**Question n. 1**

You collaborate with a company that fabricates magnetic sensors based on a non-MEMS technology. Their products have the specifications given in the Table. You have to convince them that it is worth going towards a MEMS-based magnetometer.

Parameter	Value
V_{DD}	3 V
Magnetic Field Range	±1.3 mT
Supply current (3 axes)	800 µA
Rms noise	0.3 µT
Bandwidth	50 Hz
Linearity error	1%$_{FSR}$
Max Mechanical shock	10000 g
Temperature range	-40°C +85°C

Start with an overlook of acceleration-rejecting structures, then comment on the bandwidth requirements, then on noise constraints and on architectures that improve noise density. Finally comment on consumption and achievable FSR. Use as much as possible formulas, numerical examples and sketches to assist your discussion and be convincing.

MEMS magnetometers based on the Lorentz force principle could be an attractive solution to improve the performance of existing magnetic sensors based on magnetic materials. In their differential tuning-fork-based configuration, like those shown in the figures for in-plane and out-of-plane sensing, they can reject accelerations (which are seen as a common mode) as the sign of the Lorentz current will be opposite on the two branches of the sensor. There are, however, several considerations to draw before concluding that MEMS magnetometer overcome the performance in the table.
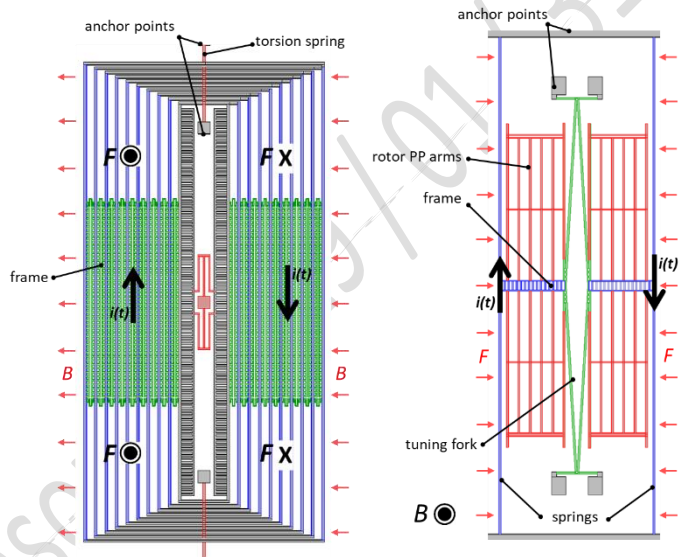
First, to guarantee a sufficiently large bandwidth at low noise density, one should exploit off-resonance operation, i.e. the current injected into the sensor should be at a frequency f_d which has an offset Δf from the sensor resonance f_0 of about three times the desired bandwidth. In this case, this value can be in the order of 150 Hz, with f_0 typically larger than 20 kHz to avoid acoustic disturbances (even higher frequencies will better reject undesired displacements caused by accelerations). With this numbers, there is still a residual gain (quantified as an effective factor $Q_{eff} = f_0/2\Delta f = 67$) and the scale factor will be:

$$SF = \frac{\Delta V_{out}}{B} = i \cdot L \frac{Q_{eff}}{2\,k} \frac{2\,C_0}{g} \frac{V_{BIAS}}{C_F}$$

Not only off-resonance operation guarantees such a large bandwidth, but it also enables to decrease the pressure, so to lower the intrinsic thermomechanical noise (by decreasing the damping coefficient) and, in general, enhances the scale-factor stability of the sensor against temperature variations. As a numerical example, we know that the NEMD is given by:

$$NEMD = \frac{\sqrt{4k_B T b}}{i \cdot L / 2} = \frac{4}{i \cdot L}\sqrt{k_B T b} = \frac{4}{i \cdot L}\sqrt{\frac{k_B T \omega_0 m}{Q}}$$

Assuming a current of e.g. 300 µA, 1-mm spring length and a Q factor of e.g. 3000 (at 20 kHz and 2 nkg mass), we get a NEMD of about 330 nT/√Hz.

