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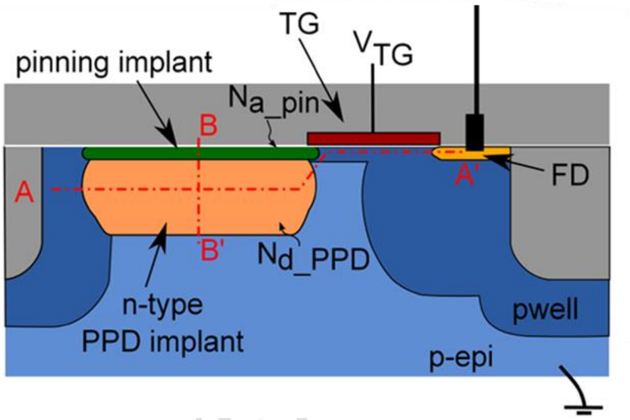
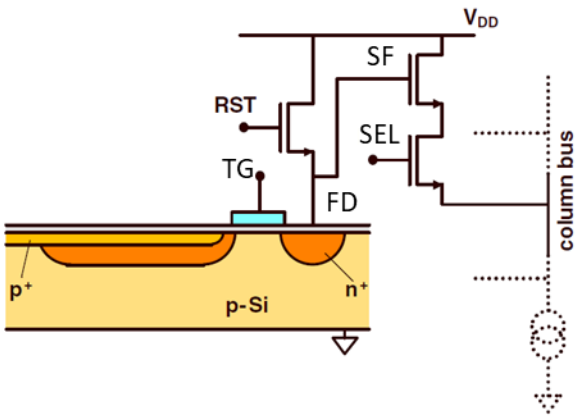
Question n. 1

Describe the technology and the topology of a 4-T active pixel sensor. Assuming a fixed pixel area, discuss advantages and drawbacks of using a 4-T topology with respect to a 3-T topology in terms of temporal noise and spatial noise. Draw a qualitative graph (but properly indicating the slopes in the different regions) of the SNR versus the photocurrent for the two sensor types compared above.



4T active pixel sensors represent the state of the art for imaging applications, due to the large dynamic range that can be achieved within relatively small changes over the simplest 3T topology. This is achieved by exploiting the pinned photodiode structure, shown in the figure. This is formed by an N-type diffusion formed over a lowly doped P-type epitaxial layer (e.g. typically 10^{15} cm^{-3}). The well is not in contact with the surface, but it is separated ("pinned") from it by a very shallow (e.g. < 50-nm thick), heavily doped (e.g. > 10^{19} cm^{-3}) P-type layer. The N-type well has no ohmic contact to a dedicated electrode, but it can be connected to an N-type implant (called floating diffusion) if the gate of a channel connecting the N-well to the floating diffusion is activated. This gives rise to the transfer from the pinned photodiode to the floating diffusion. Apart from this, the readout operation can be performed with three additional transistors (reset, source follower and readout), just like in a 3T topology, as shown aside. Details of operation are:



- RESET: asssume that the pinned PD is fully depleted by the former readout operation. With V_{TG} = 0 V, the sense (floating diffusion) node is reset to V_{DD} by closing the reset transistor (RST). The floating diffusion node FD is thus charged at $v_{FD} = V_{DD}$;
- INTEGRATION: the reset transistor is then opened, but charge is accumulated only in the pinned N_{d_PPD} region. The sense node voltage does not change.
- TRANSFER: the TG is closed: (V_{TG} risen): charge is wholly transferred to the floating diffusion (which empties the N_{d_PPD} region), causing a sudden change in the sense node (due to the current generated by photoelectrons i_{ph} and dark electrons i_d over the integration time t_{int}):

$$v_{FD} = V_{DD} - \frac{(i_{ph} + i_d)t_{int}}{C_S}$$

- READOUT: the selection transistor (SEL) closes, activating the source follower (SF) for the readout.

The key point of this structure is to decouple the functionality of electrons collection (which occurs in the pinned photodiode) from the conversion gain (which is given by the floating diffusion contact). Apart from giving advantages from the point of view of linearity and conversion gain, which are not the focus of the question, there are advantages in terms of noise for the 4T topology.

