Last Name _ Governos __ **Given Name** __ Enza __ **ID Number** __ 20190902 __

Question n. 1

You are designing pixels for an imaging sensor in mobile phone applications, and the first choice that you have to take is about the pixel size: (i) identify the three major sources that limit the resolution of your imaging system and (ii) draw a sample quoted graph of the spot size as a function of the F# number of the system. Afterwards, (iii) write the expression of the SNR and DR of the system as a function of the pixel area (assume a 4T topology). In conclusions, (iv) choose a reasonable pixel size and motivate your choice.

(i)

The sources limiting the spatial resolution of an imaging system are essentially related to optical aberrations, optical diffraction and the pixel size. The context of mobile applications (opposed to the high end market of digital cameras) is typically characterized by small available volumr, which translates in both small sensor size (thus a small pixel size for a given number of pixels), and short distance to the lens (i.e. short focal length), which is a source of aberrations as we will see in a while.

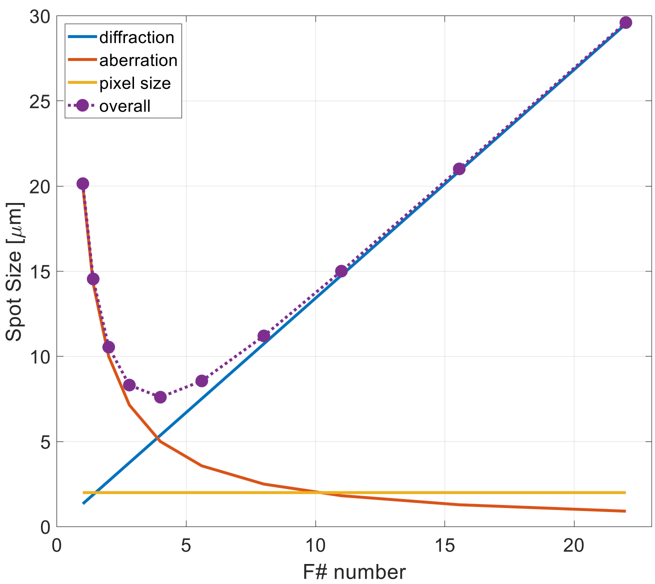
Concerning aberrations, they occur (a) due to the fact that third order terms (neglected in geometric optics) cause marginal rays to be deflected more than paraxial rays, generating a disk of lest confusion in the focal plane (for a point-like source). They also occur due to (b) differences in the refraction index at different wavelengths (chromatic aberrations). In both cases, avoiding marginal rays (i.e. closing the aperture) brings benefits in terms of aberrations (minimum spot size d_{aber}): this can be approximated by stating that the spot size goes with the inverse of the $F_{\#}$ number, $d_{aber} \propto \frac{C}{F\#}$, C being a constant value. Though using aspherical lenses and achromatic doublets bring benefits, in general aberrations will be the dominant source of resolution worsening for $F_{\#}$ lower than 4 (wide open lenses will show maximum aberrations). This is typically the case of mobile applications, where the F# is indeed low due to the short real focal length.

On the other side, diffraction arises because of the narrow aperture of the lens. For a circular aperture, the Airy spot size d_{Airy} can be quantified through the well-known formula describing the diameter of the first minimum of the diffraction pattern from a circular aperture: $d_{Airy} = 2.44\frac{\lambda}{D}f = 2.44\,\lambda\,F_{\#}$, ($D$ being the lens diameter).

Finally, the pixel size d_{pix} is itself a limit to the resolution and is independent of the $F_{\#}$ number.

(ii)

A sample graph for the spot size as a function of the $F_{\#}$ number for three effects described above is given below (a constant C of 20 µm is used for the aberrations formula. Quantitatively, for an average wavelength of 550 nm, we see that diffraction reaches a value of about 5 µm at the optimal $F_{\#}$ (in the range of 4). A sample pixel value of 2 µm is reported which seems much lower than optical effects. Let us see if there is any advantage and/or challenge in increasing its size up to values similar to the optical spot size.