Over the desired bandwidth this gives an integrated noise of about 2.5 µT$_{rms}$ which is not compatible with the target specifications. A solution could be to further reduce the pressure, but this may be complex from a process point of view. Additionally, we should consider electronics noise. With a typical circuit based on a

differential charge amplifier configuration (see the figure), and neglecting feedback resistance noise, electronic noise will be given by:

$$\sqrt{S_{Vn,B}} = \frac{\sqrt{2\,S_{Vn}\left(1+\dfrac{C_P}{C_F}\right)^2}}{SF} = \frac{\sqrt{2\,S_{Vn}\left(1+\dfrac{C_P}{C_F}\right)^2}}{i\cdot L\,\dfrac{Q_{eff}}{2\,k}\dfrac{2\,C_0}{g}\dfrac{V_{BIAS}}{C_F}} \approx \frac{\sqrt{2\,S_{Vn}}\,C_P}{i\cdot L\,\dfrac{Q_{eff}}{k}\dfrac{C_0}{g}V_{BIAS}}$$

Typical numbers (10 nV/√Hz amplifier noise, 5 pF parasitic, 250 fF C_0, 1 µm gap, $\frac{V_{DD}}{2}$ bias and other numbers as above) lead to 300 nT/√Hz, similarly to the NEMD value – but this noise will not decrease with pressure.
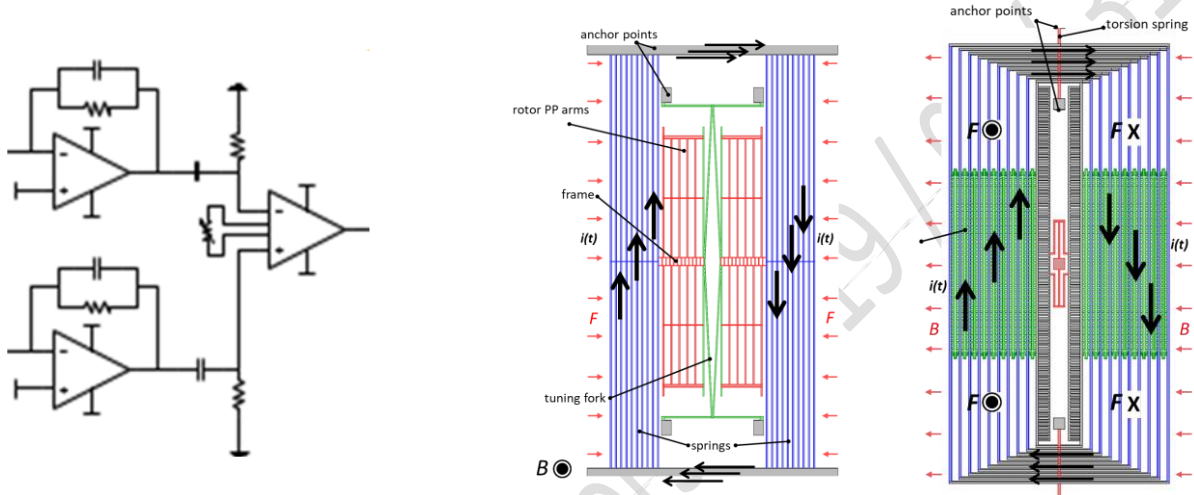
It is thus evident that we need to switch to a sensor architecture that can increase noise from all points of view: the only way is thus to boost the Lorentz force without increasing the current, which can be obtained by multi-loop Lorentz magnetometer architectures, as shown in the figure. A number of loops N_{loop} e.g. of 10 directly boosts the sensitivity, correspondingly lowering both NEMD and electronic noise.

Both the obtained noise values will thus lie in the 30 nT/√Hz range which, summed and integrated over the bandwidth, give a 300 nT$_{rms}$ noise, which is compatible with the requirements.

Note that the used Lorentz current (300 µA, which can recirculate through the three sensors) leaves much room to bias the electronics to get the desired noise value, and to bias the oscillator circuit that provides the reference drive frequency for the Lorentz current. Overall consumption can be lower than the specifications.

With the given parameters, a 1% nonlinearity error will be reached at 10% of the gap (100 nm): this value is, in turn, obtained for a field such that:

$$\frac{B_{max}iLN_{loop}Q_{eff}}{2k} = \frac{B_{max}iLN_{loop}Q_{eff}}{2\,(\omega_d)^2 m} = 100\ nm \rightarrow \quad B_{max} = 31\ mT$$

which is, once more, far beyond the specifications.

Additional final comments can be given to further improve potentialities of magnetometers:

- the oscillation drive reference can be provided by another MEMS (a resonator): this will improve stability under temperature changes, because the changes with T of f_0 and f_d will be correlated;
- monolithic implementations of a 3-axis sensor are possible (this saves a lot of Silicon area);
- further optimization of current partitioning between the sensor and the electronics can be done by writing the amplifier noise as a function of the bias current, and accounting for the oscillator current.

