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Question n. 1

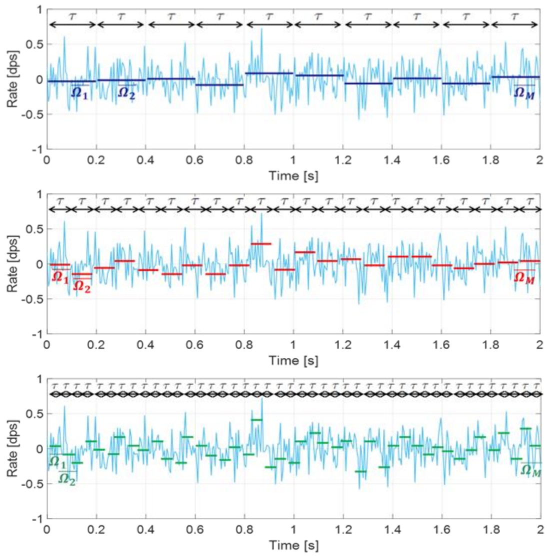
The root Allan variance is a commonly adopted technique to characterize inertial sensors. Discuss the operative measurement procedure, present typical sample graphs and indicate how to use them for a quick evaluation of the performance of inertial sensors.

What do you expect in your graphs if, while measuring the root Allan variance, you apply a temperature change to two sensors, one with good and one with poor offset drift coefficient with temperature?

The root Allan variance is a widely adopted method to characterize inertial sensors as it simultaneously and rapidly gives information about short-term (sensor noise) and long-term (offset drift and stability) performance. This is particularly intriguing for navigation applications, where short-term noise results in random walk, and offset drifts result in navigation error due to offset errors accumulation.

From an operative point of view, the Allan variance can be obtained by capturing the sensor output (under no external stimulus, rotations or accelerations) for a certain amount of time. This time slot should be significantly large to capture effects of offset drifts induced by environmental changes (e.g. temperature or humidity). So, it is typically of several tens of minutes to hours or even days, depending on the target application and on the required stability performance of the sensor.

This interval is split into a certain number M of sub-intervals of duration τ. Within each interval M, the average value of the quantity of interest (measured acceleration or rate) x_k is calculated. Similarly to a standard variance, the Allan variance accounts for the mean squared error between the considered sample and another value: this value is however the mean sample of the next slot, x_{k+1}. For the considered observation time τ, the definition thus yields:



$$\sigma_{AV}^2 = \frac{1}{2(M-1)} \sum_{k=1}^{M-1} (\overline{x_{k+1}} - \overline{x_k})^2$$

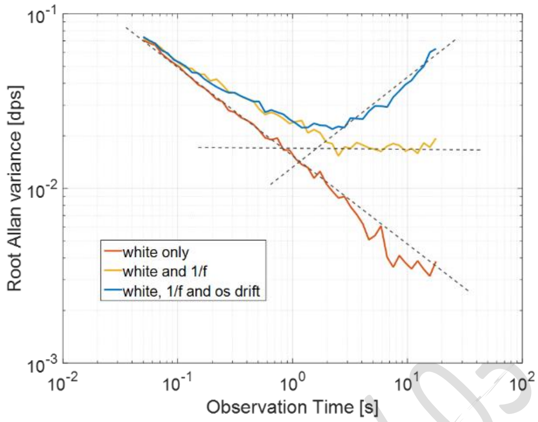
(note: for M intervals there are M-1 differences; further, the factor 2 at the denominator accounts for the fact that both the samples are noisy, unlike in a standard variance calculation).

The procedure is then repeated at several different observation times τ (as schematically shown in the figure) and the root Allan variance graph is obtained by plotting σ_{AV} versus the observation time. Usually, the maximum observation time is 1/10 of the measurement time (to allow at least 10 samples to average on), and the minimum observation time should be larger than twice the sampling time, just to cope with the sampling theorem and avoid aliasing.

Given the two average (integration) operations and the derivative (difference) operation, the AV method corresponds to apply a band-pass filter with bandwidth proportional to $1/\tau$.