(iii)

According to the discussion above, we want to verify if the two key parameters describing the sensor performance, the SNR and the DR (in particular, the maximum DR occurring at short integration times), have benefits from larger pixel size.

We first of all take into account that for a 4T topology (a) integration occurs on the floating diffusion C_{fd} capacitance, (b) the shot noise is usually low due to the pinning implant of the pinned photodiode, and (c) reset noise can be much decreased by correlated double sampling procedures.

We can thus write the following expressions, highlighting the dependence on the pixel area d_{pix}^2:

$$SNR = 20\log_{10}\frac{J_{ph}FF\, d_{pix}^2\, t_{int}}{\sqrt{q(J_{ph}FF + J_d)d_{pix}^2\, t_{int} + \frac{k_bT(C_g + C_{fd})}{RF_{kTC}} + \cdots}}$$

$$DR_{max} = 20\log_{10}\frac{Q_{max}}{Q_{min}} = 20\log_{10}\frac{V_{DD}C_{fd}}{\sqrt{\frac{k_bT(C_g + C_{fd})}{RF_{kTC}} + \cdots}}$$

(attention is focused on kTC and shot noise only). We see that the SNR benefits by a large pixel area either linearly (for short integration times) or with the square root of the area (at long integration times).

We also see that the DR is not dependent on the pixel area at short integration times: this is due to the fact that the maximum signal is integrated on the floating diffusion capacitance $C_g + C_{fd}$, and noise is also dependent on this parameter, with no dependence on the photodiode area itself.

(iv)

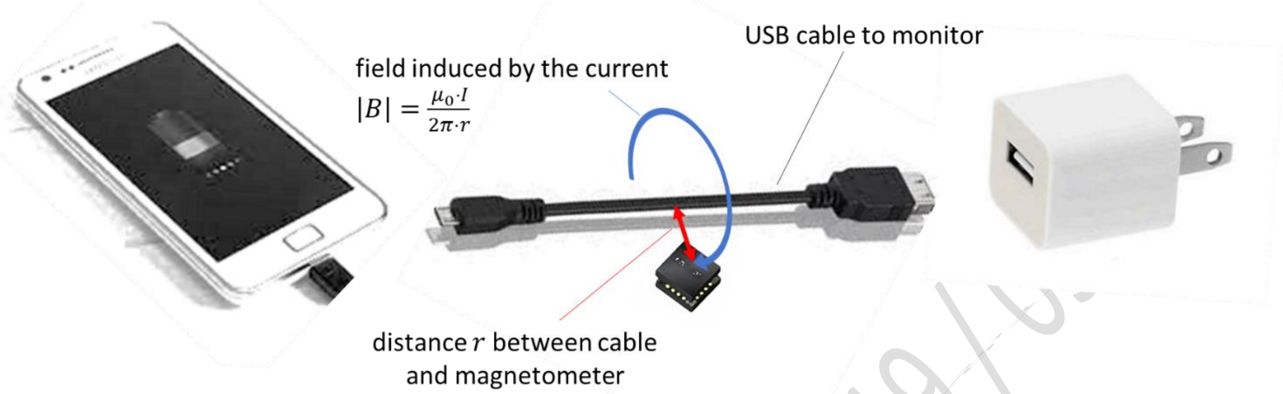
According to the discussion above, we would choose a pixel size as large as possible within the limits set by the optics, e.g. in the order of 5 µm (see again the graph at point (ii)). However, assuming a typical number of pixels that we have nowadays, a 20 Mpixel sensor (average side has 4800 pixels) would take up 2.4 cm which is too much for the available size in a mobile phone!