Question n. 2

You are doing the reverse engineering of an image sensor. The only things you know initially are the bias voltage (V_{DD} = 4 V) and the number of bits of the ADC (N_{bit} = 14), and you are given the photon transfer curve in the figure, for three different integration times.

(i) calculate the maximum sensor dynamic range;
(ii) calculate quantization noise, in units of electrons rms;
(iii) calculate the DSNU and the PRNU percentage noise contributions. Please state all the assumptions that you make for your calculations;
(iv) how can you understand that this is a 4T topology? Calculate the reset noise rejection factor.
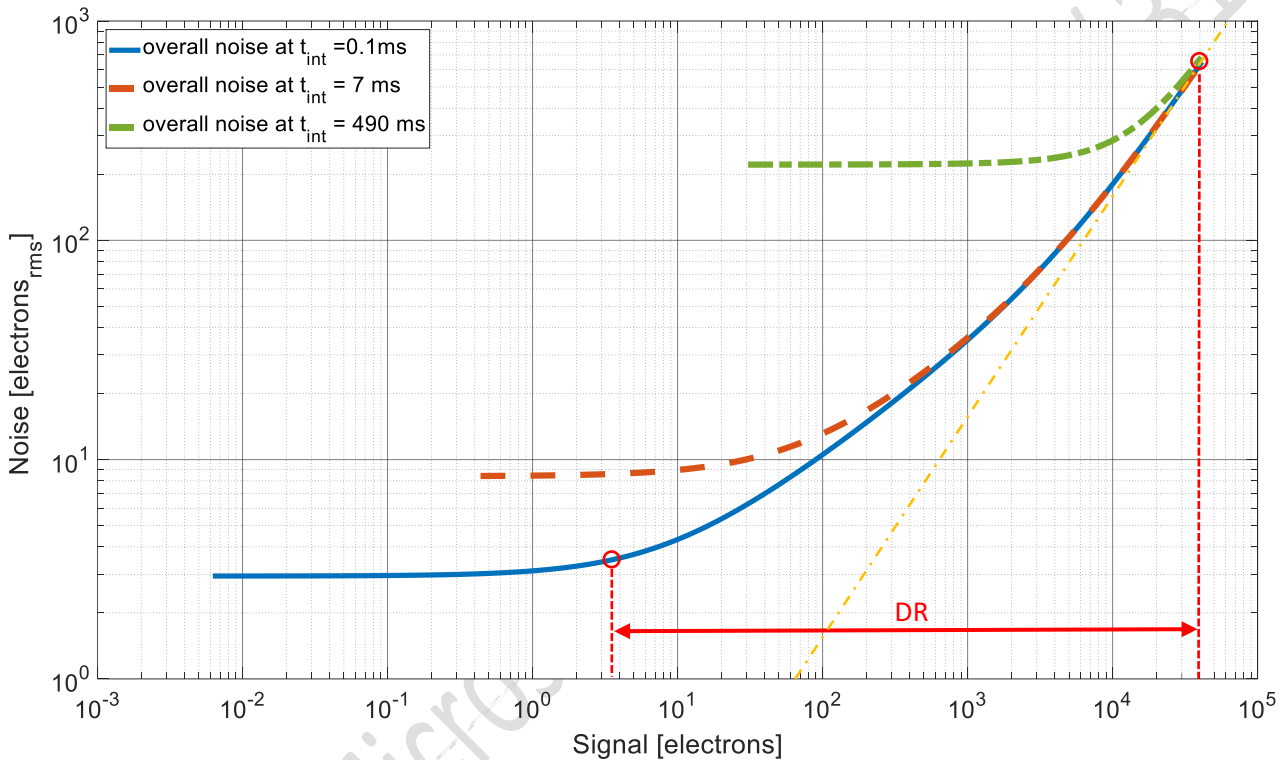
Physical Constants

q = 1.6 10^{-19} C;
T = 300 K;
k_b = 1.38 10^{-23} J/K;
ε_0 = 8.85 10^{-12} F/m;
ε_{Si} = 11.7* ε_0;

1.

The maximum sensor dynamic range is easily found by looking at the curve with minimum integration time and looking for two points on the x-axis: the signal charge corresponding to saturation (we can read 40000 electrons) and the signal charge corresponding to SNR = 1 (we read about 3.5 electrons rms). The two points are indicated by circles. The ratio of these two quantities gives the maximum DR:

$$DR_{max} = 20 \log_{10}\left(\frac{40000}{2.5}\right) = 81.2 \; dB$$

2.