A typical resulting root Allan variance graph is shown aside. One can distinguish three major regions. The first one shows a slope of -10 dB/decade, and corresponds to white noise. Indeed, the longer the observation time, the lower the bandwidth of the BPF and the larger the number of samples on which noise is averaged. This is consistent with an observed noise reduction by a factor proportional to the square root of the considered samples.

The second region is flat, and corresponds to 1/f noise contributions in the system: indeed, a BOF with a bandwidth proportional to $1/\tau$ applied to a noise contribution proportional to 1/f gives a constant value.



The third region is related to very slow changes of the output (offset drifts), which can be induced by environmental changes. Often, this can be modeled as a sort of $1/f^2$ noise, and indeed appear as a growing contribution with slope +10 dB/decade. The point in the curve that reaches the minimum, is named the stability of the sensor. E.g. the gyroscope shown in the figure has a stability of 25 mdps/√Hz at about 3 s observation time.

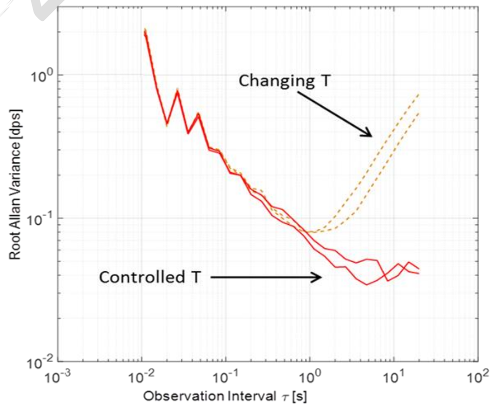
The root Allan variance can be conveniently used to measure sensor noise. Indeed, there is a direct link between the white and 1/f noise coefficients of the noise power spectral density, and the values measured by the AV graph:

$$\sigma^2_{AV,\,\Omega}(\tau) = \frac{S_{\Omega,W}}{2\,\tau}$$

$$\sigma^2_{AV,\,\Omega}(\tau) = 2\,\alpha_n\,log(2)$$

Finally, if we consider two sensors with similar noise performance, but different offset drift coefficient, we will observe that the right region of the AV graph of the "bad" sensor worsens in presence of temperature changes (as indicated in the figure), while it remains substantially stable for the sensor with "good" temperature drift coefficient.

Correspondingly, the "bad" sensor will see a stability point which is higher and/or anticipated in terms of observation time.



Last Name __*Dall'Esito* ___ **Given Name** ___*Addolorata*____ **ID Number** ___**03092018**____

Question n. 2

You are designing an ultra-low-pixel-size 3T CMOS image sensor, and you want to estimate the foreseen performance. The design parameters are given in the table:

Square pixel side L_p	1.8 µm
Fill factor FF	0.4
Silicon quantum efficiency η_{si}	0.6
Filter transmittance T_{CFA}	0.8
Depletion region depth x_{dep}	1 µm
MOS transistor length and width L_{MOS}	200 nm
MOS oxide thickness t_{Ox}	10 nm
Pixel bias voltage V_{DD}	2.5 V
Integration time t_{int}	2.9 ms
Pixel dark current i_d	0.05 fA
Photo-response nonuniformity $\sigma_{PRNU\%}$	7 %

(i) calculate the expected signal-independent noise contribution, in terms of electrons rms;
(ii) calculate the charge (in electrons) that makes the pixel saturate, and the corresponding photon flux;
(iii) draw a clearly quoted photon transfer curve (PTC), in units of electrons along both axes (in particular, quote those points where the three different noise slopes intersect);
(iv) evaluate the DR from the represented photon transfer curve.