Therefore, we will typically choose pixel size of 1 µm to 2 µm, mainly limited by the available space (sensor side of 9.6 mm for the same number of pixels as above). Thanks to the 4T topology, the DR will not suffer so much from this pixel size reduction. The use of microlenses and of electronic sharing among different pixels will boost the SNR, allowing reasonable values even at low pixel size.

Last Name _ Governos __ **Given Name** __ Enza __ **ID Number** __ 20190902 __

Question n. 2

Consider a z-axis, multi-loop MEMS magnetometer whose parameters are reported in the table. The magnetometer is used to monitor the intensity of the current flowing through a wire out of a USB power outlet, according to the Biot-Savart law. In particular, the current to monitor through the wire is expected not to exceed a maximum value of 3 A, while the minimum fluctuations to detect are in the order of 3 mA and can occur over a 50 Hz bandwidth. The magnetometer is mounted at a distance *r* = 1 mm from the cable.



(i) evaluate the minimum magnetic field to monitor, and the required Lorentz current I_{Lor} (peak value) through the MEMS to make thermomechanical noise negligible;

(ii) assuming off resonance operation, evaluate the maximum charge amplifier noise density such that also electronic noise is made negligible for the application (note that the amplifier is biased between 0 V and 6 V, with the virtual ground voltage V_{BIAS} kept at half the dynamic);

(iii) propose and draw a scheme to complete the electronic sensing chain with an additional amplifier (find its gain) and a demodulation stage, so to fully exploit the voltage supply;

(iv) Does the sensor need an initial calibration to compensate for other magnetic fields? Propose a routine for this kind of compensation.

Parameter	Symbol	Value
Parallel-plate Gap	g	1 μm
Single loop length	L_{loop}	700 μm
Number of loops	N_{loop}	12
Elastic stiffness (½ structure)	$k_{1/2}$	250 N/m
Quality factor	Q	3240
Resonance frequency	f_0	40 kHz
Rest capacitance (½ structure)	C_0	150 fF
Max current to monitor	$I_{USB,max}$	3 A
Min current to monitor	$I_{USB,min}$	3 mA
Max current bandwidth	BW	50 Hz
Distance from MEMS to wire	r	1 mm
Amplifier supply voltage	$V_{ss} - V_{dd}$	0 V - 6 V
Virtual ground voltage	V_{BIAS}	3 V
Feedback Capacitance	C_f	15 fF
Parasitic Capacitance	C_P	3.25 pF

Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K;
$\varepsilon_0 = 8.85\ 10^{-12}$ F/m;
$\mu_0 = 4\pi\ 10^{-7}$ N/A^2;
T = 300 K;

(i)

We know that the noise equivalent magnetic field density (NEMD) of a MEMS magnetometer can be expressed by the following equation, in case of a multi-loop sensor:

$$\sqrt{S_{Bn,M}} = \frac{4}{I_{Lor} N_{loop} L_{loop}} \sqrt{k_B T b}$$

where $b_{1/2}$ is the damping coefficient, which can be obtained as $b_{1/2} = k_{1/2}/(\omega_0 Q)$.

By equating this expression (in units of T/√Hz) to the required noise density (the minimum measurable field, 600 nT, over a bandwidth of 50 Hz), we obtain the required Lorentz current as:

$$\frac{4}{I_{Lor} N_{loop} L_{loop}} \sqrt{k_B T b_{1/2}} = \frac{\mu_0 I_{USB,min}}{2\pi\, r\, \sqrt{BW}} \quad \rightarrow \quad I_{Lor} = \frac{8\pi\, r\, \sqrt{BW}\ \sqrt{k_B T b_{1/2}}}{\mu_0 I_{USB,min}\, N_{loop} L_{loop}} = 200\ \mu A$$

(note: the so calculated damping coefficient is for half the structure, as all calculations are done for half the structure. If you used an additional factor 2 or √2, you were not penalized).