As we are given the supply voltage and the number of bits, we can easily calculate the LSB in units of V:

$$LSB_V = \frac{V_{DD}}{2^{N_{bit}}} = \frac{4V}{2^{14}} = 244 \ \mu V$$

To pass into units of charge, we need to know the overall capacitance C_{int} onto which the charge is integrated. This can be found by looking at the saturation point, where we know that

$$Q_{max} = \Delta V_{max} \cdot C_{int} \approx V_{DD} \cdot C_{int}$$

(where we have approximated the total possible voltage sweep with the entire voltage supply V_{DD}). We thus find the integration capacitance by inverting the formula above:

$$C_{int} = \frac{Q_{max}}{V_{DD}} = \frac{40000 \ q}{4 \ V} = 1.6 \ fF$$

At this point we can find quantization noise as:

$$\sigma_{quant,q} = \frac{V_{DD}}{2^{N_{bit}}\sqrt{12}} C_{int} = \frac{LSB_V}{\sqrt{12}} C_{int} = \frac{244 \ \mu V}{\sqrt{12}} 1.6 \ fF = \ 1.12 \ 10^{-19} \ C = 0.7 \ e_{rms}$$

This indicates that quantization noise, a signal-independent contribution, is negligible with respect to other signal-independent contributions which give 3 electrons rms even at the shortest integration time.

3.

Let us start with DSNU. Its expression in terms of charge is given by:

$$\sigma_{DSNU,q} = \frac{i_d t_{int} \sigma_{DSNU,\%}}{q}$$

which indicates that we have two unknowns, the dark current i_d and the percentage DSNU itself, $\sigma_{DSNU,\%}$, which is what we need to find. We can thus make a few hypotheses which will be later verified.

The hypotheses are based on the fact that the dark current shot noise goes with the square root of t_{int}, while the DSNU goes linear with t_{int}. As a consequence, we expect that:

- at low integration times, signal-independent noise contributions will be dominated by those terms which are independent of the integration time itself (e.g. reset noise);
- at intermediate integration times, it might be that dark current shot noise will become relevant;
- at long integration times, DSNU should definitely dominate over other terms.

From the curve at the lowest integration time, we thus assume reset noise to weigh about 3 electrons rms. We then look at the curve shown for intermediate integration times and assume that in its flat region dark current shot noise dominates over DSNU:

$$(8.5\ e_{rms})^2 = \frac{q i_d t_{int}}{q^2} + (3\ e_{rms})^2 \quad \rightarrow \quad i_d = q\frac{(8.5\ e_{rms})^2 - (3\ e_{rms})^2}{7\ ms} = 1.45\ fA$$

With this value, and assuming that DSNU dominates for the longest integration time, we can find the percentage DSNU as:

$$\sigma_{DSNU,\%} = \frac{220q}{i_d\ t_{int}} = \frac{220q}{1.45\ fA\ \ 490\ ms} = 0.05 = 5\%$$

As a verification, we recalculate dark shot noise and DSNU noise at all the integration times and check if our hypotheses were correct:

-   at 0.1 ms:

$$\sigma_{dark,q} = \frac{\sqrt{q i_d t_{int}}}{q} = 0.9\ e_{rms} \qquad\qquad \sigma_{DSNU,q} = \frac{i_d t_{int}\sigma_{DSNU,\%}}{q} = 0.04\ e_{rms}$$

-   at 7 ms:

$$\sigma_{dark,q} = \frac{\sqrt{q i_d t_{int}}}{q} = 8\ e_{rms} \qquad\qquad \sigma_{DSNU,q} = \frac{i_d t_{int}\sigma_{DSNU,\%}}{q} = 3\ e_{rms}$$

-   at 490 ms:

$$\sigma_{dark,q} = \frac{\sqrt{q i_d t_{int}}}{q} = 66\ e_{rms} \qquad\qquad \sigma_{DSNU,q} = \frac{i_d t_{int}\sigma_{DSNU,\%}}{q} = 222\ e_{rms}$$

Considering that noise contributions compare each other quadratically, our hypotheses are verified: dark-current related noise contributions are negligible at the shortest integration time, dark shot noise dominates at intermediate integration times ($8^2 >> 3^2$) and DSNU dominates at the longest integration time ($222^2 >> 66^2$). Percentage DSNU is, in conclusions, of about 5%.

(note: making these hypotheses has led to the solution avoiding the need of solving a system of two equations - DSNU and dark noise at two integration times - which could have been another way to solve this point).

