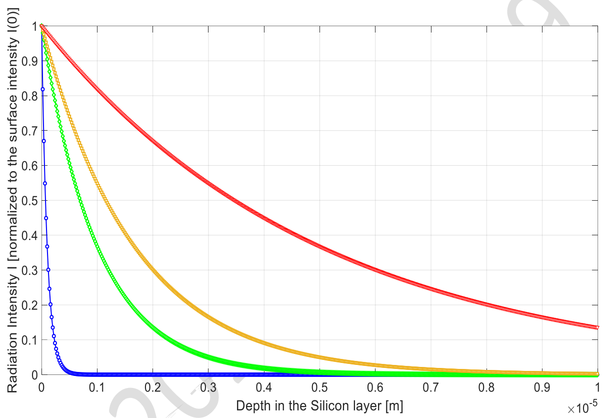
Last Name __Navrò__ **Given Name** __Graziano__ **ID Number** __20190109__

Question n. 1

You are a technological consultant for a startup company working on high-dynamic-range CMOS image sensors targeting more than 70 dB of DR. You are asked to provide convincing advices on the *technological requirements* to optimize the CMOS process that the company needs to develop. Discuss and motivate these requirements, accounting for physical principles, technological feasibility and performance trade-offs.

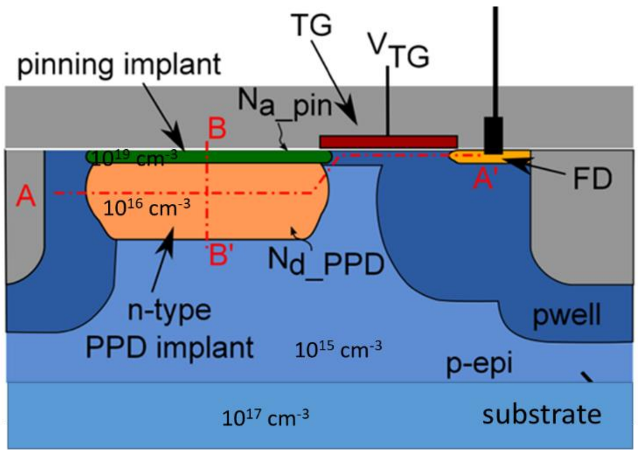
The physical principles of radiation absorption indicate that light at the longest wavelengths in the visible range (about 700 nm) is absorbed (with a probability larger than 90%) <u>within 10-12 μm</u>. This implies that the <u>thickness of the active layer</u> should be at least in this range. This active layer will be typically a carefully grown, <u>high-quality</u> (i.e. low defects) <u>epitaxy</u>, to keep low the dark-carrier generation rate. Going to larger thicknesses gives no advantages, as the increase in light absorption at long wavelengths is much less than linear, and thus low compared to the increase in dark current.



Within such a thickness we should create a collecting junction: among the possible options, in view of an HDR application, it is recommended to use a process that enables <u>pinned photodiodes</u>. Such photodiodes are formed by a wholly depleted <u>N-type region</u>, surrounded by the <u>lowly-doped P-type epitaxial layer</u> and a <u>shallow P++ surface implant</u>. The typical doping of the high-quality epitaxy is of e.g. 10^{15} cm^{-3}. This implies that, on average, the N-type pinned implant should be at least an order of magnitude more doped (e.g. 10^{16} cm^{-3}).

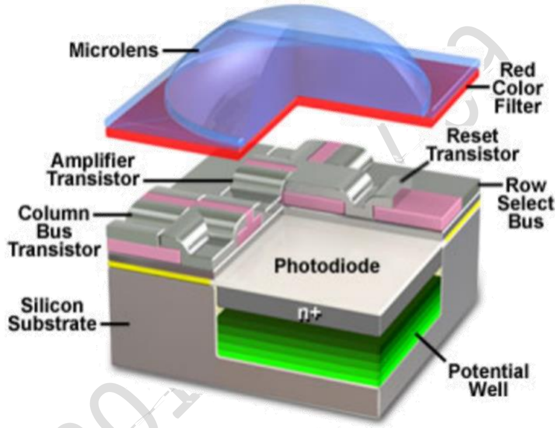
Assuming maximum voltages of e.g. 3 V for the process the N-type layer gets fully depleted if its depth is: $x_{dep,N} = \sqrt{\frac{2\epsilon_{Si}(V_{bi}+3\,V)}{q\,N_{D_{PD}}}} \approx 700\ nm$, *while most of the depletion region will still extend in the P-type epitaxial layer:* $x_{dep,P} = \sqrt{\frac{2\epsilon_{Si}(V_{bi}+3\,V)}{q\,N_{A_{epi}}}} \approx 2.2\ \mu m$. *The pinned photodiode will thus collect both by drift within these depleted regions, and by diffusion from the non-depleted P-type epitaxial layer: for this reason, it is recommended that the <u>substrate is of a heavily doped P-type</u>, such that a small barrier prevents photogenerated electrons to diffuse from the epitaxial layer into the substrate.*