(ii)

In order to refer electronic noise backward to input-referred magnetic field noise, one needs (a) to calculate the charge amplifier output noise and (b) to divide it by the sensitivity. We thus start by calculating the sensitivity, assuming a differential tuning fork, multi-loop MEMS magnetometer. The calculation includes a factor 2 for the differential sensing and an additional factor 2 for the two halves of the tuning fork structure. A factor 2 at the denominator accounts for the distributed Lorentz force. Operating off-resonance at three times the required bandwidth yields an effective gain factor $Q_{eff} = \frac{40\ kHz}{2\cdot 150\ Hz} = 133$.

$$\frac{\Delta V}{\Delta B} = \underbrace{2}_{\substack{differential\\ sensing}} \cdot \underbrace{\frac{I_{Lor} Q_{eff} N_{loop} L_{loop}}{2\, k_{\frac{1}{2}}}}_{\substack{from\ force\\ to\ displacement}} \cdot \underbrace{\frac{2C_0}{g}}_{\substack{from\ displacement\\ to\ capacitance\ (2\ halves)}} \cdot \underbrace{\frac{V_{BIAS}}{C_F}}_{\substack{from\ capacitance\\ to\ voltage}} = 53.6\ \frac{V}{T}$$

The electronic noise for a charge amplifier configuration includes the amplifier noise and the noise of the feedback resistor. However, the latter can be made negligible by choosing a suitably large resistor value (the pole will be at low frequency and the gain at 40 kHz will be dominated by the capacitive impedance). As a consequence, we write the output ($\sqrt{S_{Vn,O}}$) and input-referred ($\sqrt{S_{Bn,E}}$) electronic noise density as:

$$\sqrt{S_{Bn,E}} = \frac{\sqrt{S_{Vn,O}}}{\frac{\Delta V}{\Delta B}} = \frac{\sqrt{2\, S_{Vn,I} \left(1 + \frac{C_P + C_0}{C_F}\right)^2}}{\frac{\Delta V}{\Delta B}} = \frac{\mu_0 I_{USB,min}}{2\pi\, r\, \sqrt{BW}}$$

(the factor 2 accounts for the two amplifiers) from which we can derive the required amplifier noise:

$$\sqrt{S_{Vn,I}} = \frac{\mu_0 I_{USB,min}}{2\pi\, r\, \sqrt{BW}} \frac{\frac{\Delta V}{\Delta B}}{\sqrt{2}\left(1 + \frac{C_P + C_0}{C_F}\right)} = 14 \frac{nV}{\sqrt{Hz}}$$

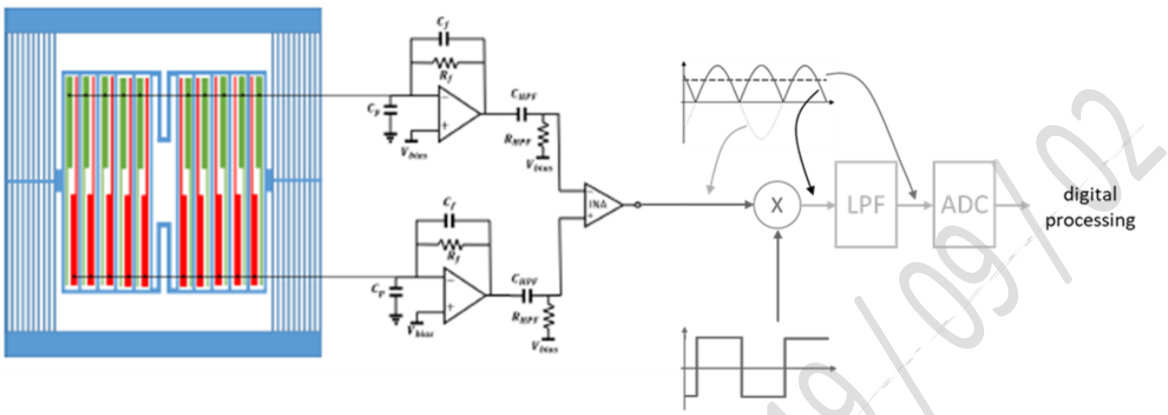
(iii)

The stage that completes the electronic gain chain needs to bring the differential signal into a single-ended one, and thus will be likely an instrumentation amplifier (INA) configuration. The INA optimum gain will bring the charge amplifier output in presence of the maximum field to measure (the FSR) to the supply voltage (we assume no other external fields for this calculation, see the next point for further details).