Finally, for PRNU we just look at the steepest part of the curve: with any of the techniques that can be used (e.g. looking at one point on that part of the curve, or taking a line tangent to that part of the curve with a +20 dB/dec slope, see the figure), we can evaluate the PRNU:

$$\sigma_{PRNU,q} = \frac{i_{ph} t_{int}\sigma_{PRNU_\%}}{q} \quad \rightarrow \quad \sigma_{PRNU_\%} = q\frac{\sigma_{PRNU,q}}{i_{ph} t_{int}} = \frac{1}{70} = 1.4\%$$

4.

The fact that this is a 4T topology is easily understood by looking at the kTC noise and comparing it to the kTC noise that would be obtained from the overall integration capacitance C_{int} calculated at point 2 above:

$$\sigma_{kTC_{int},q} = \frac{\sqrt{k_B T C_{int}}}{q} = 16\ e_{rms}$$

while from the graph at the minimum integration time we have already evaluated a dominating kTC noise value of 3 electrons rms (with quantization noise and dark-related noises negligible).

This means that kTC noise due to the overall integration capacitance is rejected by a factor about 5.5 in linear quantities (about 30 in power units). This is possible thanks to correlated double sampling, a technique that can be adopted only with 4T topologies.

By the way, such a high DR (see point 1) can be effectively achieved only with 4T topologies and at large pixel areas. The sensor is probably belonging to a high-end semiprofessional camera.

Question n. 3

The company you work for wants to finalize the design of a dual-mass yaw gyroscope and your boss wants you to take some final decisions on a few electro-mechanical and system-level parameters. The target values of the project are reported in the Table and an overview of the mechanical structure and the sense chain is given in the Figure. You are required to:

(i) compute the maximum displacement in the sense direction per unit angular rate, compatible with your constraints;

(ii) compute the required number of drive comb fingers (half structure), under the constraint of a 500-mV sinusoidal drive-actuation voltage v_{da};

(iii) compute the intrinsic noise equivalent rate density (intrinsic NERD). Then, find the minimum number of parallel-plate cells (in the half structure) so that the electronics noise contribution is lower than the intrinsic one. Finally, compute the total noise density;

(iv) draw a quoted plot of the output voltage waveform $v_{out}(t)$ when a test input angular rate $\Omega(t) = \Omega_t \cdot \sin(2\pi f_\Omega t)$ (parameter values in table) is applied to the sensor.
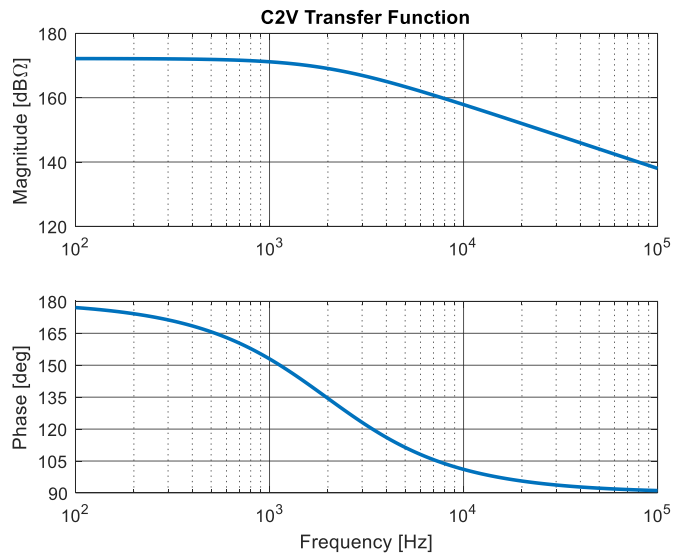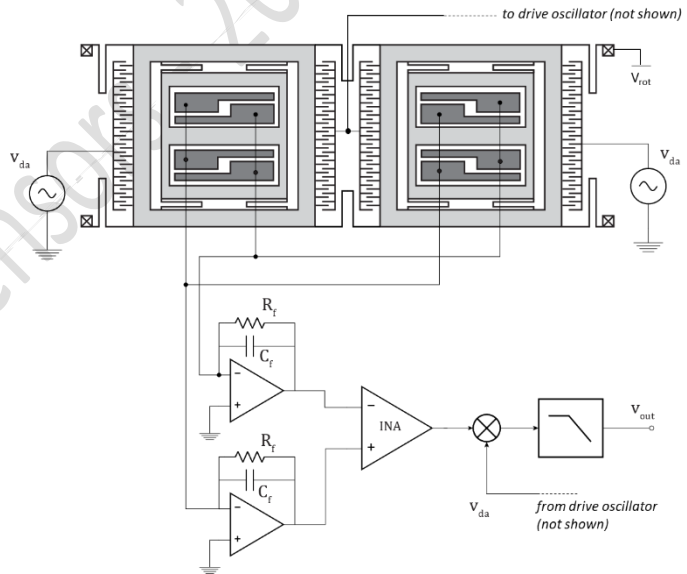
Physical Constants

