Last Name _PROVA_ **Given Name** __SELVAGGIA_ **ID Number** _20180628__

Question n. 1

Explain what the so-called *quadrature error* in MEMS gyroscopes is. Focus at least on (i) its origin, (ii) its input-referred expression, (iii) its effects on the system performance and (iv) its compensation techniques.

(i)

An imperfect MEMS process may result in a drive resonator whose excited motion follows a direction not exactly orthogonal to the sense mode. As a consequence, there is a continuous oscillating motion along the sensing direction, during drive motion, even in absence of external angular rate. The origin of this phenomenon lies mostly in etching nonuniformities (asymmetric spring terms, asymmetric comb finger gaps) and in the skew angle issue (non-orthogonality of the sidewalls). As the Coriolis force is proportional to the velocity of the drive motion, while this contribution is proportional to the displacement, the latter is in quadrature with the signal, so one can look for ways to compensate it.

(ii)

An input referred expression for quadrature can be found by defining an equivalent quadrature force F_q that acts in the sense direction, proportional to the motion in the drive direction through a cross-spring term k_{ds}:

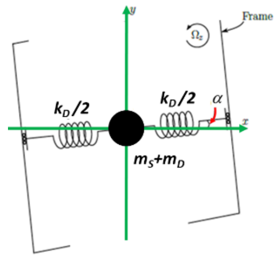
$$F_q = k_{ds}\, x$$

This can be inserted in the sense motion equation:

$$m_S\ddot{y} + b_s\dot{y} + k_s y + k_{ds}x = -2m_s\Omega\dot{x}$$

After simplifications, one finds that the equivalent, input-referred quadrature can be written as:

$$B_q = \frac{k_{ds}}{2\, m_S\, \omega_D}$$



Which can be finally rearranged by accounting for the dependence of the cross-spring term k_{ds} on the angle α representing the deviation of the drive motion from the ideal trajectory:

$$B_q \approx \frac{\alpha}{2}\omega_D$$

This tells us that – though it is good to have a high resonance frequency to stay out of the audio bandwidth and far from environmental vibrations – the choice of the operating frequency cannot be unbounded towards high values.

(iii)

If demodulation is operated by an ideal waveform $\cos(\omega_D \cdot t)$ (noiseless and with the correct and constant phase), in principle quadrature error can be bypassed. If saturation within the electronic chain before demodulation is avoided, then quadrature does not represent an issue.

However, a real demodulation waveform $\cos(\omega_D \cdot t + \varphi_{err} + \varphi_{noise})$ always includes a phase error (which may drift in time) and is affected by phase noise. The result thus yields:

$$V_{dem} = S\left[\Omega\cos(\omega_D\, t) + B_q\,\sin(\omega_D\, t)\right]\cdot\cos(\omega_D t + \varphi_{err} + \varphi_{noise})\cdot LPF \approx$$

$$\approx S\left[\Omega\cos(\varphi_{err} + \varphi_{noise}) + B_q\,\sin(\varphi_{err} + \varphi_{noise})\right] = S\cdot\Omega\; + S\cdot B_q\cdot\varphi_{err} + S\cdot B_q\cdot\varphi_{noise}$$

The expression above shows how mechanical nonidealities (quadrature), coupled to electronics demodulation nonidealities (phase offset and noise) bring to the output additional noise and additional offset terms. Note also that if the relative phase of the demodulation waveform changes, then an offset drift is observed.

(iv)

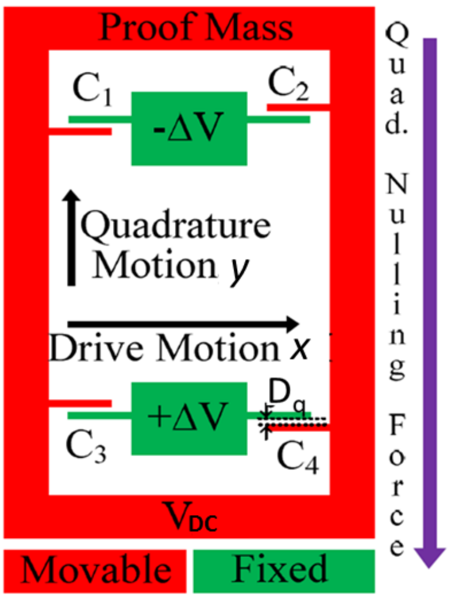
To mitigate quadrature, one can pursue one of the following strategies:

- design optimization (symmetricity, good spring topology, wide springs, optimized resonance frequency…);
- electronic compensation: inject a signal equal and opposite with respect to quadrature in the sense chain. This requires a one-time calibration. Residual drifts due to imperfect calibration are not compensated by this technique;
- electromechanical compensation: according to the Tatar scheme, shown below, a suitable design of additional electrodes within the Coriolis frame leads to a force that:

$$F_{QC} = -4\frac{\varepsilon_0\cdot h}{D_Q^2}V_{DC}\,N_{QC}\,\Delta V\,x$$

  - is proportional to the drive amplitude x;
  - is orthogonal to the drive direction;
  - has a modulus that depends of the values of V_{DC}, ΔV and the number of plates N_{QC};
  - has a sign which can be set by choosing the sign of ΔV.

  This force can be thus initially trimmed to compensate quadrature.



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Question n. 2

A manufacturer of CMOS digital imaging cameras based on the 3T topology sells two products which have the identical parameters given in the Table, except for the presence of a CFA for camera #1, and the absence of a CFA for camera #2 (which is thus a black&white camera). For both the cameras:

(i) evaluate the maximum dynamic range (neglect quantization noise);

(ii) consider now quantization noise: draw a quoted graph of the maximum DR as a function of the n. of bits of the ADC. Choose the number of bits such that the maximum DR remains larger than 62 dB;

(iii) evaluate now the average SNR under a 555-nm, monochromatic optical intensity (power per unit area) of 36 mW/m^2, evenly impinging on the whole sensor.

Pixel side (square shape)	3 µm
Fill factor	0.5
Microlenses	Yes
Silicon quantum efficiency (at 555 nm)	0.7
RGB Filter transmittance (at 555 nm)	0.2 – 0.5 – 0.1
RGB arrangement	Bayer GRGB
Dark current density	0.4 mA/m^2
Parasitic capacitance at gate node	0.466 fF
Depletion region width	1 µm
Supply voltage	3 V
Integration time range of the camera	1 ms – 10 s

Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K

$q = 1.6\ 10^{-19}$ C

$h = 6.626\ 10^{-34}$ Js

$c = 3\ 10^{8}$ m/s

$\varepsilon_{Si} = 11.7$

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

(i)

The DR is a characteristic parameter of the pixel, independent of the amount of incoming signal and so, in this case, independent of the type of camera that we consider. Its calculation requires the estimation of the maximum and minimum measurable signals, limited by saturation and signal-independent noise respectively.

The presence of microlenses indicates that the area for the depletion capacitance calculation and the dark current should take into account the factor FF = 0.5. We evaluate both quantities as:

$$i_d = j_d \cdot A \cdot FF = j_d \cdot l_p^2 \cdot FF = 0.4\ 10^{-3}\frac{A}{m^2}\ (3\ 10^{-6})\ m^2 \cdot 0.5 = 1.8\ fA$$

$$C_{dep} = \frac{(\epsilon_0 \epsilon_{si} \cdot A \cdot FF)}{x_{dep}} = \frac{\left(8.85\ 10^{-12}\dfrac{F}{m}\ \ 11.7\ \ (3\ 10^{-6})\ m^2 \cdot 0.5\right)}{10^{-6}\ m} = 0.47\ fF$$

The total integration capacitance becomes thus:

$$C_{int} = C_{dep} + C_g = 0.93\ fF$$

At this point we can write the maximum DR, which should be calculated at the minimum integration time (i.e. when shot noise is at its minimum value):

$$DR = 20\log_{10}\left(\frac{V_{dd}\,C_{int}}{\sqrt{k_B T C_{int} + q\; i_d t_{int,min}}}\right) = \mathbf{62.75\;dB}$$

(ii)

With a negligible n. of bits, the DR will reach its maximum value. If the n. of bits is too small, quantization noise will begin affecting the DR. The formula above should be modified by accounting for quantization noise as:

$$\sigma_{q,C}^2 = \frac{LSB^2}{12}\,C_{int}^2 = \frac{V_{dd}^2}{2^{2n}12}\,C_{int}^2$$