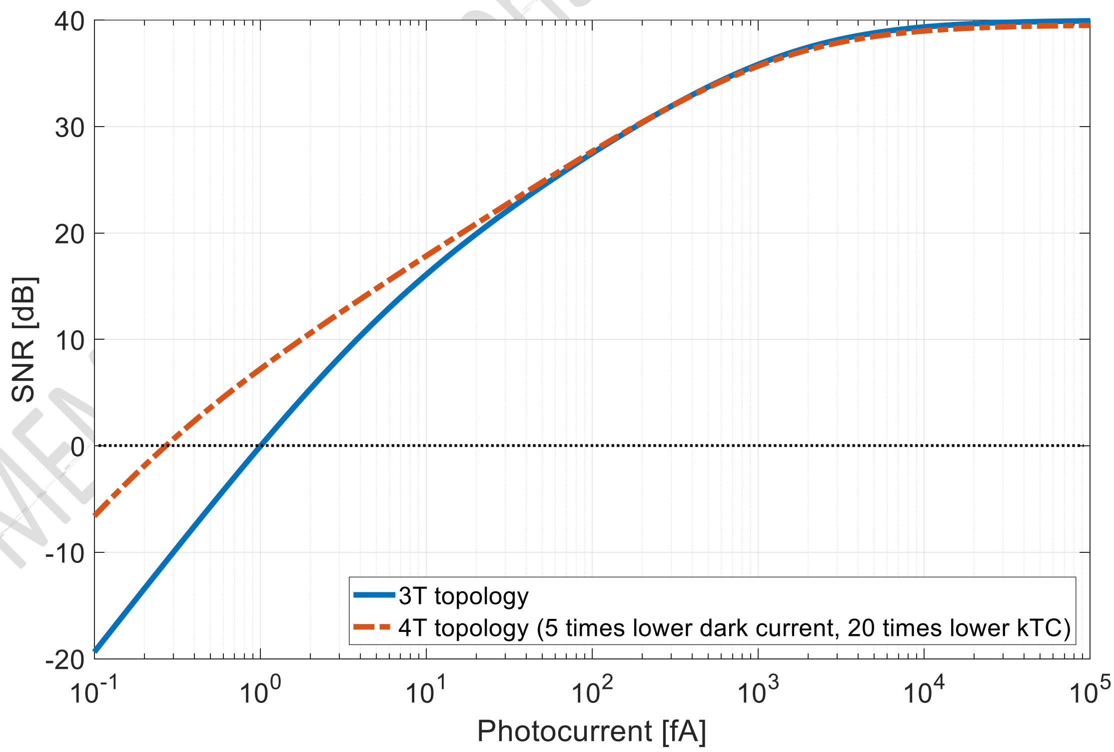
Concerning temporal noise, there are two main reasons why the 4T topology is better than the 3T one. In terms of dark current shot noise, the pinning implant has the effect of lowering the lifetime of electrons generated close to the surface. As this area is typically full of defects (interface between Si and SiO2, so interruption of Silicon crystal regularity, and high doping in addition), the dark current generation in this region is usually very large, and dominant over the bulk contribution from a regularly grown epitaxial layer with few defects. In the 4T topology, thanks to the pinning layer, this dark current contribution is not captured, with an overall reduction of the dark current which can be as large as a factor 10 with respect to a 3T topology of the same photodiode area.

In terms of reset noise, the possibility to operate a correlated double sampling (CDS), i.e. sampling of offset and noise during light integration and then subtraction of this value to the signal captured during the readout (which includes again offset and noise, but also the signal) reduces kTC noise by a relevant factor (ideally infinite, apart from nonidealities in the two capacitances which are used to store the two sampled values). A rejection factor of kTC noise up to 50 can be obtained with respect to a 3T topology.

Finally, what about spatial (FPN) noise? Terms related to the pixel electronic offset in the DSNU will be compensated, again thanks to CDS. The DSNU will be similar to a 3T topology in %, but as the dark current decreases, its absolute value will decrease too.

In terms of PRNU instead, no advantages will be obtained and some disadvantages may be given due to the additional capacitance used to store the information for the CDS operation.