ε_0 = 8.85 10^{-12} F/m

k_B = 1.38 10^{-23} J/K

T = 300 K

Target linearity error $\varepsilon_{lin\%}$	0.5%$_{FSR}$
Target **FSR**	2000 dps
Process height **h**	20 μm
Drive resonance f_d	19.6 kHz
Sense resonance f_s (at V$_{rot}$=15 V)	20 kHz
Process minimum gap **g**	1.5 μm
Parallel plate length L_{pp}	200 μm
Drive mass (½ structure) m_d	2 nkg
Sense mass (½ structure) m_s	8 nkg
Drive Quality Factor Q_d	15000
Sense Quality Factor Q_s	800
Input referred quadrature error B_q	300 dps
Rotor voltage V_{rot}	15 V
AC drive voltage v_{da}	500 mV
Feedback capacitance C_F	200 fF
Feedback resistance R_F	400 MΩ
INA gain G_{INA}	1
Multiplier gain G_{mux}	2 V^{-1}
Amplifier voltage noise S_{nv}	(8 nV/√Hz)^2
Parasitic capacitance C_P	2 pF
Test signal input AC rate Ω_t	100 dps
Test signal input frequency f_Ω	10 Hz

1.

We are dealing with a standard dual-mass mode-split gyroscope with a capacitive differential read-out. Usually, the maximum displacement in the sense (y) direction is limited by either the saturation of the electronics or the maximum acceptable non-linearity of the parallel plate electrodes. In this case, we do not have enough data to check the saturation of the electronics and so we focus on the non-linearity requirement. We know that the maximum linearity error ϵ is given by:

$$\epsilon_{lin} = \left(\frac{y_{max}}{g}\right)^2,$$

Where the power of two is due to the differential read-out. Inverting that formula, we obtain:

$$y_{max} = g_{pp}\sqrt{\epsilon_{lin}} = 1.5 \text{ μm} \times \sqrt{0.005} \approx 106 \text{ nm}.$$

2.

With all other electro-mechanical parameters fixed, the number of comb fingers N_{CF} is proportional to the drive displacement x_d on the proof masses. From the previous point we can extract the optimal sensitivity as the one that matches y_{max} with the maximum input angular rate Ω_{FSR}.

Let's start by computing the optimal sensitivity S_y as:

$$S_y = \frac{y_{max}}{\Omega_{FSR}} = \frac{106 \text{ nm}}{2000 \text{ dps}} \approx 53 \frac{\text{pm}}{\text{dps}}.$$

Knowing the split frequency $f_{split} = f_s - f_d = 400$ Hz, we can find the required x_d as:

$$S_y = \frac{x_d}{\omega_{split}} \Rightarrow x_d = S_y \omega_{split} \frac{180}{\pi} \approx 7.63 \text{ μm}.$$

Finally, from the relationship between driving voltage and drive displacement, we can find N_{CF}:

$$x_d = v_{da}V_{rot}\left(2N_{CF}\frac{\epsilon_0 h}{g}\right)\frac{Q_d}{k_d} = v_{da}V_{rot}\left(2N_{CF}\frac{\epsilon_0 h}{g}\right)\frac{Q_d}{\omega_d^2(m_s + m_d)} \Rightarrow N_{CF} = \frac{g\omega_d^2(m_s + m_d)x_d}{2v_{da}V_{rot}N_{CF}\epsilon_0 hQ_d}$$
$$\approx 43.6 \text{ (43 or 44)}.$$

3.

Firstly, let us compute the intrinsic Noise Equivalent Rate Density ($NERD_m$) that is due to the white force noise $S_{nF} = 4k_BTb = 4k_BT\frac{\omega_s m_s}{Q_s}$. Dividing this term by the sensitivity in terms of force we obtain:

$$NERD_m = \sqrt{4k_BT\frac{\omega_s m_s}{Q_s}} \cdot \frac{1}{2m_s\omega_d x_d} \cdot \frac{180}{\pi} \approx 549 \frac{\text{μdps}}{\sqrt{\text{Hz}}}$$

