Last Name _Di Tredici_ **Given Name** _Venere_ **ID Number** __20180713__

Question n. 1

Discuss noise in MEMS accelerometers, indicating the different physical sources and which design parameters you can act on (with corresponding trade-offs) to optimize the result as a function of the final application.

The first contribution to the total noise of an accelerometer is the thermomechanical noise. Gas molecules inside the MEMS package, randomly excited by thermal agitation, hit the seismic mass applying a random "fluctuation" force that results in mechanical noise. It can be demonstrated that the noise force density (in [N/√Hz]) associated to this statistical phenomenon is given by:

$$\sqrt{S_{F,th}} = \sqrt{4k_B T b}$$

where k_B is the Boltzmann constant and b is the damping coefficient of the structure. This contribution can be input-referred as follows:

$$NEAD = \sqrt{S_{a,in}} = \sqrt{\frac{4k_B T b}{m^2}} = \sqrt{\frac{4k_B T \omega_0}{Q\, m}}$$

The thermomechanical noise can be improved by raising the accelerometer mass (as expected for an inertial sensor…) but it can be difficult if area occupation constraints are tight.

The damping coefficient, typically dominated by parallel plates squeezed-film damping, can be lowered by acting on the MEMS encapsulating pressure. Here, another trade off arises: optimization of the bandwidth requires ideally a Q in the order of 0.5 (in between overdamped and underdamped conditions) while, on the other hand, noise optimization requires high Q (and so, a low b).

For what concerns electronic noise, the critical stage is represented by the front-end amplifier. From the noise analysis of the trans-capacitance stage, we know that the input-referred feedback resistor (R_f) noise is given by:

$$\sqrt{S_{R,out}} = \sqrt{\frac{4k_B T}{R_f}} \cdot \frac{1}{sC_f} \cdot \frac{1}{SF}$$

where C_f is the feedback capacitance and SF is the sensitivity in [V/g]. The resistance can be typically designed high enough (e.g. with MOS pseudo-resistors) in order to make this contribution negligible.

Considering now the operational amplifier, we can typically neglect its input-referred current noise, while we should consider its voltage noise, that can be brought to the system input as:

$$\sqrt{S_{v,out}} = \sqrt{S_{v,in}} \cdot \left(1 + \frac{C_p}{C_f}\right) \cdot \frac{1}{SF}$$

This contribution can be minimized by lowering S_{vin} (at the cost of increased power consumption), decreasing C_p (designing carefully the interconnections between MEMS and electronics) and raising the sensitivity (coping as well with FSR specifications due to linearity constraints).

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Question n. 2

A 3T-topology CMOS camera features the parameters indicated in the Table. You have to test it, comparing predictions and experiments.

Pixel size $\boldsymbol{l_p}$	2 µm
Fill factor ***FF***	0.35
Silicon quantum efficiency $\boldsymbol{\eta_{si}}$	0.6
Filter transmittance $\boldsymbol{T_{CFA}}$	0.6
Pixel well p-type doping $\boldsymbol{N_A}$	$4 \cdot 10^{21}$ m^{-3}
Pixel built-in voltage $\boldsymbol{V_{BI}}$	0.7 V
Pixel bias voltage $\boldsymbol{V_{DD}}$	3 V
Parasitic capacitance at the gate node $\boldsymbol{C_G}$	0.5 fF
Integration time $\boldsymbol{t_{int}}$	10 ms
Pixel dark current $\boldsymbol{i_d}$	0.2 fA
Pixel microlenses	Yes

(i) Calculate the pixel conversion gain (in µV/electron) assuming initially a very low input photon signal.

(ii) Assuming a linear response for the pixel, with the conversion gain calculated above, evaluate the input photon flux (number of photons per unit area and unit time) impinging on the pixel surface that makes the output voltage change by half of the maximum swing.

(iii) You now use a calibrated photon source to shine the camera with the just calculated photon flux. However, you find that the output voltage drops by less than the value calculated above. After explaining why, estimate roughly how much less this drop can be with respect to predictions.

(iv) Evaluate the signal-to-noise ratio (SNR) in the conditions considered at point (iii).

Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K
$h = 6.63\ 10^{-34}$ J s
$c = 3\ 10^{8}$ m/s
$q = 1.6\ 10^{-19}$ C
$\varepsilon_0 \cdot \varepsilon_{Si} = 8.85\ 10^{-12} \cdot 11.7$ F/m
T = 300 K

(i)

For small input signals, the pixel can be considered as a reverse biased PN junction with a voltage equal to V_{DD}. The junction depletion capacitance can be calculated starting from the expression of the depletion region of a unilateral junction:

$$x_{dep} = \sqrt{\frac{2\epsilon_0\epsilon_{Si}(V_{DD}+V_{BI})}{q\ N_A}} = 1.1\ \mu m$$

Taking into account the fact that the diode occupies only an area equal to the overall pixel area multiplied by the fill factor, the depletion capacitance can be calculated as:

$$C_{dep} = \frac{\epsilon_0\epsilon_{Si}A_{pd}}{x_{dep}} = \frac{\epsilon_0\epsilon_{Si}A_{pix}FF}{x_{dep}} = \frac{\epsilon_0\epsilon_{Si}\ l_p^2\ FF}{x_{dep}} = 0.13\ fF$$

So that the overall capacitance across which the charge is integrated (process of direct integration) is:

$$C_{int} = C_g + C_{dep} = 0.63\ fF$$

The conversion gain, in terms of output voltage change per single collected electron, is easily calculated as:

$$CG = \frac{q}{(C_g + C_{dep})} = 253\ \frac{\mu V}{e^-}$$

(ii)

For an ideal pixel (i.e. neglecting transistors voltage drops) the dynamic is the supply voltage, so half the dynamic is V_{DD}/2=1.5 V. The number of electrons that cause such a drop is:

$$N_{e^-} = \frac{\Delta V_{out}}{CG} = \frac{1.5\ V}{252\ \frac{\mu V}{e^-}} = 5930\ e^-$$

We then know that the relationship between the number of impinging photons and the number of collected electrons passes through the quantum efficiency. Note that the presence of microlenses let us neglect the loss caused by the non-unitary fill factor:

$$Q_{tot} = qN_{e^-} = \phi_{in}\, q\, l_p^2\, \eta_{Si}\, T_{CFA}\, t_{int} \rightarrow \phi_{in} = \frac{N_{e^-}}{l_p^2\, \eta_{Si}\, T_{CFA}\, t_{int}} = 4.1 \cdot 10^{17} \frac{ph}{s\, m^2} = 4.1 \cdot 10^{5} \frac{ph}{s\, \mu m^2}$$

(iii)

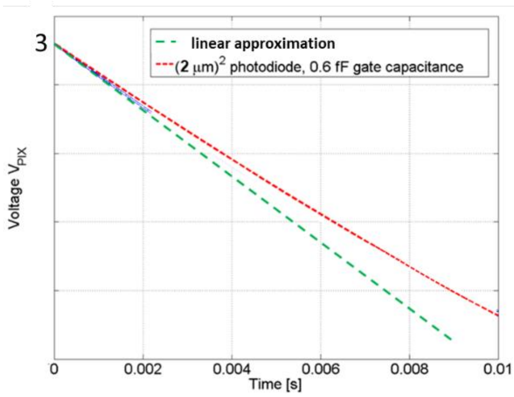
The assumption done in the calculations so far is that the pixel response is linear. This is not true, as the depletion capacitance changes during integration because of the changing voltage across the junction. As an example, when the pixel voltage has effectively undergone a 1.5-V drop, the conversion gain becomes:

$$CG = \frac{q}{\left(C_g + C_{dep,1.5}\right)} = \frac{q}{\left(C_g + \frac{\epsilon_0 \epsilon_{Si}\, l_p^2\, FF}{x_{dep,1.5}}\right)} = \frac{q}{\left(C_g + \frac{\epsilon_0 \epsilon_{Si}\, l_p^2\, FF}{\sqrt{\frac{2\epsilon_0 \epsilon_{Si} (\frac{V_{DD}}{2} + V_{BI})}{q\, N_A}}}\right)} = \frac{q}{\left(C_g + 0.17\, fF\right)} = 238\ \frac{\mu V}{e^-}$$

Indicating that, effectively, as long as the pixel integrates light its conversion gain decreases and thus the slope of the voltage ramp decreases (see the figure aside). It is hard to estimate the effective decrease for the input photon flux calculated at point (ii). We know that with the just calculated conversion gain the drop would be:

$$\Delta V_{out} = N_{e^-}\, CG = 5930 \cdot 238 \frac{\mu V}{e^-} = 1.41\ V$$

The actual voltage drop will be in between 1.5 V and the value calculated above.



(iv)

In the just-examined conditions, the SNR can be calculated as:

$$SNR = 20 \log_{10} \frac{q\, N_{e^-}}{\sqrt{q(i_d\, t_{int} + q\, N_{e^-}) + k_B T C_{int}}} = 37.7\ dB$$

Note that there is no significant different if we assume the depletion capacitance for small signals or for large signals, as the capacitance at the integration node is dominated by the gate parasitic.

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Question n. 3

A MEMS gyroscope based on a tuning-fork topology, operated in mode-split conditions, is coupled to the driving/readout circuit as shown. In the design phase of the system, there are some parameters that still need to be dimensioned.