Physical Constants
$k_b = 1.38\ 10^{-23}$ J/K
$q = 1.6\ 10^{-19}$ C
$\varepsilon_0 \cdot \varepsilon_{Si} = 8.85\ 10^{-12} \cdot 11.7$ F/m
$\varepsilon_0 \cdot \varepsilon_{Ox} = 8.85\ 10^{-12} \cdot 4$ F/m
T = 300 K

(i)

Signal-independent noise contributions are represented by reset noise, dark current shot noise and dark-signal nonuniformity. About the latter we are given no information, so we assume it to be negligible. Dark current shot noise is immediately calculated in terms of number of electrons as:

$$\sigma_{el,dark} = \frac{\sqrt{q \cdot i_d \cdot t_{int}}}{q} = 0.95\ e_{rms}$$

For the calculation of reset noise, we need to evaluate the capacitance affecting the anode node, formed by the depletion capacitance and the MOS capacitance (we assume it as the full gate to channel capacitance). Taking into account the fill factor for the depletion capacitance, they are calculated as:

$$C_{dep} = \frac{\epsilon_0 \epsilon_{Si} A_p FF}{x_{dep}} = \frac{\epsilon_0 \epsilon_{Si} \left(L_p\right)^2 FF}{x_{dep}} = 0.134\ fF$$

$$C_{MOS} = \frac{\epsilon_0 \epsilon_{Ox} (L_{MOS})^2}{t_{ox}} = 0.142\ fF$$

So that the reset noise contribution in terms of electrons can be calculate as:

$$\sigma_{el,kTC} = \frac{\sqrt{k_B T \left(C_{dep} + C_{MOS}\right)}}{q} = 6.68\ e_{rms}$$

We see that reset noise dominates over dark current shot noise at the given integration time. The overall signal-independent shot noise is the quadratic sum of these two contributions (6.75 e_{rms}), which substantially equals the reset noise.

(ii)

Neglecting overdrive drops across the transistors, we assume the full voltage drop to be V_{DD}. In this conditions, the maximum number of electrons that can be measured by the pixel is readily calculated as:

$$N_{el,max} = \frac{Q_{max}}{q} = \frac{V_{DD}\left(C_{dep} + C_{MOS}\right)}{q} = 4309\ e$$

From this number and from the equation that relates the photon flux to the photocurrent of the pixel we can calculate the maximum acceptable photon flux at the given integration time:

$$i_{ph,max} = \Phi_{max}\, q\, \eta\, T_{Si}\, (L_P)^2$$

$$\frac{i_{ph,max} t_{int}}{q} = \frac{Q_{max}}{q} = N_{el,max} = \Phi_{max}\, \eta\, T_{Si}\, (L_P)^2\, t_{int}$$

$$\Phi_{max} = \frac{N_{el,max}}{\eta\, T_{Si}\, (L_P)^2\, t_{int}} = 9.55\; 10^{17}\, \frac{ph}{s\, m^2}$$

In this calculation there is an assumption that the pixels feature microlenses, so the FF does not appear in the formula. The assumption is reasonable because the pixel size is very small. In absence of microlenses, the same calculation would yield a photon flux increased by 1/0.4, i.e. $2.39\; 10^{18}\, \frac{ph}{s\, m^2}$.

(iii)

To draw the graph, we calculate the relevant points. This can be done by equating the three different noise contributions.

Signal independent noise crosses photon shot noise when the signal (in terms of charge) is:

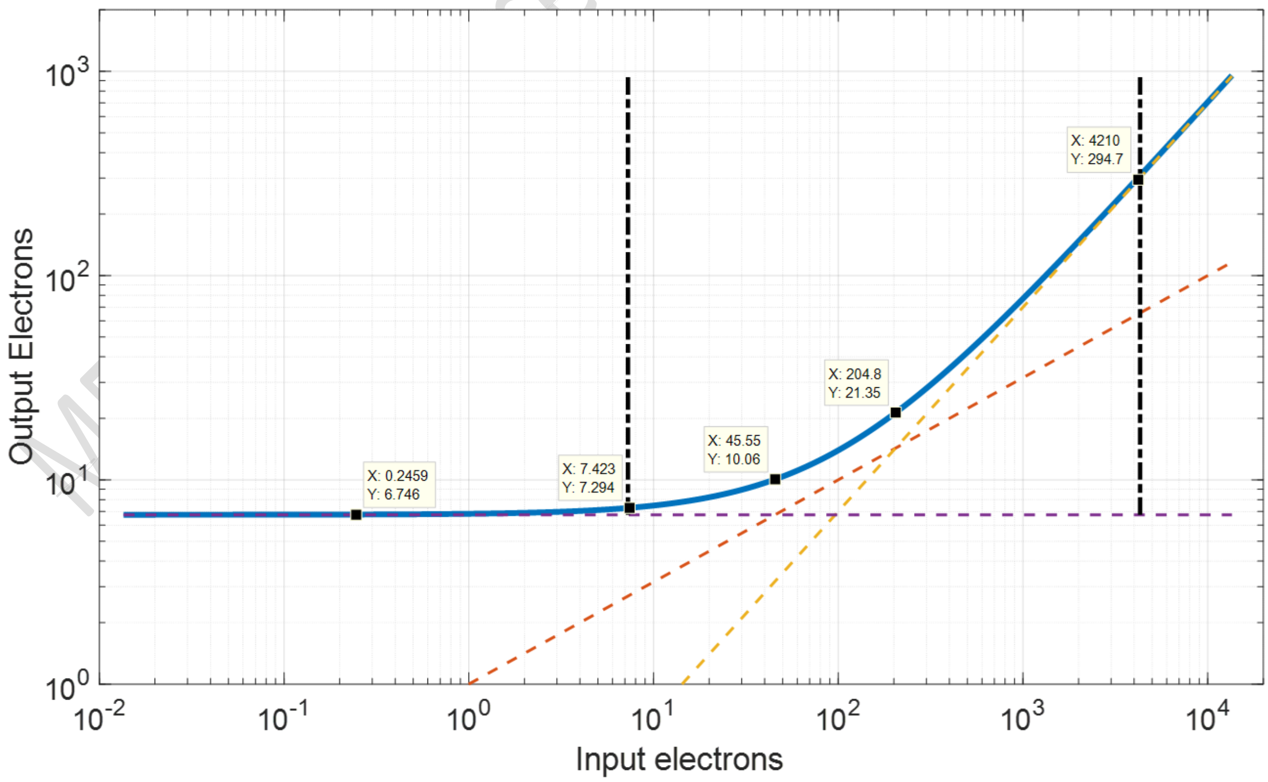
$$\frac{\sqrt{q\, i_{ph} t_{int}}}{q} = 6.75\; e_{rms} \;\rightarrow\; \frac{i_{ph} t_{int}}{q} = (6.75\,)^2 = 45.5\; e$$

In this point, the overall noise is √2*6.75 e_{rms} = 9.54 e_{rms}.

Photon shot noise equals PRNU noise when the signal (in terms of charge) is:

$$\frac{\sqrt{q\, i_{ph} t_{int}}}{q} = \sigma_{PRNU,\%} \frac{i_{ph} t_{int}}{q} \;\rightarrow\; \frac{i_{ph} t_{int}}{q} = \frac{1}{\sigma^2_{PRNU,\%}} = 204\; e$$

In this point, the overall noise is $\frac{\sqrt{2q\, i_{ph} t_{int}}}{q}$ = √408 e_{rms} = 20.2 e_{rms}. Considering the signal independent noise floor and the saturated, both already calculated, the graph is readily drawn. The black lines indicate the points where S=N and of saturation.



(iv)

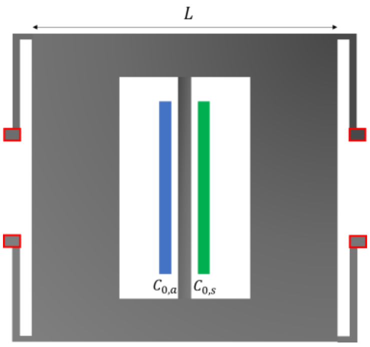
The DR is immediately evaluated from the graph by taking the ratio between the signal corresponding to saturation and the signal corresponding to S=N. Note that this signal is slightly larger than the value of signal-independent noise: this means that photon shot noise is (though very small) not completely negligible in this condition. The result is:

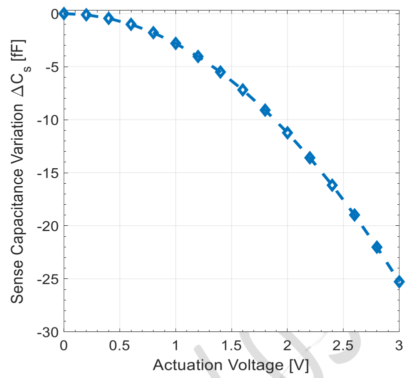
$$DR = 20\log_{10}\frac{N_{el,max}}{N_{el,min}} = 20\log_{10}\frac{4309}{7.3} = 55.5\ dB$$

Last Name ___***Dall'Esito***___ **Given Name** ___***Addolorata***___ **ID Number** ___**03092018**___

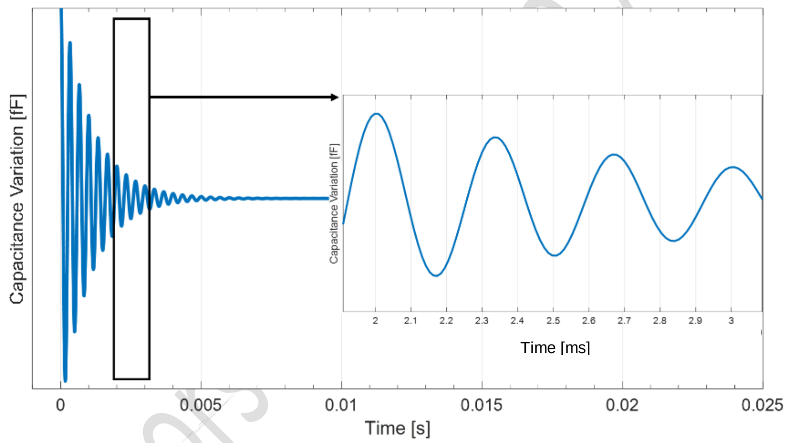
Question n. 3

You are analyzing the results of the electromechanical characterization of a square-shaped MEMS accelerometer, performed using one set of parallel plate electrodes as the actuation port, and the other set as the sensing port:





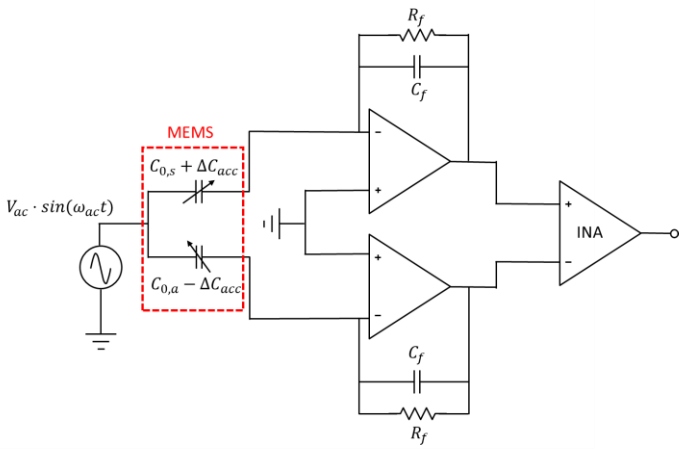
- **(i)** from the shown ring-down measurement, extract the accelerometer stiffness ***k***;
- **(ii)** can you also determine the intrinsic ***NEAD*** (in [µg/√Hz])?
- **(iii)** from the given C-V curve, derive the rest gap of the parallel plate electrodes.



At this point, you can couple your sensor with the electronic readout chain shown in figure, now using both parallel plates for differential sensing:

- **(iv)** find the value of V_{ac} that optimizes the output swing at the rail-to-rail INA output.