In this case, the maximum field to measure corresponds to 600 µT and the corresponding differential output voltage at the charge amplifiers is (using the sensitivity calculated above) 32.2 mV. This is an amplitude of an AC signal centered on the virtual ground of the amplifiers (3 V), and the available AC voltage dynamic range is also 3 V (an AC signal with peak to peak values of 6 V). The required gain is thus:

$$G_{INA} = \frac{3V}{32.3\ mV} = 93$$

The chain can be completed, as shown in the drawing, by a synchronous demodulator, whose reference AC frequency is provided by a signal coming from the oscillator that generates the Lorentz current. An ADC will follow, to digitize the signal.



(note: a passive AC coupling is placed at the charge amplifiers output to avoid their offset difference being amplified by the INA).

(iv)

For sure there may be other concurring and disturbing magnetic fields. This can come from other electronic tools or – even if the system is far from other disturbing means – from the unavoidable Earth field.

Having values between 10 and 100 µT, the Earth field is exactly in the range that we want to measure. Its intensity along the sensing axis changes as a function of the position on Earth and of the orientation of the measuring system with respect to the Earth field direction. There are at least to ways to compensate the field:

- the first one consists in measuring the field when no current is flowing through the USB cable, to store this value as a digital number in a memory, and then to subtract it at the end of the measurement. The advantage of this technique is that it is easy and adds almost no extra power consumption. The drawback is that it should be implemented every time the orientation of the cable is changed, and there is a need to switch off the USB current to perform it correctly;
- an alternative strategy could be the use of a pair of magnetometers aligned on the same plane and positioned symmetrically around the wire. In this case, they will sense an opposite field induced by the USB current, but the same field generated by Earth. By taking the difference of their output, the Earth field will be cancelled. The advantage of this technique is that it does not require any calibration, the USB current can be always left on, and the measurement is independent of the orientation of the USB wire with respect to the Earth field. The disadvantage lies in a more complex system, with twice the volume and twice the consumption.

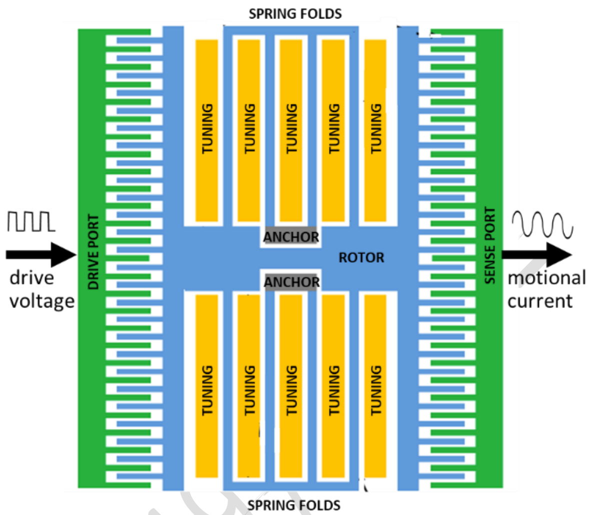
Note that the Earth field, whatever the compensation technique, should be accounted for in the calculation of the INA gain to avoid saturation. The INA gain, thus, should be probably decreased a little.

Last Name _ Governos __ **Given Name** __ Enza __ **ID Number** __ 20190902 __

Question n. 3

You are required to design a MEMS oscillator like the one depicted in the figure, according to the specifications given in the Table and in the following questions.