In summary, the SNR of the 4T topology will be better than the 3T one in the region where signal-independent noise is dominant (i.e. at low integration times). It will be equal – for the same integration time and pixel area – where photon noise (i.e. the physics of the Poisson process) dominates. It will be very similar (maybe a little worse) where PRNU dominates. Overall, the DR will be increased because of the improvements at the low end (i.e. at low signal values). A sample comparative graph for the two topologies is shown below (note: pixel saturation is not shown to best highlight the effect of PRNU, but assuming the same pixel area, it will be similar for the two pixel topologies). Note how the 4T topology crosses the 0-dB value at signals lower than the 3T, thus guaranteeing a larger DR.



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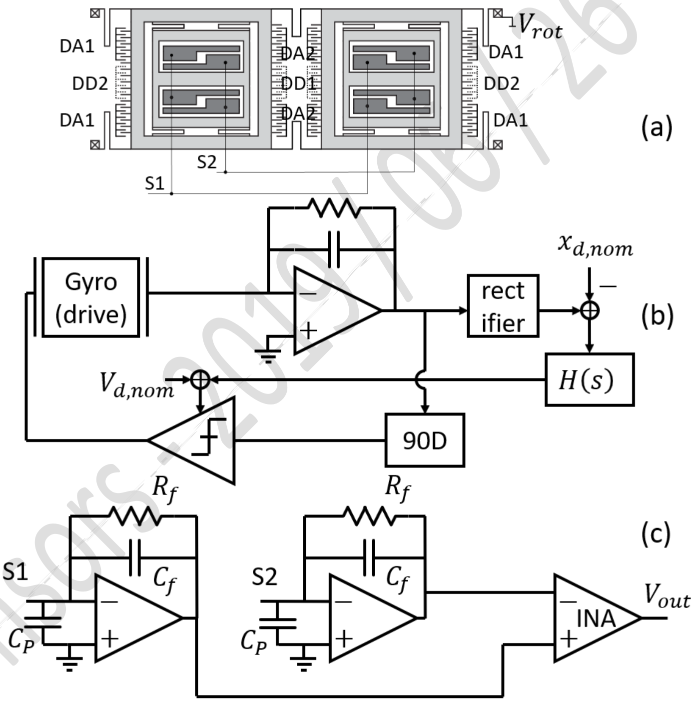
Question n. 2

You have been given the task to finalize the design of a z-axis MEMS gyroscope. The system is composed by a dual-mass tuning fork structure (Fig. a), where the drive loop is actuated in a push-pull configuration by two square-waves in anti-phase (±Vd) at the two drive actuation ports (DA1 and DA2), and the motional current is read-out differentially at the two drive-detection ports (DD1 and DD2). The differential drive loop, together with the AGC secondary loop, is sketched with its single-ended equivalent for the sake of clarity in Fig. b. A capacitive front-end then reads the signal along the sense axis at the S1 and S2 ports (Fig. c).

Data are given for half of the structure. You are asked to:

Parameter	Symbol	Value
Gap (PP and CF)	g	1.5 µm
Process height	h	25 µm
Split Frequency	Δf	500 Hz
Drive Frequency (anti-phase mode)	f_d	25 kHz
Sense mass	m_s	8 nkg
Drive Frame mass	m_d	2 nkg
Drive Quality Factor	Q_d	10 000
Sense Quality Factor	Q_s	600
Full-Scale Range	FSR	2500 dps
Quadrature	B_q	0 dps
Nominal drive amplitude	$V_{d,nom}$	1 V
Rotor Voltage	V_{rot}	10 V
Sense Rest Capacitance (single-ended, total)	C_s	200 fF
Feedback Capacitance	C_f	400 fF
Parasitic Capacitance	C_P	2 pF



(i) find the number of comb fingers N_{CF} required for each of the drive actuation ports, to obtain a sensitivity $S_y = 55.5$ pm/dps (be careful: the actuation voltages are two square waves between $\pm V_{d,nom}$);

(ii) what is the maximum linearity error ϵ_{lin} of this sensor?

(iii) find the maximum voltage noise density of the operational amplifiers in the sense chain to make the electronic noise equal to the intrinsic noise of the sensor. Neglect other noise sources;

(iv) what is the relative change in sensitivity $\Delta S_y / S_y$ in case of a reduction of both Q_d and Q_s by 20% (e.g. due to a temperature increase)? Consider the filtering block within the AGC loop with a transfer function $H(s) = \frac{k_{AGC}}{1+\frac{s}{\omega_p}}$, where $k_{AGC} = 10$ and assume that the drive mode impedance at resonance is modeled by a 10 $M\Omega$ motional resistance;

(v) how does the answer to question (iv) change if $H(s) = \frac{k'_{AGC}}{s}$ (where $k'_{AGC} = 1$, if needed)?

Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K;

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m;

T = 300 K;

(i) The required number of comb finger at the drive actuation ports is linked to the sensitivity through the drive displacement x_d.

Let us begin by computing the required displacement x_d for the target sensitivity S_y:

$$S_y = \frac{x_d}{\Delta\omega} \Rightarrow x_d = S_y \cdot \Delta\omega = 55.6\frac{\text{pm}}{\text{dps}} \cdot \frac{180}{\pi} \cdot 2\pi \cdot 500\text{Hz} = 10\ \mu\text{m}.$$

Considering that the anti-phase drive mode stiffness $k_d = \omega_d^2(m_s + m_d) = 246.7\frac{N}{m}$, the number of comb-finger N_{CF} that generates the target displacement with the nominal actuation voltage $V_{d,nom}$ is:

$$x_d = 2\frac{4}{\pi}V_{d,nom}\left(\frac{2V_{rot}N_{CF}\epsilon_0 h}{g}\right)\frac{Q_d}{k_d} \Rightarrow N_{CF} = \frac{x_d g\pi}{16V_{d,nom}V_{rot}\epsilon_0 hQ_d} = 33.$$

(ii) As illustrated in subfigure c), the system employs a differential capacitive readout. The linearity error ϵ_{lin} is then given by:

$$\epsilon_{lin} = \left(\frac{\Delta y_{max}}{g}\right)^2 = \left(\frac{S_y \cdot FSR}{g}\right)^2 = 0.86\%.$$

(iii) The intrinsic noise of the sensor is given by the $4k_BTb_s$ force white noise power spectral density. Once this is divided by the rate-to-force sensitivity ($F/\Omega = 2m_s\omega_d x_d$), we obtain the noise equivalent rate density:

$$NERD_m = \sqrt{2 \cdot \frac{4k_BT}{4\,Q_s m_s \omega_d x_d^2}}\frac{180}{\pi} = 590\ \frac{\mu\text{dps}}{\sqrt{\text{Hz}}}.$$

To find the equivalent voltage noise of the operational amplifiers $S_{n,OA}$ required to have a total noise equally split between sensor and front-end, we must compare these two values at the same node of the chain (e.g. in terms Ω).

To do so, we need to compute the electronic noise at the output of the charge amplifier, and then divide it by the voltage sensitivity of the sensor. In formulas:

$$NERD_e = \sqrt{2} \cdot S_{n,OA}\left(1 + \frac{C_P + C_s}{C_f}\right)\frac{C_f}{V_{rot}}\frac{1}{\frac{2C_s}{g}}\frac{1}{S_y} = NERD_m,$$

Reversing the formula, we arrive to the final result:

$$S_{n,OA} = \frac{2 \cdot NERD_m V_{rot} C_s S_y}{\sqrt{2} \cdot \left(1 + \frac{C_P + C_s}{C_f}\right)C_f g} = 23.8\frac{\text{nV}}{\sqrt{\text{Hz}}}.$$

(iv) A change in Q_s has actually (almost) no effect, since the mode-split operation rejects very well any reasonable change in the quality factor of the sense mode.

As for the drive quality factor Q_d, if no AGC loop were present, the relative change of Q_d would cause a corresponding relative change in the drive displacement amplitude x_d and therefore in the sensitivity.

Actually, the role of the AGC loop is indeed to sense this perturbation on the displacement amplitude and act on the driving amplitude to compensate for it.

The 20% open loop error on the sensitivity is therefore reduced by a factor of $1 + G_{loop,agc}$. To compute it, we need to properly consider the transfer between the amplitudes of the signals involved in this AGC loop, therefore:

$$G_{loop,agc} = \frac{4/\pi}{R_m} \cdot \frac{1}{\omega_d C_f} \cdot \frac{2}{\pi} \cdot k_{AGC} = 12.9.$$

The factors $4/\pi$ and $2/\pi$ are given by the square wave driving and by the average value of the rectified sine wave. As the AGC loop operates on the amplitude of signals at resonance, the equivalent RLC of the drive mode is just simplified to R_m.