Conversely, the heavily doped P++ pinning implant should be <u>very shallow</u>: its purpose it to <u>block dark generated carriers at the dirty Si-SiO$_2$ interface</u> and to prevent their collection by the pinned photodiode. At the same time, this layer should not absorb blue light, which would be otherwise lost. For this reason, thicknesses lower than 50 nm are recommended for this layer.



With all such process features, a <u>pinned photodiode</u> can be obtained as depicted in the figure. The transfer gate is a standard MOS gate, which, when activated, connects the pinned photodiode region to the floating diffusion, for the photocarriers readout. By reducing both (i) collection of surface-generated dark current, and (ii) reset noise (thanks to correlated double sampling, CDS, which is enabled by the 4-T topology), the pinned photodiode enables to reach DR larger than 70 dB, overcoming typical limitations of the 3T topology.

Additional technological features that should be adopted are common to all other types of CMOS-image-sensor processes (e.g. the 3T topology based on the standard PN junction):



- (i) the presence of micro-color filter arrays (CFA) enables to get a color image out of an array of natively monochrome, quantum, photodiodes;
- (ii) the presence of microlenses bypasses the problem of a low fill factor, i.e. a low active area over the total pixel area;
- (iii) the presence of infrared/ultraviolet blocking filters avoids generation of photoelectrons by radiation which is not visible to the human eye;

Within all these different features, it is also fundamental to minimize process nonuniformities (both optical and electronic), not to impair the achieved DR at pixel level due to fixed pattern noise (FPN). Nonuniformities from pixel to pixel can indeed occur in terms of

- Silicon doping
- planar geometry (mask errors or misalignments)
- layer thickness (poor planarity)

All such effects have an impact on percentage photo-response (mostly quantum efficiency) and dark-signal (mostly dark current) nonuniformities, which are two sources of FPN that affect the overall sensor DR and should be minimized.

Note: alternative paths for the answer, deviating from the "technological requirements" and focusing more on the DR formula, 4T topology and working principle, CDS… are appreciated – though conceptually not aligned with the key point of this question.

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Question n. 2

You are designing a system formed by a parallel-plate MEMS accelerometer and an integrated readout circuit for high-full-scale-range (FSR) applications. The target parameters for your design are given in the Table. First consider the situation given in Fig. 1 (neglect the central driving electrodes, disconnected from the circuit):

(i) given the target FSR, find the optimum resonance frequency f_0 for the accelerometer;
(ii) find the optimum sensitivity after the low-pass filter, i.e. at the point indicated as V_{out} in the circuit of Fig 1 (express it in [mV/g]);
(iii) find the corresponding optimum value for the feedback capacitance C_F of the charge amplifiers.

Target full-scale range (FSR)	16 g
Target linearity error	1 %
Rest capacitance (sense electrodes)	250 fF
Rest capacitance (drive electrodes)	632 fF
Parallel plates gap	1.59 µm
Test signal amplitude	0.5 V
Test signal frequency	500 kHz
Elastic stiffness	5 N/m
Supply voltage of the sensing circuit	±2 V
Gain of sensing INA stage	10
Gain of driving INA stage (Fig. 2)	100
Multiplier gain	2 V^{-1}
DC voltage V_a (Fig. 2)	1 V

After a suggestion from a colleague of yours, you now consider the alternative circuit depicted in Fig. 2:

(iv) what does the feedback mechanism do when an acceleration occurs on the sensor? Explain in detail the working principle and clarify which is the physical quantity controlled by the loop;
(v) calculate the new sensitivity, once again in [mV/g] at the point indicated as V_{out} in Fig. 2 (hint: base your starting equations on the considerations of point (iv) and neglect the voltage signal v_t in your calculation).