Parameter	**Symbol**	**Value**
Fold Length	L	100 µm
Fold Width	W	2 µm
Gap	g	1 µm
Process Height	h	25 µm
Mass	m	0.73 nkg
Rotor Voltage	V_{ROT}	15 V
Tuning Voltage	ΔV_t	0 V
Feedback Capacitance	C_f	200 fF
Drive Amplitude	V_d	1 V
Temperature Range	-	-40 °C / +85 °C
Room Temperature	T_0	27 °C
Quality Factor at T_0	Q	500
Number of tuning electrodes	N_t	8



(i) find the resonant frequency of the structure;

(ii) find the motional resistance that gives you a sinusoidal signal at the charge amplifier output that is at least 1 V across the whole temperature range. Then, find the minimum number of comb fingers that satisfy the condition;

(iii) considering a standard deviation for the etching resolution σ_x = 25 nm and a standard deviation for the process height σ_h = 1 µm, find the new target resonant frequency so that you can trim each device to have the nominal f_0 (as calculated at point (i) above), exploiting the N_t tuning electrodes. What is the maximum tuning voltage V_t you need to apply to compensate ±3σ?

(iv) How does the answer to point (iii) change if you have a very large spread on the etching σ_x = 150 nm? Is this design able to cope with such poor matching?

Physical Constants

ε_0 = 8.85 10^{-12} F/m
k_B = 1.38 10^{-23} J/K
q = 1.6 10^{-19} C
T = 300 K
E = 179 GPa

(i)

The moving shuttle of the structure is connected to the anchor by 4 springs. Each of these springs is made by 2 folds in series. Therefore, the we can compute the mechanical stiffness k_m as:

$$k_m = Eh\left(\frac{W}{L}\right)^3 \cdot \frac{4}{2} = 71.60\,\frac{\mathrm{N}}{\mathrm{m}}$$

Then, we can find the resonant frequency f_0:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_m}{m}} = 50\ \mathrm{kHz}$$

(ii)

The voltage at the charge amplifier output is in our case:

$$V_{out,CA} = \frac{4}{\pi} V_d \cdot \frac{1}{\omega_0 C_f R_m}$$

Where the number of comb-fingers is inside the R_m term.

We can then extract directly what is the maximum R_m that we have to guarantee:

$$R_{m,max} = \frac{4}{\pi} \frac{V_d}{V_{out,CA}} \cdot \frac{1}{\omega_0 C_f} = 20.26\ \mathrm{M\Omega}$$

We have a fixed driving amplitude and we have to guarantee a minimum voltage amplitude at the charge amplifier output. Therefore, the worst-case condition to analyze is when the R_m is the largest, i.e. at the maximum temperature $T_{max} = 85$ °C of the range we are told to consider.

To begin, we can find the value of the quality factor at T_{max} as:

$$Q(T) = \frac{const}{\sqrt{T}} \Rightarrow Q(T_0)\sqrt{T_0} = Q(T_{max})\sqrt{T_{max}} \Rightarrow Q(T_{max}) = Q(T_0)\sqrt{\frac{T_0}{T_{max}}} = 457$$

Now we can express the R_m at T_{max} as:

$$R(T_{max}) = \frac{b_{max}}{\eta^2} = \frac{\omega_0 m}{Q(T_{max})} \cdot \frac{1}{\eta^2}$$

From this expression we can find the required value for η, and then find the minimum number of comb finger N_{CF}:

$$\eta = \sqrt{\frac{\omega_0 m}{Q(T_{max}) R_{m,max}}} = 156 \times 10^{-9} \frac{\mathrm{A}}{\mathrm{m/s}}$$

Since the transduction factor for a single comb-finger η_{1cf} is:

$$\eta_{1cf} = \frac{2V_{ROT}\epsilon_0 h}{g} = 6.63 \times 10^{-9} \frac{\mathrm{A}}{\mathrm{m/s}}$$

the required number of comb-finger N_{CF} for each port is:

$$N_{CF} = \frac{\eta}{\eta_{1cf}} = 24$$

(iii)