So, the relative change on the sensitivity is much smaller than the open-loop one and is equal to:

$$\frac{\Delta S_y}{S_y} = \frac{\Delta x_d}{x_d} = \frac{\Delta Q_d}{Q_d} \cdot \frac{1}{1 + G_{loop,agc}} = 1.44\%.$$

Which is an acceptable scale factor change across the temperature range for typical consumer applications.

(v) In case an integral gain is used within the AGC loop, the loop gain is given by the following expression:

$$G_{loop,agc} = \frac{4/\pi}{R_m} \cdot \frac{1}{\omega_d C_f} \cdot \frac{2}{\pi} \cdot \frac{k_{AGC}^{'}}{s}.$$

This integral gain causes the loop gain in DC (at zero frequency) to be infinite, therefore the Q_d relative change is completely compensated (assuming that temperature changes are very very slow, so pseudo-DC).

In fact, an integral control loop can react on the required control signal (the voltage driving amplitude) even when the sensed error is zero (unlike the proportional control loop used in question iv).

The relative change in Qs is rejected by the mode-split operation just like in the previous case. Therefore:

$$\frac{\Delta S_y}{S_y} \approx 0.$$

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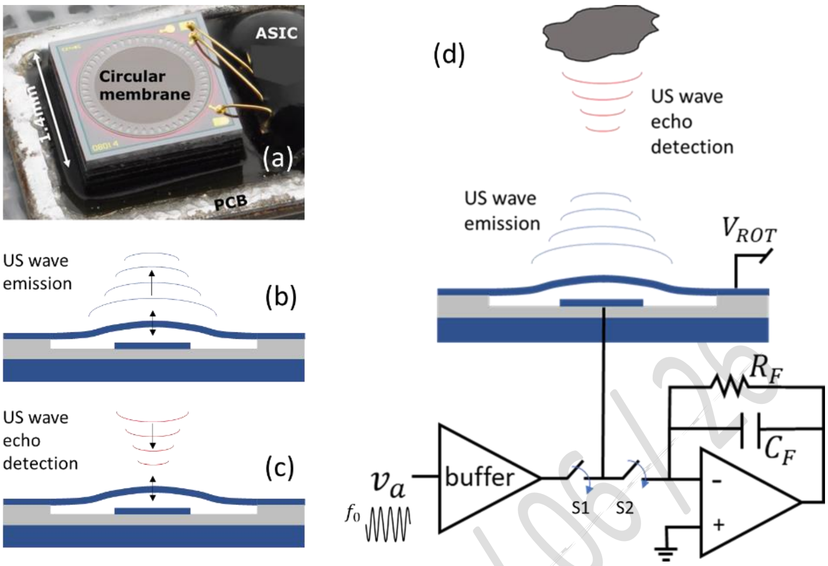
Question n. 3

Consider a circular suspended MEMS membrane (Fig. 1a), used as an ultrasonic transducer for obstacle recognition in autonomous driving applications. Such a membrane can emit an ultrasound wave when actuated by a proper electrostatic force at its resonance frequency in the ultrasonic range (see the cross-section in Fig. 1b). Dually, the membrane can be excited by an incoming ultrasonic wave (Fig. 1c), and its deflection can be sensed using a capacitive sensing interface.



To use it as a transducer, the membrane can be connected to the circuit shown in Fig. 1d, where the switch S1 is closed during wave emission, and the switch S2 is closed during wave detection. The same capacitive electrode, designed underneath the membrane, can be used thus as either the actuation or detection port in two consecutive intervals.

The presence of an object and its distance can be detected by measuring the echo wave return time. To operate properly, the echo wave should not return back to the membrane before the emission period is concluded. Some of the system parameters are reported in Table 1. You are asked to:

Parameter	**Value**
resonance frequency f_0	150 kHz
membrane (and electrode) diameter d	1 mm
mass m	18 nkg
gap g	1 µm
actuation voltage v_a	200 mV
rotor voltage V_{ROT}	5 V
feedback capacitance C_F	2 pF
parasitic capacitance C_P	50 pF
amplifier voltage noise density S_{Vn}	(5 nV/√Hz)^2
electronic filtering bandwidth BW_{eln}	5 kHz
ultrasonic wave speed v_{us}	343 m/s
ultrasonic wave attenuation β_{us}	6 dB/m
acceptable linearity error $e_{\%}$	5%