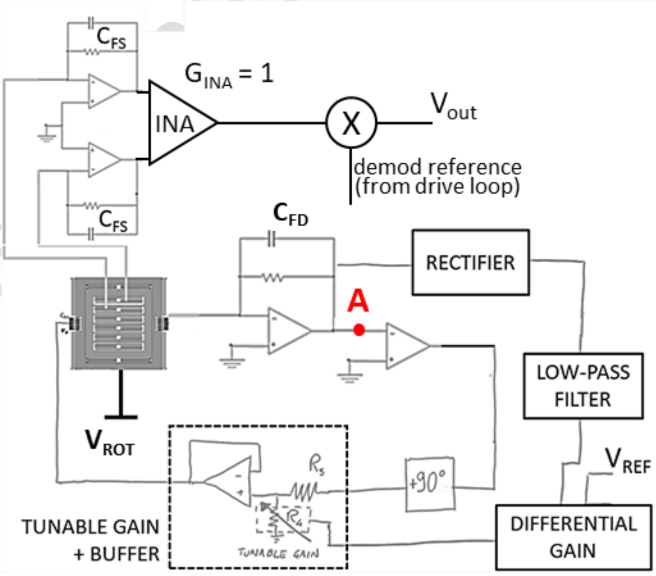
Target full-scale range Ω_{FSR}	±2000 dps
Maximum quadrature $\Omega_{B,max}$	±5000 dps
Drive displacement x_d	5 µm
Mode split value Δf_{ms}	1.2 kHz
Sense PP capacitance (s.e.) C_{0PP}	200 fF
PP gap g	2 µm
Circuit supply voltage V_{BIAS}	±3 V
Rotor voltage V_{rot}	12 V
Amplifier voltage noise S_{Vn}	$(36\ nV/\sqrt{Hz})^2$
Parasitic capacitance C_P	0.5 pF
Drive-detection transduction η_D	$80\cdot10^{-9}$ A/(m/s)

(i) Assuming initially no quadrature compensation, calculate the optimum scale factor, such that the output voltage dynamic is best exploited. Then, find the corresponding optimum value of the feedback capacitance C_{FS} of the sense chain.

(ii) Calculate the optimum drive capacitance C_{FD} to minimize phase noise, the latter being given by the expression (in [rad²/Hz]): $S_{pn} = \frac{S_{vn_{out,CA,drive}}}{(v_{out,CA,drive})^2/2}$, where the numerator $S_{vn_{out,CA,drive}}$ indicates the noise power voltage density (in [V²/Hz]) at the output of the charge amplifier of the drive loop (point A), and the term $v_{out,CA,drive}$ indicates the amplitude of the voltage sinusoidal waveform in the same point A of the circuit. In the found optimum condition, evaluate the phase noise value S_{pn}.

(iii) Consider now the sense chain. Calculate the input-referred noise contribution given by the voltage noise of the charge amplifiers of the sense chain (neglect thermomechanical noise).

(iv) Assume now that you want to design quadrature compensation electrodes. With the results of points (ii) and (iii), find the acceptable residual quadrature B_q such that the input-referred noise contribution induced by phase noise and quadrature matches the input-referred noise found for the sense chain at point (iii).



Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K

T = 300 K

(i)

In the optimum conditions, the signal (and quadrature) will fill the voltage dynamics. As the two contributions are phase shifted by 90°, their sum should be intended as a sum of the modulus and is thus done quadratically. This yields:

$$SF_{dps} = \frac{V_{DD}}{\sqrt{\Omega_{max}^2 + B_{q,max}^2}} = \frac{3\ V}{\sqrt{2000^2 + 5000^2}\ dps} = 0.56\frac{mV}{dps}$$

We know that the scale factor (the sensitivity) is related to the feedback capacitance of the sense chain through the formula:

$$\frac{V_{out}}{\Omega} = SF_{dps} = 2\frac{V_{BIAS}}{C_{FS}}\frac{C_0}{g}\frac{x_D}{\Delta\omega_{MS}}\frac{\pi}{180}$$

By inverting the equation and forcing the sensitivity calculated above, we find:

$$C_{FS} = 2V_{BIAS}\frac{C_0}{g}\frac{x_D}{\Delta\omega_{MS}}\frac{1}{SF_{dps}}\frac{\pi}{180} = 50\,fF$$

(ii)