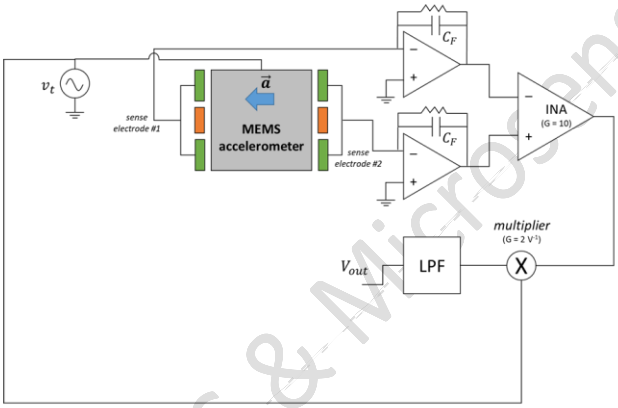

Fig. 1

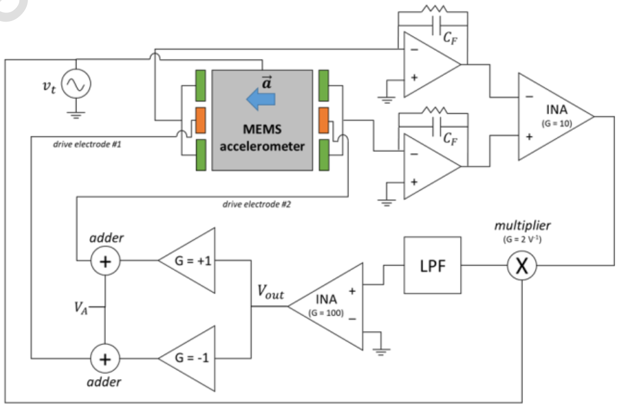

Fig. 2

Physical Constants
g = 9.81 m/s²

1.
A parallel-plate MEMS accelerometer shows typically a limitation to the FSR given by the nonlinearity of the parallel plates. The relationship between the maximum displacement x_{max} and the linearity error ϵ_{lin}, and the relationship between the same maximum displacement and the acceleration corresponding to the FSR (that generates it) are:

$$\epsilon_{lin} = \left(\frac{x_{max}}{x_0}\right)^2 \qquad x_{max} = \frac{FSR}{\omega_0^2}$$

where ω_0 is the resonance frequency accounting also for electrostatic softening. In the specific case, the electrostatic stiffness (with a high-frequency test signal v_t with amplitude V_t) is:

$$k_{elec} = 2\frac{V_t^2 C_{0s}}{g^2}\frac{1}{2} = 0.025\ N/m$$

which is negligible compared to the mechanical value given in the data (we thus neglect electrostatic softening for the sake of simplicity). We thus simply get:

$$f_0 = \frac{\omega_0}{2\,\pi} = \frac{1}{2\,\pi}\sqrt{\frac{FSR}{x_0\sqrt{\epsilon_{lin}}}} = \frac{1}{2\,\pi}\sqrt{\frac{16\cdot 9.8\ m/s^2}{1.59\ \mu m\ \sqrt{0.01}}} = 5\ kHz$$

2.

The optimum sensitivity corresponds to the situation where the maximum acceleration generates the maximum signal that avoids saturation in the circuit. The specific circuit is based on a small-amplitude, high-frequency signal v_t which modulates the sensing signal around 500 kHz. This signal is in turn demodulated back to baseband with a multiplier and a low-pass filter.