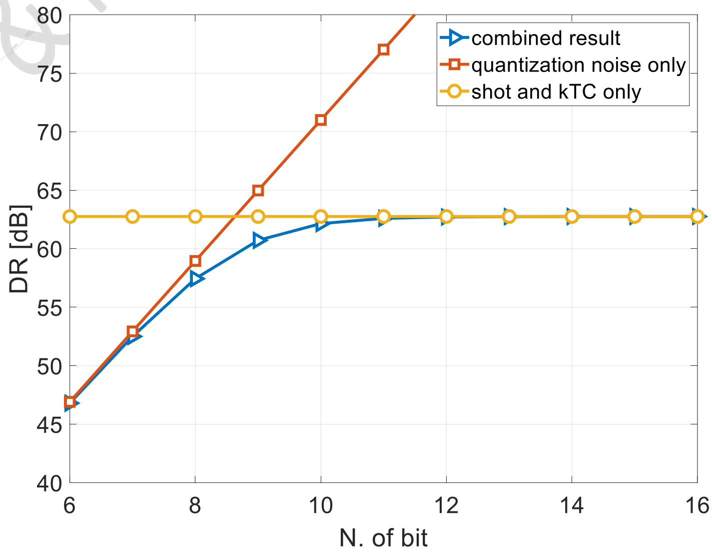
$$DR = 20\log_{10}\left(\frac{V_{dd}\,C_{int}}{\sqrt{k_B T C_{int} + q\; i_d t_{int,min} + \frac{V_{dd}^2}{2^{2n}12}\,C_{int}^2}}\right)$$

Inverting the formula above and setting DR = 62 dB, one obtains a minimum number of **10 bits** to satisfy the condition.

For what concerns the graph, assuming that quantization noise dominates, the DR expression becomes linear with 2^n:

$$DR_{quant-only} = 20\log_{10}\left(\frac{V_{dd}\,C_{int}}{\frac{V_{dd}}{2^n\sqrt{12}}\,C_{int}}\right) = 20\log_{10}\left(2^n\sqrt{12}\right)$$

The graph is thus the combination of DR as calculated at point (i), and DR as expressed by the simplified formula above:



(iii)

For the last point, one can calculate the photocurrent on the four different pixels as:

$$i_{ph} = \Re \cdot T \cdot A \cdot I$$

where *I* is the impinging optical intensity, *A* is the full pixel area (the FF issue is bypassed through microlenses), *T* is the filter transmittance (1 for the BW case) and $\Re$ is the responsivity:

$$\Re = \frac{\eta q \lambda}{hc} = 0.313\ A/W$$

The currents are thus

$$i_{ph,BW} = 0.313 \frac{A}{W}\ (3\mu m\ )^2 \cdot 36 \frac{mW}{m^2} = 101\ fA$$

$$i_{ph,R} = 0.313 \frac{A}{W}\ (3\mu m\ )^2 \cdot 36 \frac{mW}{m^2} 0.2 = 20\ fA$$

$$i_{ph,G} = 0.313 \frac{A}{W}\ (3\mu m\ )^2 \cdot 36 \frac{mW}{m^2} 0.5 = 50\ fA$$

$$i_{ph,B} = 0.313 \frac{A}{W}\ (3\mu m\ )^2 \cdot 36 \frac{mW}{m^2} 0.1 = 10\ fA$$

Noise can be calculated using the above evaluated current values for the photon shot noise contribution. We choose the same integration time as for the other points (you could calculate this point at any integration time, as this was not specified). We also use the just-evaluated n. of bits (10) for quantization noise.

In the calculation of the SNR, when looking at different GRGB pixels, signals sum up linearly (twice the G contribution and once the B and R contributions), while noise sums up quadratically. As for the CFA case we are considering four pixels, to make a fair comparison in terms of SNR we should bin four pixels also for the BW case. We thus get:

$$SNR_{BW-4pixel} = 20 \log_{10} \left(\frac{404\ fA}{\sqrt{4k_B T C_{int} + q(404\ fA + i_d) + 4\sigma_{q,C}^2}} \right) = 32.9$$

$$SNR_{CFA-4pixel} = 20 \log_{10} \left(\frac{130 fA}{\sqrt{4k_B T C_{int} + q(130\ fA + i_d) + 4\sigma_{q,C}^2}} \right) = 26.3$$

As expected, CFA noise has, on average, a worse SNR than the BW case.