Then, let us find the required number of parallel plates N_{PP}. We are given the data to compute the electronics noise due to the feedback resistor R_f and to the amplifier voltage noise S_{nV}. By properly bringing these noise contributions to the charge amplifier output and input referring it we are able to find the unknown N_{PP}. The total noise referred to the (differential) output of the charge amplifier $S_{nV,CA}$ is:

$$S_{nV,CA} = \sqrt{2S_{nV}^2\left(\frac{C_f + C_P + 2N_{PP}C_{PP}}{C_f}\right)^2 + 2\frac{4k_BT}{R_f} \cdot \frac{1}{\omega_d C_f}},$$

where C_{PP} is the rest capacitance of one single parallel plate electrode. Since $C_{PP} = \frac{\epsilon_0 hL_{PP}}{g} = 23.6$ fF $\approx \frac{C_P}{100}$, we can assume that, with a reasonable value for N_{PP}, the contribution of the sense rest capacitance in parallel with the parasitic capacitance will be negligible. Therefore, we will use this approximation in the following and we can already compute $S_{nV,CA}$:

$$S_{nV,CA} = \sqrt{2S_{nV}^2\left(\frac{C_f + C_P}{C_f}\right)^2 + 2\frac{4k_BT}{R_f} \cdot \frac{1}{\omega_d C_f}} = 384 \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

To input-refer this noise as angular rate, we have to find the (differential) voltage sensitivity S_v:

$$S_V = \frac{\partial y}{\partial \Omega} \cdot \frac{\partial C}{\partial y} \cdot \frac{\partial V}{\partial C} = S_y \cdot 2N_{PP} \frac{C_{PP}}{g} \cdot 2 \frac{V_{rot}}{C_f}.$$

Therefore, the input referred electronic noise $NERD_e$ is:

$$NERD_e = \frac{S_{nV,CA}}{S_V} = S_{nV,CA} \cdot \frac{1}{S_y \cdot 2N_{PP} \frac{C_{PP}}{g} \cdot 2 \frac{V_{rot}}{C_f}}.$$

Imposing $NERD_e < NERD_m$ we can find the required N_{PP}:

$$\frac{S_{nV,CA}}{S_y \cdot 2N_{PP} \frac{C_{PP}}{g} \cdot 2 \frac{V_{rot}}{C_f}} < NERD_m \Rightarrow N_{PP} > \frac{gC_f S_{nV,CA}}{4C_{PP}V_{rot}S_y NERD_m} \approx 2.8 \to 3.$$

N_{PP} is a reasonable and relatively low value, so the approximation discussed above is valid. With this number of parallel plates, the overall noise becomes $549 \frac{\mu dps}{\sqrt{Hz}} \sqrt{2} = 780 \frac{\mu dps}{\sqrt{Hz}}$.

4.

Firstly, let us compute the sensitivity at the v_{out} node $S_{v_{out}}$:

$$S_{v_{out}} = S_V G_{INA} \left(\frac{1}{2} G_{mux}\right) = \frac{\partial y}{\partial \Omega} \cdot \frac{\partial C}{\partial y} \cdot \frac{\partial V}{\partial C} \cdot G_{INA} \left(\frac{1}{2} G_{mux} v_d\right) = S_y \cdot 2N_{PP} \frac{C_{PP}}{g} \cdot 2 \frac{V_{rot}}{C_f} \cdot G_{INA} \left(\frac{1}{2} G_{mux} v_d\right)$$

$$= 375 \frac{\mu V}{dps}.$$

Then, from the Bode plot of the charge amplifier transfer function we can notice that the phase shift of this stage is quite far from the ideal +90° (or -270°). From the plot one can estimate a phase shift ϕ_{CA} between +95° and +100°. Actually, we have all the data for a precise computation of ϕ_{CA}:

$$\phi_{CA} = 180 - \tan^{-1}\left(\frac{f_d}{p_{fb}}\right) = 180 - \tan^{-1}(2\pi R_f C_f f_d) = 95.8\,°$$

This affects the demodulation performed by the multiplier! If we arbitrarily assign a phase of 0° to v_{da}, i.e. $v_{da} \propto \cos(\omega_d t + 0)$, we can assign a phase to all the relevant quantities of this system:

- $x_d \propto \cos\left(\omega_d t - \frac{\pi}{2}\right)$, because the mechanical transfer function x/F has a 0° phase shift at low frequency, a -180° shift at high frequency and a -90° shift at resonance (which is the relevant one, since the drive resonator operates at resonance).
- $F_{co} \propto \cos(\omega_d t + 0)$, because the Coriolis force is proportional to the drive velocity, which is the derivative of the displacement. The derivative adds +90° with respect to the phase of the displacement.
- $y_{co} \propto \cos(\omega_d t + 0)$, because the sense resonator operates off-resonance. Since $f_d < f_s$, this second-order mechanical transfer function does not add any phase shift.
- v_{CA} and $v_{INA} \propto \cos(\omega_d t + 0)$, because the capacitive gain does not shift the output voltage phase with respect to the displacement (or the sense capacitance variation). v_{da} is in phase with the INA voltage and is a correct choice for demodulating the Coriolis signal.
- The quadrature force is, by definition, in quadrature with the Coriolis force, and therefore the related displacement and voltage are in quadrature with their Coriolis counterparts.