In terms of modulus, we can consider that the sinewave signal with an amplitude v_{INA} at the INA output is multiplied by another sinewave v_t with amplitude 0.5 V. At the multiplier output we thus have

$$v_{mult} = (2\ V^{-1})\cdot V_{INA}\cos(2\pi 500kHz\ t)\cdot V_t\cos(2\pi 500kHz\ t) = 2\ V^{-1}\ V_{INA}V_t\ \frac{1}{2}[\cos(0) + \cos(2\pi 1MHz\ t)] = 0.5\ V_{INA}[\cos(0)+\cos(2\pi 1MHz\ t)]$$

After the low-pass filter, the component at 1 MHz is filtered and the quasi DC signal V_{out} is proportional to $0.5\ V_{INA}$, which is thus lower than the INA output. The critical saturation point in the circuit is thus the INA output itself. This voltage, in the optimized situation, should cover the electronic FSR (2V) under an acceleration FSR (16 g). At the INA output, the sensitivity S_{INA} should be thus 125 mV/g, which at the point indicated as V_{out} becomes S = 62.5 mV/g.

3.

The optimum sensitivity should match the following equation:

$$S = S_{INA}\cdot 0.5 = 2\frac{C_0}{x_0}\frac{V_t}{C_F}\frac{1}{\omega_0^2}\ G_{INA}\cdot 0.5 = 62.5\frac{mV}{g}$$

which yields an optimum feedback capacitance C_F of 125 fF.

4.

To study the behavior of the circuit of Fig. 2, it is worth adding some considerations on the phases of the signal up to the multiplier. Assuming a 0° reference phase for the test signal v_t, we know that the currents $i_{1,2}$ at the charge amplifier inputs show a +90° phase ($i_{1,2} = C_{1,2(t)}\frac{dv_t}{dt}$), and the voltages at the charge amplifiers outputs show a -180° phase (integration of the current, and negative input of the amplifier). The INA takes the difference between the voltages: if the accelerometer is centered, the INA will show a null output. If the inertial force pushes the mass rightwards, the INA output will show a -180° phase with respect to v_t (thus, a negative output after the multiplication and LPF). If the inertial

force pushes the mass leftwards, the INA output will be in phase with v_t (thus, a positive output after the multiplication and LPF).

At this point, the second INA amplifies this signal (by comparing it to 0) into what is called V_{out} in this new circuit. This signal is summed and subtracted to V_A, yielding a net electrostatic force F_{elec} on the accelerometer through the driving electrodes:

$$F_{elec} = \left[\frac{(V_A + V_{out})^2}{2} - \frac{(V_A - V_{out})^2}{2}\right]\frac{C_{0d}}{x_0} = \frac{2V_A V_{out} C_{0d}}{x_0}$$

The force is directed in such a way that the acceleration action is counterbalanced. This essentially means that the reaction circuit acts in such a way that the inertial force is compensated by the electrostatic force. Indeed, this technique is known as force-feedback.

5.

Assuming a high-enough loop gain, the sensitivity is thus found by setting a perfect balance between the electrostatic force applied by the feedback at the drive electrodes, and the inertial force. As suggested by the text, we neglect the small signal v_t in the calculation:

$$\frac{2V_A V_{out} C_{0d}}{x_0} = m \cdot a$$

Which immediately yields the sensitivity:

$$\frac{V_{out}}{a} = \frac{m\, x_0}{2V_A C_{0d}} = \frac{k\, x_0}{\omega_0^2 2V_A C_{0d}} = 6.4\frac{mV}{m/s^2} = 62.5\frac{mV}{g}$$

The sensitivity is the same as for the circuit of Fig. 1, however with this configuration the MEMS is kept always centered, improving the system linearity.

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Question n. 3

You are designing a dual-mass Y-axis gyroscope, whose simplified structure is depicted in the Figure. A set of comb-finger electrodes (not shown) allows the anti-phase motion of the proof masses along the drive X-axis, whereas the angular rate along the Y-axis is sensed with a torsional mode through the S1-S2 parallel-plate vertical electrodes placed underneath the moving mass. The parameters of the system are given in the Table.