The solution of this point starts by making explicit the expression of phase noise as a function of the amplifier noise and of the conversion from drive motion into signal at the point A. Note that the signal corresponds to the motional current i_m multiplied by the drive feedback impedance (1/ sC_{FD}):

$$S_{Pn} = \frac{S_{Vn}\left(1+\frac{C_P}{C_{FD}}\right)^2}{\left(v_{out,CA,drive}\right)^2/2} = \frac{S_{Vn}\left(1+\frac{C_P}{C_{FD}}\right)^2}{\left(\frac{i_m}{s\,C_{FD}}\right)^2/2} = \frac{S_{Vn}\left(1+\frac{C_P}{C_{FD}}\right)^2}{\left(\frac{\eta_d s\,x_D}{s\,C_{FD}}\right)^2/2} = \frac{S_{Vn}\left(1+\frac{C_P}{C_{FD}}\right)^2}{\left(\frac{\eta_d x_D}{C_{FD}}\right)^2/2}$$

The dependence on the drive detection capacitance can be made clearer as:

$$S_{Pn} = \frac{S_{Vn}}{(\eta_d x_D)^2/2}\left(\frac{1+\frac{C_P}{C_{FD}}}{\frac{1}{C_{FD}}}\right)^2 = \frac{S_{Vn}}{(\eta_d x_D)^2/2}(C_{FD}+C_P)^2$$

Which indicates that the minimum feedback capacitance for the drive amplifier will minimize phase noise. However, we should be sure that the output voltage of this amplifier does not saturate. So, the minimum drive capacitance is the one that makes the amplifier output signal correspond exactly to the circuit supply (in other words, this will give a sinewave at the drive amplifier output with an amplitude at the saturation limit):

$$\frac{\eta_d x_D}{C_{FD}} = V_{BIAS} \rightarrow C_{FD} = \frac{\eta_d x_D}{V_{BIAS}} = 133\,fF$$

As this capacitance is compatible with typically available values, we choose to operate with the found C_{FD}. In this condition, phase noise is minimized and, from the expressions above, turns out to be:

$$S_{Pn} = \frac{S_{Vn}}{(\eta_d x_D)^2/2}\,(C_{FD}+C_P)^2 = 1.3\cdot 10^{-14}\frac{rad^2}{Hz}$$

(iii)

Input-referred-noise generated by the sense-chain amplifier voltage noise can be calculated by bringing this noise contribution to the circuit output, and then dividing by the scale factor (the factor 2 accounts for the differential configuration, with two uncorrelated noise sources):

$$S_{Vn,CA,out} = 2S_{Vn}\left(1+\frac{C_P}{C_{FS}}\right)^2 \rightarrow \sqrt{S_{Vn,CA,dps}} = \sqrt{\frac{2S_{Vn}\left(1+\frac{C_P}{C_{FS}}\right)^2}{SF_{dps}^2}} = 1\frac{mdps}{\sqrt{Hz}}$$

(note how, using the sensitivity in [V/dps], we directly obtain the input noise density in dps/√Hz).

(iv)

We know that phase noise can bring an additional input-referred noise contribution $\sqrt{S_{Pn,in}}$ (in dps/√Hz) through quadrature. Its expression (writte in dps$_{rms}$ through the bandwidth) can be written as:

$$\sqrt{S_{Pn,in}}\sqrt{BW} = \sin\left(\sqrt{S_{Pn}\,BW}\right) \cdot B_q = \sin(\varphi_{noise}) \cdot B_q \qquad (see^*)$$

where BW is the frequency of the system output filter, which limits the bandwidth. By assuming small phase noise (i.e. small phase fluctuations compared to 360°, which is reasonable), we can approximate the sine function with its argument:

$$\sqrt{S_{Pn,in}}\sqrt{BW} \approx \sqrt{S_{Pn}\,BW} \cdot B_q \;\rightarrow\; \sqrt{S_{Pn,in}} = \sqrt{S_{Pn}BW} \cdot \frac{B_q}{\sqrt{BW}} = \sqrt{S_{Pn}} \cdot \frac{B_q}{\sqrt{Hz}}$$

(note that now the expression $\sqrt{S_{Pn}\,BW}$ should be intended as dimensionless, as it is the approximated result of a sine operation: therefore the simplification of BW leaves the $\sqrt{Hz}$ at the denominator)**.

We need to force this contribution to be identical to the noise density found at point (iii) above. This equivalence can be calculated as:

$$\sqrt{S_{Pn}} \cdot \frac{B_q}{\sqrt{Hz}} = \sqrt{S_{Vn,CA,dps}} \rightarrow B_{q,max} = \frac{\sqrt{S_{Vn,CA,dps}}\,\sqrt{Hz}}{\sqrt{S_{Pn}}} = 5\;dps$$

phase noise, indicated by this expression, is discussed in the sixth lesson about gyroscopes in the course slides

**note that we are approximating phase noise as white. In reality, phase noise will be itself filtered by the drive loop bandwidth, but this goes beyond the purpose of this exercise and of the course.*