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Question n. 3

Consider the accelerometer and the related front-end electronics sketched in figure 1.

(i) Using the data reported in the Table, and knowing that the maximum acceptable linearity error is equal to 1%, determine the optimum sensitivity, expressed in [mV/g];

(ii) Find the required input voltage noise density of the front-end operational amplifiers in order to obtain a system resolution of 50 µg/√Hz.

Consider now a mechanical offset of the proof mass, x_{os} = 100 nm, affecting the parallel plate cells as depicted in figure 2.

(iii) Consider two cases: a 20 g and a -20 g acceleration, both directed along the x axis. Calculate the differential capacitance variation in the two cases.

(iv) Starting from results of point (iii), consider a vibration (i.e. a sinusoidal acceleration) of amplitude 20g at about 28 kHz. What is the input-referred system output (in g) after the 50-Hz low-pass-filter (LPF), if no other accelerations are applied to the sensor?

Accelerometer parameters	
Parallel-plate rest gap	1.5 µm
Mass	5 10^{-9} kg
AC bias amplitude (V_{AC})	1 V
Required FSR	± 5 g
Parallel plates area (single-ended)	60000 $(\mu m)^2$
Quality factor	1.5
Electronics parameters	
Feedback capacitance	1 pF
Parasitic capacitance	10 pF
INA Gain	1

Physical Constants

q = 1.6 10^{-19} C
k_b = 1.38 10^{-23} J/K
T = 300 K (if not specified)
ε_0 = 8.85 10^{-12} F/m

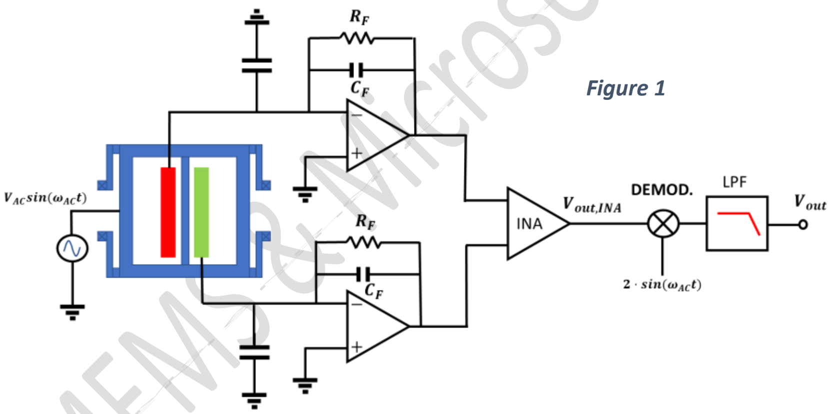

Figure 1

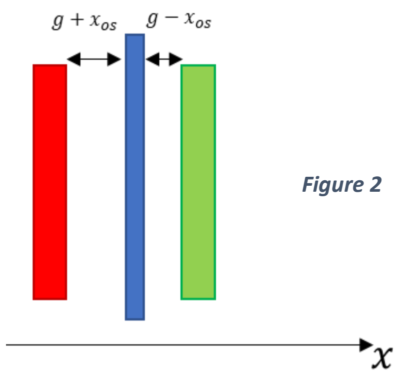

Figure 2

(i) Knowing the rest gap and the linearity error specification at the FSR, we can derive the full-scale displacement of the accelerometer:

$$x_{FSR} = g \cdot \sqrt{\frac{\epsilon_{lin,\%}}{100}} = 150\ nm$$

And consequently, the accelerometer resonant frequency:

$$\frac{x_{FSR}}{a_{FSR}} = \frac{1}{\omega_0^2} \rightarrow f_0 = 2\pi \sqrt{\frac{a_{FSR}}{x_{FSR}}} = 2.8\ kHz$$