Process height $\boldsymbol{h}$	24 µm
Springs length $\boldsymbol{L_{sp}}$	100 µm
Springs width $\boldsymbol{w_{sp}}$	2 µm
Tuning fork length $\boldsymbol{L_{tf}}$	100 µm
Tuning fork width $\boldsymbol{w_{tf}}$	2 µm
Drive and sense mass (½ structure) $\boldsymbol{m}$	5 nkg
Sense resonance frequency $\boldsymbol{f_s}$	21.8 kHz
Sense Quality Factor $\boldsymbol{Q_s}$	1000
Lever for Coriolis force $\boldsymbol{l_{Co}}$	100 µm
Sense electrodes distance $\boldsymbol{l_{el}}$	150 µm
Additional electrodes distance $\boldsymbol{l_0}$	50 µm
Moment of inertia (full structure) $\boldsymbol{J}$	$6.7\cdot10^{-17}$ kg m^2
Max input angular rate $\boldsymbol{\Omega_{max}}$	1000 dps
Max quadrature $\boldsymbol{B_{q,max}}$	1000 dps
Rotor voltage $\boldsymbol{V_{DC}}$	10 V
Maximum voltage $\boldsymbol{\Delta V}$	5 V
Additional electrodes width $\boldsymbol{W}$	50 µm
Vertical gap $\boldsymbol{g}$	2 µm
Sense electrode area $\boldsymbol{A_0}$	$1\cdot10^{-9}$ m^2

Initially neglect the four additional electrodes biased at $\pm\Delta$V. You are required to:

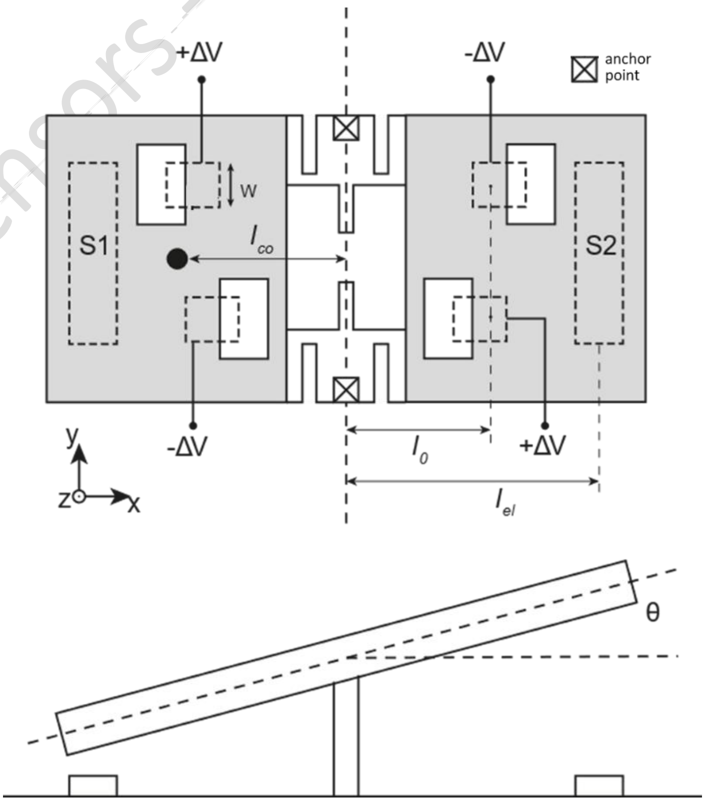
(i) find the drive amplitude x_d in order to obtain a sensitivity $S_\theta = \frac{\partial\theta}{\partial\Omega} = 1.7\,\frac{\mu\mathrm{rad}}{\mathrm{dps}}$, where θ is the rotation angle of the sense frame as in the bottom figure (hint: recall the formula relating the moment of inertia *J* and the resonance frequency f_s, and neglect electrostatic softening of the sense mode);

(ii) with S_θ given in the point above, find the sensitivities in terms of vertical displacement $S_z = \frac{\partial z}{\partial\Omega}$ and of single-ended capacitive variation $S_c = \frac{\partial C}{\partial\Omega}$;

(iii) find the maximum linearity error.

Now consider the additional four electrodes underneath the structure, biased at $\pm\Delta$V:

(iv) what is the purpose of these four electrodes? Is a maximum bias voltage $\pm\Delta V_{max} = \pm 5$ V enough to reach the purpose?



Physical Constants

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

E = 160 GPa

1.

We are dealing with a gyroscope that is driven in-plane along the x axis to sense an angular rate in the y direction. The two masses are for sure driven in anti-phase so that the Coriolis force will pull up one mass and down the other mass. In this way, an external acceleration is sensed as a common-mode

signal and rejected. Notice that the sense mode is a torsional mode: the Coriolis force applies a torque with a lever l_{co} to each mass.