(i) evaluate the lumped-model stiffness of the membrane, the maximum displacement and the maximum Q factor to cope with a 5% capacitive linearity error during resonant actuation mode (you can assume the membrane motion as a parallel-plate rigid translation, for the sake of simplicity);

(ii) the emission operation consists in closing the switch S1 and applying a voltage sinewave at resonance (v_a) as depicted in Fig. 1d: assuming to let the membrane motion grow up to 63% of the regime value, evaluate the *minimum distance* at which an object can be correctly detected;

(iii) calculate the thermomechanical and electronic noise contributions (neglect the feedback resistor) in terms of membrane displacement [m_{rms}], and identify the minimum overall displacement of the membrane that can be correctly measured by the sensing interface;

(iv) given that the attenuation coefficient of an ultrasonic wave in air is β_{us} = 6 dB/m (intensity halves every traveled meter), and assuming no energy losses during wave reflection, estimate the *maximum distance* that can be measured by the ultrasonic sensor, and its dynamic range.

Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$k_B = 1.38\ 10^{-23}$ J/K

T = 300 K

(i) The lumped model stiffness can be immediately evaluated as the mass and the resonance frequency of the membrane are known. This stiffness is intended as the ratio between the force and the average membrane motion:

$$k = \omega_0^2 \cdot m = (2\pi \cdot f_0\,) \cdot m = 15989\ N/m$$

The linearity error of a single-ended parallel plate configuration like the one discussed in this example can be evaluated as:

$$\epsilon_{\%} = \frac{\Delta C_{real} - \Delta C_{lin}}{\Delta C_{real}} = \frac{\frac{\epsilon_0 A}{g}\left(\frac{g}{g-x} - 1\right) - \frac{\epsilon_0 A}{g^2} x}{\frac{\epsilon_0 A}{g}\left(\frac{g}{g-x} - 1\right)} = \frac{\frac{x}{g-x} - \frac{x}{g}}{\frac{x}{g-x}} = 1 - \frac{\frac{x}{g}}{\frac{x}{g-x}} = 1 - \frac{g-x}{g} = \frac{x}{g}$$

By setting this value to 5%, we find a maximum displacement of:

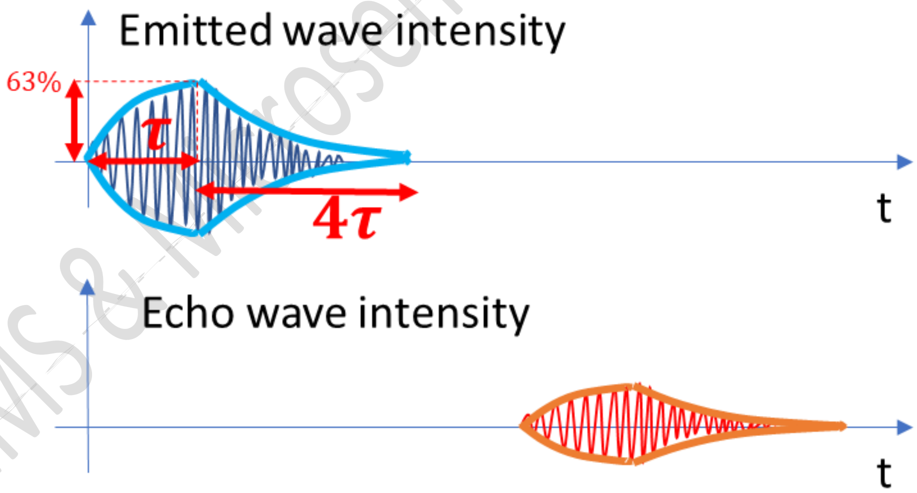
$$\frac{x}{g} = 0.05 \ \rightarrow \ x_{max} = 0.05 \cdot 1\ \mu m = 50\ nm$$

Note that we have used the case where the membrane approaches the electrode, which is obviously the worst one. Once the maximum displacement is found, one can calculate the maximum Q factor to avoid a displacement larger than x_{max} under the driving conditions described in the figure.

$$F_{elec} \approx \frac{C_0}{g} V_{rot} v_a \ \rightarrow \ x_{max} = \frac{F_{elec}\, Q_{max}}{k} \rightarrow \ Q_{max} = \frac{x_{max} k}{F_{elec}} = 115$$

(ii) A growth of the 63% in an exponential rise is obtained just after one time constant τ. However, right after the drive circuit is disconnected the membrane will not go immediately to its rest position, but will decay with the same exponential constant.