Actuation rest capacitance $C_{0,a}$	200 fF
Sense rest capacitance $C_{0,s}$	200 fF
Accelerometer side length L	300 µm
Effective density ρ_{eff} (already includes holes of PP cells)	$0.8 \cdot 2320$ kg/m^3
Process height H	30 µm
Required FSR	32 g
Feedback capacitor C_f	200 fF
INA Gain G_{ina}	5
INA supply voltage V_{DD}	±3 V



Physical Constants

q = 1.6 10^{-19} C
k_b = 1.38 10^{-23} J/K
T = 300 K (if not specified)
ε_0 = 8.85 10^{-12} F/m

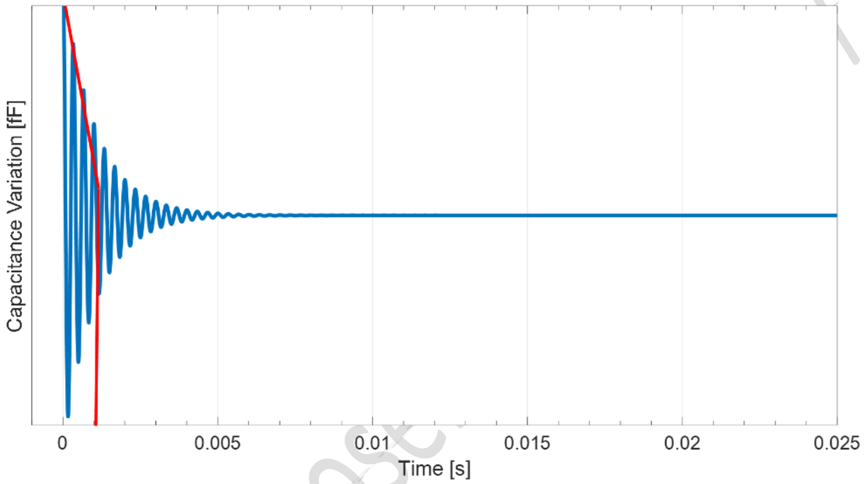
(i) First of all, the mass of the accelerometer can be calculated as:

$$m = \rho_{eff} \cdot L^2 \cdot H = 5\, nkg$$

In the zoomed area of the ring-down graph, we can count 3 periods in 3 ms and hence the resonant frequency f_0 turns to be equal to 3 kHz. The stiffness is thus readily obtained:

$$\omega_0 = \sqrt{\frac{k}{m}} \rightarrow k = \omega_0^2 m = 1.78\, N/m$$

(ii) From the ring down measurement, we can graphically extract the information about the exponential decay time constant, obtaining a $\tau \sim 1$ ms:



Thus, the corresponding quality factor can be calculated using the relation:

$$\tau = \frac{Q}{\pi f_0} \rightarrow Q \sim 10$$

and, consequently, the NEAD is readily obtained:

$$NEAD = \sqrt{\frac{k_b T \omega_0}{mQ}} = 8\, \mu g/\sqrt{Hz}$$

(iii) The C-V characterization curve is obtained applying an increasing voltage V_a to the actuation port and reading the capacitive variation at the sense electrode. Given the voltage drop, the rotor experiences an electrostatic force:

$$F = \frac{V_a^2}{2} \cdot \frac{C_0}{g}$$

Consequently, the static displacement of the proof mass will be:

$$x = \frac{F}{k} = \frac{V_a^2}{2} \cdot \frac{C_0}{g} \cdot \frac{1}{k}$$

The linearized capacitance variation at the sense port is given by:

$$C_s = C_{0,s} - \Delta C_s = C_{0,s} - \frac{dC}{dx} \cdot x = C_{0,s} - \frac{C_0}{g} \cdot x$$

Substituting the expression of the displacement into the last equation, and rearranging the terms, the gap can be obtained as:

$$g = \sqrt{\frac{C_0^2 V_a^2}{2k\Delta C_s}}$$

Choosing a point on the graph and substituting its coordinates in the equation, *g = 2 µm* is obtained.

(iv) The well-known accelerometer readout scheme presents a sensitivity equal to:

$$S = \frac{1}{\omega_0^2} \cdot \frac{2C_0}{g} \cdot \frac{V_{ac}}{C_f} G_{ina}$$

The INA output has to reach the supply voltage for an input acceleration equal to the full-scale-range. Thus:

$$FSR \cdot S = V_{DD} \rightarrow V_{ac} = 0.68\, V$$