To compute the drive displacement x_d that gives us the correct sensitivity S_θ, let's find the expression of this sensitivity. Of course, we cannot use the usual formula $S = x_d/\Delta\omega$ because this is a displacement sensitivity ([m/dps]) for an in-plane gyro, not an angular sensitivity ([rad/dps]) for a torsional out-of-plane gyro. In fact, the effect of the Coriolis force is different: it applies a double torque T_{co} to the structure, given by:

$$T_{co} = F_{co} l_{co} + F_{co} l_{co} = 2 \cdot 2 m v_d \Omega \cdot l_{co} = 4 l_{co} m \omega_d x_d \Omega.$$

This is because F_{co} acts on the barycenter of the mass with a lever l_{co}, multiplied by 2 because Coriolis applies a torque on both masses (note: if you did not multiply by 2, you will not be penalized as it is considered a minor error).

T_{co} is then turned into angle θ through the torsional mode equation. By analogy[1], we can apply the usual formulas of a linear mass-spring-damper system provided we substitute $F \rightarrow T;\ k_m \rightarrow\ k_\theta; x \rightarrow \theta; m \rightarrow J$. Therefore:

$$\theta = T_{co} \frac{Q}{k_\theta} \ \rightarrow\ S_\theta = \frac{\partial\theta}{\partial\Omega} = \frac{4 l_{co} m \omega_d x_d Q}{k_\theta} \left[\frac{\mathrm{rad}}{\frac{\mathrm{rad}}{\mathrm{s}}}\right].$$

To find x_d we need to find k_θ, ω_d. If the gyro is mode-matched, Q will be the sense quality factor; if the gyro operates in mode-split, Q will be the the effective Q. We need to verify this.

First off, let us find k_θ. Applying the analogy with the linear mass-spring system:

$$\omega_s = \sqrt{\frac{k_\theta}{J}} \rightarrow k_\theta = \omega_s^2 \cdot J = 1.25 \cdot 10^{-6}\ \mathrm{Nm}$$

Then, let us find the drive stiffness $k_{d,ap}$ (anti-phase mode) and the drive resonance frequency. The stiffness of the half-structure is:

$$k_{d,ap} = k_{sp} + k_{tf} = Eh\left(\frac{W_{sp}}{L_{sp}}\right)^3 \cdot \frac{2}{2} + Eh\left(\frac{W_{sp}}{L_{sp}}\right)^3 \cdot \frac{2}{1} = (30.72 + 61.44)\frac{\mathrm{N}}{\mathrm{m}} = 92.16\frac{\mathrm{N}}{\mathrm{m}},$$

Where we have 2 anchor springs in parallel, each made of the series of 2 folds, and 2 tuning forks in parallel, each with a single beam. The drive resonance frequency f_d is:

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k_{d,ap}}{m}} = \frac{1}{2\pi}\sqrt{\frac{92.16\frac{\mathrm{N}}{\mathrm{m}}}{5\ \mathrm{nkg}}} = 21.608\ \mathrm{kHz}$$

From the sense resonance frequency datum, we can compute the split frequency Δf_{split}:

$$\Delta f_{split} = f_s - f_d = (21.8 - 21.608)\ \mathrm{kHz} = 192\ \mathrm{Hz}.$$

[1] Recall the differential equation of a torsional system $J\ddot{\theta} + b_\theta\dot{\theta} + k_\theta\theta = T_{ext}$, very similar to the usual linear one.

Therefore, the gyro is operated in mode-split since the split frequency is much larger than the -3dB bandwidth of the sense mode (10.9 Hz). The effective quality factor Q_{eff} is:

$$Q_{eff} = \frac{f_s}{2\Delta f_{split}} = \frac{21800}{2 \cdot 192} = 56.8$$

By inverting the sensitivity formula, we can get the drive displacement:

$$x_d = \frac{S_\theta k_\theta}{4 l_{co} m \omega_d Q_{eff}} \frac{180}{\pi} = 8\ \mu\text{m}.$$

Notice that you could have used the moment of inertia of the half structure and the torque applied to the half structure to get the same result.

2.