We can thus represent the situation of the emitted wave burst (and the membrane motion during emission) as below.



After letting the wave rise to 63% of its maximum value, to let the burst wave decrease significantly we have to wait 4-5 additional time constants. The total time duration of the emission can be thus assumed as 5 (or 6) time constants. As the time constant is given by:

$$\tau = \frac{Q}{\pi \cdot f_0} = 244\ \mu s$$

The system should not receive the echo signal before $t_{tot} = 1.2\ ms$, otherwise the membrane motion caused by the echo wave will be overlapping with the motion still caused by the drive excitation, giving a wrong overall signal. We thus find immediately the minimum distance of object recognition, as the velocity of sound (and ultrasound) is known:

$$2 \cdot d_{min} = v_{us} \cdot t_{tot} \;\rightarrow\; d_{min} = 21\ cm$$

(iii) The thermomechanical noise calculation can be done like for other MEMS sensors, the noise power spectral density $S_{n,F}$ in terms of force being:

$$\sqrt{S_{n,F}} = \sqrt{4k_B T b}$$

where *b* is the damping coefficient, which can be directly evaluated as the resonance frequency, mass and quality factor of the membrane are known:

$$b = \frac{\omega_0 m}{Q} = 1.4 \cdot 10^{-4}\,\frac{kg}{s}$$

As the membrane is operating at resonance, the displacement induced by the force will be amplified by the quality factor, approximately across the full width at half maximum (FWHM). The bandwidth (half of the FWHM) and the overall rms noise in terms of displacement are thus calculated as:

$$BW = \frac{f_0}{2Q} = 4\ kHz \;\rightarrow\; \sigma_{x,tm} = \sqrt{S_{n,F}}\,\frac{Q}{k} \cdot \sqrt{BW} = 720\ fm_{rms}$$

The electronic noise expression is given below, together with the transfer function from displacement to output voltage of the charge amplifier. We can thus back refer electronic noise in terms of displacement, so to compare it with thermomechanical noise and to find out overall noise in terms of displacement:

$$S_{v,out,CA} = S_{v,in,CA}\left(1 + \frac{(C_P + C_0)}{C_F}\right)^2$$

$$\frac{\delta v_{out,ca}}{\delta x} = \frac{V_{rot}}{C_F}\frac{C_0}{g}$$

$$\sigma_{x,eln} = \frac{\sqrt{S_{v,in,CA}}\left(1 + \frac{(C_P + C_0)}{C_F}\right)}{\frac{V_{rot}}{C_F}\frac{C_0}{g}}\sqrt{BW_{eln}} \; = 600\ fm_{rms}$$

$$x_{min} = \sigma_{x_{tot}} = \sqrt{\sigma_{x,eln}^2 + \sigma_{x,tm}^2} = 940\ fm_{rms}$$

(iv) The maximum distance d_{max} is the one such that the reflected wave comes back with an intensity equal to the minimum measurable signal. As we have calculated this signal in terms of membrane displacement, we can use this value for the calculation. Assuming that I_{max} is the maximum intensity of acoustic pressure emitted by the membrane (i.e. the one corresponding to the maximum membrane displacement), we can write:

$$I_{min} = I_{max} \cdot 10^{-\frac{6\frac{dB}{m}\cdot 2d_{max}}{20}}$$

As a consequence of the proportionality between membrane displacement and emitted pressure:

$$d_{max} = -\frac{1}{2}\frac{20}{6}\log_{10}\frac{I_{min}}{I_{max}} = \frac{1}{2}\frac{20}{6}\log_{10}\frac{I_{max}}{I_{min}} = \frac{1}{2}\frac{20}{6}\log_{10}\frac{0.63\ x_{max}}{x_{min}} = 7.5\ m$$

The corresponding dynamic range of the ultrasonic sensor, i.e. the ratio between the maximum measurable distance and the minimum measurable distance, is thus finally found as:

$$DR = 20 \cdot \log_{10} \frac{d_{max}}{d_{min}} = 31.13\ dB$$