characterized by four transistor gates and three shared transistors, which occupy the area indicated in the data. A single pixel, thus, has a remaining available area for the PPD:

$$A_{PPD-to} = A_{pix-to} - \frac{A_{eln}}{4} = L_{pix-to}^2 - \frac{A_{eln}}{4} = 7.02 \cdot 10^{-13}\, m^2$$

$$A_{PPD-wa} = A_{pix-wa} - \frac{A_{eln}}{4} = L_{pix-wa}^2 - \frac{A_{eln}}{4} = 3.95 \cdot 10^{-12}\, m^2$$

(ii)

The size of the Airy disk is for the two sensors:

$$d_{Airy} = 2.44\, \lambda\, F_{\#} = 2.44\, \lambda \frac{f}{D_{lens}} \qquad \rightarrow \qquad d_{Airy-to} = 4.5\, \mu m \qquad d_{Airy-wa} = 2\, \mu m$$

The tele-objective system is thus sensor limited, while the wide-angle system is well designed. We are neglecting here aberrations, which will be probably relevant in mobile applications due to the small lens aperture. Such effects can be mitigated by adopting system of lenses made by aspheric optical elements (for spherical aberrations) and forming achromatic doublets (for chromatic aberrations).

(iii)

The distance of the scene allows calculating the magnification factors of the two systems:

$$m_{to} = \frac{f_{to}}{d_{scene}} = 0.0028 \qquad m_{wa} = \frac{f_{wa}}{d_{scene}} = 0.0013$$

Each point in the scene is sending the mentioned flux towards the lenses (the solid integration angle is already accounted for in the provided photon flux per unit area). As a pixel captures photons from an area of the scene magnified through the square of the magnification factor, we can calculate the overall number of photons per second that belongs to a single pixel, and then the photocurrent, as:

$$\phi_{to} = \phi_{ph}\, A_{PPD-to}\, / m_{to}^2 \qquad \phi_{to} = \phi_{ph}\, A_{PPD-wa} / \, m_{wa}^2$$

$$i_{ph-to} = q\, \phi_{to}\, \eta = 74\, aA \qquad i_{ph-wa} = q\, \phi_{wa}\, \eta = 2\, fA$$

$$Q_{ph-to} = i_{ph-to} t_{int} = 3.7\, e^- \qquad Q_{ph-wa} = i_{ph-wa} t_{int} = 101\, e^-$$

where, for the photocurrent calculation, we have taken into account the quantum efficiency η. Note how the photocurrent (and thus the number of electrons) of the tele-objective sensor is much lower than for the wide angle one.

(iv)

The SNR is readily calculated in terms of charge as:

$$SNR = 20 \log_{10} \frac{Q_{ph}}{Q_n} = 20 \log_{10} \frac{Q_{ph}}{\sqrt{q\, t_{int}(i_{ph} + i_d) + k_b T C_{sense}}} =$$

$$= 20 \log_{10} \frac{Q_{ph}}{\sqrt{q\, (Q_{ph} + t_{int} J_d A_{PPD}) + k_b T C_{sense}}}$$

where for each of the two sensors one should take into account the corresponding value of photo-charge and pinned diode area. The calculation yields an SNR of -3 dB and +19 dB, respectively, for the tele-objective and the wide-angle cameras.

The main problem in this design is the desire of a too small angle for a tele-objective lens, which, at the state of the art, is difficult to achieve due to small spacing constraints between sensor and lens. The consequence is that the pixel size becomes small and the SNR is degraded.

Alternative options to improve the design are:

- use different number of pixels for the two sensors (reduce the pixel number for the tele-objective sensor, increasing the pixel area and thus the SNR). This poses some difficulties for post-processing operations when one needs to blend information from the two cameras, however, as these have now different number of pixels;
- use a different integration time for the two sensors: this helps in improving the SNR of the tele-objective, if its integration time can be increased. With 200 ms integration time the to sensor reaches almost the same performance as the wa sensor. This will however degrade the DR of the to sensor;
- obviously, another option is to allow a larger focal length for the to sensor… which will however make your mobile phone thicker…