Firstly, we need to evaluate the variation of f_0 caused by imperfections in the etching. Since f_0 is proportional to the square root of k_m, which is in turn proportional to the third power of the width of the springs W, we can write:

$$\frac{df}{f} = \frac{1}{2}\frac{dk}{k} = \frac{3}{2}\frac{dW}{W}$$

A spring is etched by two sides, therefore the standard deviation of its width $\sigma_W = 2\sigma_x$. Notice that f_0 has no dependence on the process height h, therefore we can neglect the σ_h datum.

In the end, we can finally write:

$$\frac{df}{f} = \frac{3}{2}\frac{dW}{W} = \frac{3}{2}\frac{2\sigma_x}{W} = 0.0375$$

If we define the maximum variation of frequency Δf as the $\pm 3\sigma$ boundary:

$$\Delta f = f_0 \cdot \frac{df}{f} = 11.25 \text{ kHz}$$

Then we can find a good target frequency $\widetilde{f}_0$ as:

$$\widetilde{f}_0 = f_0 + \Delta f = 61.25 \text{ kHz}$$

This is due to the fact that we can only down-tune the frequency of the resonator. Therefore, a -3σ device is already at the desired 50 kHz f_0, whereas a $+3\sigma$ device will be down-tuned by the maximum 11.25 kHz.

As for the maximum tuning voltage to be applied, we can find the maximum electrostatic stiffness Δk to be added to the mechanical stiffness, and from there compute the maximum tuning voltage to be applied.

Since $\frac{df}{f} = \frac{1}{2}\frac{dk}{k}$, we can find immediately Δk as:

$$\frac{\Delta f}{f_0} = \frac{1}{2}\frac{\Delta k}{k_m} \Rightarrow \Delta k = 2\frac{\Delta f}{f_0} k_m = 32.22 \text{ N/m}$$

From the electrostatic stiffness formula and considering that we have $N_t = 8$ tuning electrodes whose length is about L (the length of the fold of the springs), we can find:

$$k_{el,max} = 2\,\Delta V_{t,max}^2 \frac{C_t N_t}{g^2} \Rightarrow \Delta V_{t,max} = \sqrt{\frac{k_{el,max} g^3}{2\epsilon h L N_t}} = 9.54 \text{ V}$$

To be more precise, we could consider $h = 17\ \mu\text{m}\ (-3\sigma_h)$ in the last equation, but the difference is relatively small.

(iv)

The far worse etching spread will cause the f_0 to spread much more, as well as all the drive/sense gaps of the comb-finger ports and the tuning gaps of the parallel-plate.

Even in the worst-case of the largest f_0, the system is able, by applying a sufficient tuning voltage, to compensate for the spread. If we do the computation of Q3 with the new numbers in this worst case:

$$\Delta f = 67.5 \text{ kHz}$$

$$g = 1\ \mu\text{m} - 3\sigma_x = 0.55\ \mu\text{m}$$

$$\Delta V_{t,max} = 20.79 \text{ V}$$

In principle, this is feasible, even though with increasing tuning voltages the compensated frequency becomes more and more sensitive to the tuning voltage: as a result it becomes more difficult to match precisely the f_0 of 50 kHz.

In the case where we end up with narrower springs and larger gaps, the R_m increases. In the case of comb-finger actuation, we know that $R_m \propto g^2$. Then:

$$\frac{dR_m}{R_m} = 2\frac{dg}{g} = 2\frac{0.45\ \mu\text{m}}{1\ \mu\text{m}} = 90\%$$

An increase in R_m lowers the loop gain of the whole oscillator, but with a square-wave driving we already have a very large loop gain at startup, that will be compressed to 1 when the output of the MEMS driver will clamp to the supply voltages.

In the end, even though some performance worsening will arise, the system is actually able to cope with this situation.