The vertical displacement z at the center of the sense electrode is given by:

$$z = l_{el} \cdot \tan\theta \approx l_{el} \cdot \theta.$$

Therefore:

$$S_z = \frac{\partial z}{\partial \Omega} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial \Omega} = l_{el} \cdot S_\theta = 255\ \frac{\text{pm}}{\text{dps}}.$$

As for the capacitive sensitivity, considering that a vertical parallel plate behaves just like the usual structure:

$$S_C = \frac{\partial C}{\partial \Omega} = \frac{\partial C}{\partial z} \cdot \frac{\partial z}{\partial \Omega} = \frac{\epsilon_0 A_{el}}{g^2} \cdot S_\theta = 570\ \frac{\text{zF}}{\text{dps}}.$$

3.

We just need to apply the formula for the linearity error of a differential read-out:

$$\epsilon_{lin} = \left(\frac{\Delta z_{max}}{g}\right)^2 = \left(\frac{S_z \cdot \Omega_{max}}{g}\right)^2 = 0.0166\ (1.66\%).$$

If you did so, you will be awarded full points. The most correct answer would have been to consider also the motion induced by the maximum quadrature rate $B_{q,max}$:

$$\epsilon_{lin} = \left(\frac{\Delta z_{max}}{g}\right)^2 = \left(\frac{S_z \cdot \sqrt{\Omega_{max}^2 + B_{q,max}^2}}{g}\right)^2 = 0.0332\ (3.32\%).$$

4.

The four electrodes implement a Tatar scheme for out-of-plane modes. By tuning the ΔV voltage you can introduce and finely tune an x-z (or x-θ) cross-stiffness that can compensate the motion introduced by quadrature errors. The main difference with respect to the "standard" in-plane implementation is that this scheme applies a torque (rather than just a force) to the MEMS: it is important to consider the position of the four electrodes (i.e. the lever l_0 of the applied quadrature compensating force).

Let's prove this statement by computing the quadrature compensating torque T_{QC} applied by the 4 electrodes, L_{ov} being the rotor-electrode overlap at rest and x being the (anti-phase mode) drive displacement2:

$$\begin{aligned} T_{QC} = F_1 l_0 - F_2 l_0 + F_3 l_0 - F_4 l_0 = \\ &= \frac{1}{2}\frac{\epsilon_0 W}{g^2} l_0 [(V_{DC} - \Delta V)^2 (L_{ov} + x) - (V_{DC} + \Delta V)^2 (L_{ov} + x) \\ &+ (V_{DC} + \Delta V)^2 (L_{ov} - x) - (V_{DC} - \Delta V)^2 (L_{ov} - x)] \end{aligned}$$

By doing the computation, you find out that all the terms proportional to V_{DC}^2, ΔV^2 and L_{ov} disappear (like in the conventional Tatar scheme) and you get:

$$T_{QC} = \frac{1}{2}\frac{\epsilon_0 W}{g^2} l_0 [2 V_{DC} \Delta V (-4x)] = \frac{4 \epsilon_0 W V_{DC} \Delta V l_0}{g^2} \cdot x.$$

So, this is an architecture that applies an angular displacement ($\theta = T_{QC} \cdot k_\theta / Q_{eff}$ proportional to the drive displacement, just like the quadrature error.

The quadrature torque T_Q is:

$$T_Q = 2 m \omega_d x_d B_q (2 l_{co})$$

The maximum T_Q to be compensated can be obtained by substituting $B_Q = B_{Q,max}$ and the maximum T_{QC} that can be applied is found by substituting $x = x_d$ and $\Delta V = \Delta V_{max}$. $\Delta V_{max} = 5$ V is enough to compensate the quadrature if the following ratio is smaller than 1:

$$\frac{T_{Q,max}}{T_{QC,max}} = 4 m \omega_d x_d B_{q,max} l_{co} \cdot \frac{g^2}{4 \epsilon_0 W V_{DC} \Delta V l_0 x_d} = \frac{g^2 m \omega_d B_{q,max} l_{co}}{\epsilon_0 W V_{DC} \Delta V l_0} = 4.28 > 1$$

This result tells us that the compensating torque T_{QC} is not enough (by a factor of about 4) to compensate the quadrature error.

[2] $F_1, \dots, F_4$ are numbered left to right, top to bottom.