Ideally, a demodulation using the drive actuation voltage should recover the Coriolis signal and reject the quadrature-induced signal. But we can notice from the Bode plot of the charge amplifier transfer function that the phase shift of this stage is quite far from the ideal +90° (or -270°). This phase shift is experienced by the Coriolis signal but not by the drive actuation voltage used for demodulation, and so the demodulation will be imperfect! From the plot one can estimate a phase shift ϕ_{CA} between +95° and +100°. Actually, we have all the data required for a precise computation of ϕ_{CA}:

$$\phi_{CA} = 180 - \tan^{-1}\left(\frac{f_d}{p_{fb}}\right) = 180 - \tan^{-1}\left(2\pi R_f C_f f_d\right) = 95.8°$$

Let's call the deviation of this phase from the ideal +90° phase error $\phi_{err} = \phi_{CA} - 90° = +5.8°$.

To study the effect of this phase error, we can explicitly write the operation performed by the demodulation accounting for ϕ_{err}:

$$v_{out} \propto \left[\Omega \cos(\omega_d t + \phi_{err}) + B_q \sin(\omega_d t + \phi_{err})\right] \cos(\omega_d t)$$

where we considered the signal at the INA output proportional to the term within square brackets and the drive voltage proportional to the cos factor. By performing the multiplication and neglecting the $2\omega_d$ terms, filtered out by the LPF, we obtain:

$$v_{out} \propto \Omega \cos(\omega_d t + \phi_{err} - \omega_d t) + B_q \sin(\omega_d t + \phi_{err} - \omega_d t) = \Omega \cos(\phi_{err}) + B_q \sin(\phi_{err}),$$

where $\cos(\phi_{err}) \approx 0.995$ can be neglected. Therefore, at the output one can observe the following signal:

$$v_{out} = S_{v_{out}}[\Omega_t \cos(2\pi f_\Omega t) + B_q \sin(\phi_{err})] = [37 \cos(2\pi f_\Omega t) + 11] \text{ mV}.$$
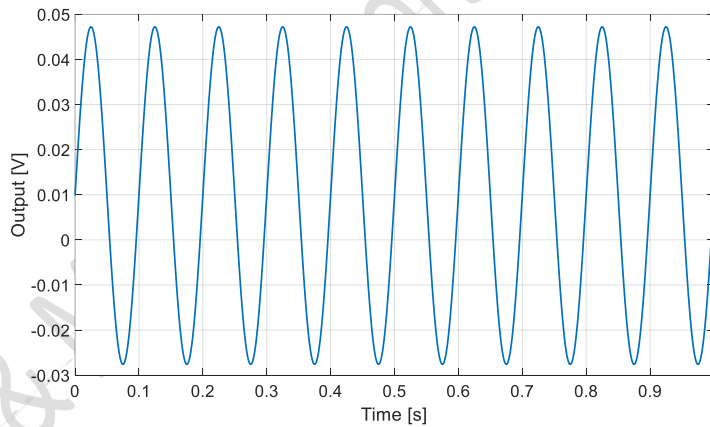
As an extra, one can even consider the phase shift given by the sense mechanical transfer function y/F. One can write this transfer function as:

$$\frac{Y(j\omega)}{F_y(j\omega)} = \frac{1}{k_s} \cdot \frac{1}{-\omega^2 + \dfrac{j\omega_s \omega}{Q_s} + \omega_s^2},$$

whose phase shift ϕ_{ds} at the drive resonance frequency is given by the arc tangent of the ratio of the real and imaginary parts:

$$\phi_{ds} = -\tan^{-1}\left(\frac{\dfrac{\omega_s \omega_d}{Q_s}}{\omega_s^2 - \omega_d^2}\right) \approx -1.77°.$$

Considering also this phenomenon leads to a slightly lower $\tilde{\phi}_{err} = \phi_{err} + \phi_{ds} \approx 4°$: this does not significantly affect the Coriolis signal, but reduces to some extent the quadrature offset seen at the output.