At this point, the calculation of the sensitivity is straightforward:

$$S = \frac{1}{\omega_0^2} \cdot \frac{\Delta C}{\Delta x} \cdot \frac{\Delta V}{\Delta C} = \frac{1}{\omega_0^2} \cdot \frac{2C_0}{g} \cdot \frac{V_{ac}}{C_f} \cdot \mathrm{G_{INA}} = \ 14\ \frac{mV}{g}$$

(ii) We can start calculating the thermomechanical noise contribution:

$$NEAD = \sqrt{\frac{4k_B T \omega_0}{mQ}} = 20\ \mu g/\sqrt{Hz}$$

This term will not be the dominant one in the noise budget. Let's consider only the operational amplifier voltage noise:

$$S_{eln,in} = \sqrt{2S_v} \cdot \frac{\left(1 + \frac{C_p}{C_f}\right)}{S} = 50\frac{\mu g}{\sqrt{Hz}} \rightarrow S_v = 45\ nV/\sqrt{Hz}$$

(iii) First, it can be noted that the input acceleration of ±20 g is outside the ±5 g linear range, so we cannot linearize the capacitance variation without a non-negligible error. The displacement induced by this kind of acceleration is equal to:

$$= \frac{\pm 20 \cdot 9.8\ m/s^2}{\omega_0^2} = \ \pm 600\ nm$$

We can simply calculate the differential capacitance variation (around the new working point given by the mechanical offset), for an acceleration of +20 g:

$$\Delta C_{1,+20g} = \frac{\epsilon_0 A_{pp}}{g + x_{os} + x_{+20g}} - \frac{\epsilon_0 A_{pp}}{g + x_{os}}$$

$$\Delta C_{2,+20g} = \frac{\epsilon_0 A_{pp}}{g - x_{os} + x_{+20g}} - \frac{\epsilon_0 A_{pp}}{g - x_{os}}$$

$$\Delta C_{diff,+20g} = \Delta C_{2,+20g} - \Delta C_{1,+20g} = \ -90.5\ fF$$

And for an acceleration of -20g:

$$\Delta C_{1,-20g} = \frac{\epsilon_0 A_{pp}}{g + x_{os} + x_{-20g}} - \frac{\epsilon_0 A_{pp}}{g + x_{os}}$$

$$\Delta C_{2,-20g} = \frac{\epsilon_0 A_{pp}}{g - x_{os} + x_{-20g}} - \frac{\epsilon_0 A_{pp}}{g - x_{os}}$$

$$\Delta C_{diff,-20g} = \Delta C_{2,-20g} - \Delta C_{1,-20g} = 199.1\ fF$$

Thus, the mechanical offset determines an asymmetry in our sensitivity.

(iv) The external acceleration is at 28 kHz, one decade beyond the accelerometer resonant frequency. Given the -40 dB/dec attenuation of the MEMS transfer function for $\omega > \omega_0$, the displacement at 28 kHz will be:

$$x_{28\ kHz,\pm 20g} = \frac{\pm 20 \cdot 9.8\ m/s^2}{\omega_0^2} \cdot \frac{1}{100} = \pm 6\ nm$$

We can calculate the two capacitance variations for the peak value of the sinusoidal acceleration with the same procedure of point (iii), using this time the updated displacement value:

$$\Delta C_{diff,28kHz,+20g} = \Delta C_{2,28kHz,+20g} - \Delta C_{1,28kHz,+20g} = 2.872\ fF$$

$$\Delta C_{diff,28kHz,-20g} = \Delta C_{2,28kHz,-20g} - \Delta C_{1,28kHz,-20g} = -2.868\ fF$$

The distorted sinusoidal capacitance variation will have a non-null mean value:

$$\Delta C_{mean} = \frac{\Delta C_{diff,28kHz,+20g} + \Delta C_{diff,28kHz,-20g}}{2} = 0.023\ fF$$

That will be not filtered by the LPF. Thus, the offset given by the external vibration corresponds to an acceleration of:

$$a_{in,VRE} = \Delta C_{mean} \cdot \frac{V_{AC}}{C_f} \cdot \frac{1}{S} = 162\ \mu g$$

Comparable with the system resolution. This kind of error is known as VRE (Vibration Rectification Error).