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Question n. 1

After giving the expression of the signal to noise ratio in a CMOS image sensor, draw a typical SNR vs input photocharge graph in presence of all the noise sources that you have studied in the course, for a 3T topology. Describe clearly how the sensor dynamic range can be extracted from such a graph. Finally, indicate qualitative variations in the graph that you can expect when passing to a 4T topology.

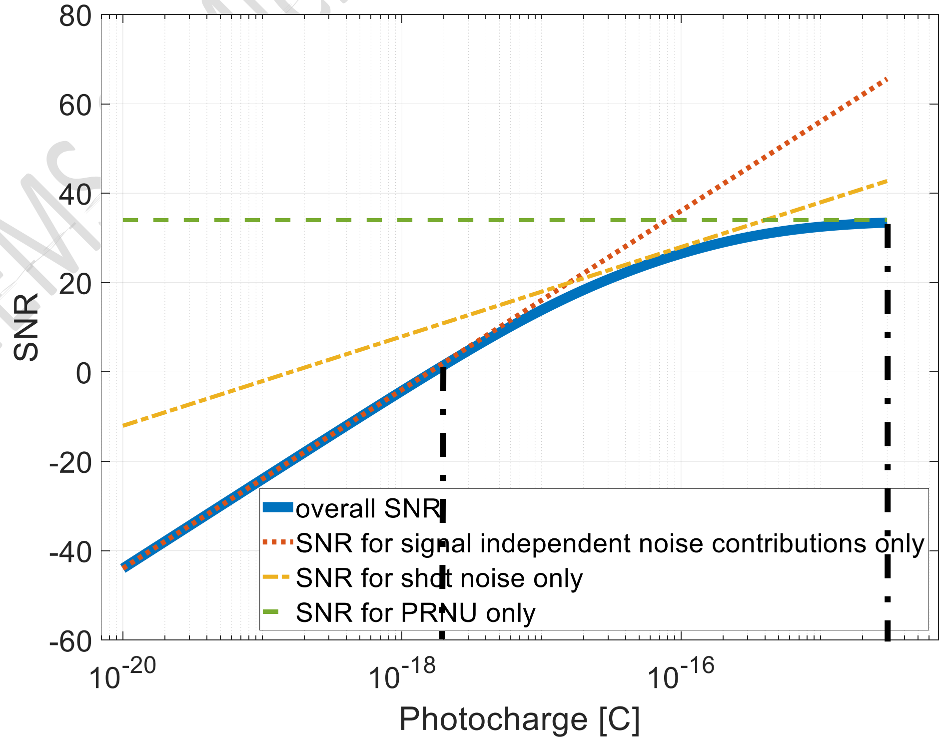
The SNR can be easily described with an equation, in terms of charge, which uses the photocurrent integrated over the integration time at the numerator, and all (spatial and temporal) noise contributions at the denominator:

$$SNR = 20\log_{10}\frac{i_{ph}\cdot t_{int}}{\sqrt{q(i_{ph}+i_d)t_{int}+k_bT(C_g+C_{dep})+\sigma_q^2+(i_d t_{int}\,DSNU_{\%})^2+(i_{ph}t_{int}\,PRNU_{\%})^2}}$$

$$=\frac{Q_{ph}}{\sqrt{q(Q_{ph}+Q_d)+k_bT(C_g+C_{dep})+\sigma_q^2+(Q_d\,DSNU_{\%})^2+(Q_{ph}\,PRNU_{\%})^2}}$$

We can highlight three different kinds of terms:

- signal independent noise includes kTC noise, dark current shot noise, quantization noise and DSNU. All these contributions are independent of the input photocharge. Therefore, the SNR grows linearly with photocharge when this noise contributions dominate, which generally happens at low signals;
- a signal dependent noise source associated to photocurrent shot noise: the SNR goes with the square root of the photocharge when this contribution dominates;
- a signal dependent noise source associated to photoresponse nonuniformity. The SNR does not change with photocharge when



this contribution dominates, which generally occurs at large signal.

A sample plot of the SNR vs the photocharge is reported above. Note that this plot is NOT the photon transfer curve, though it is very similar to it in terms of content and information that it brings.

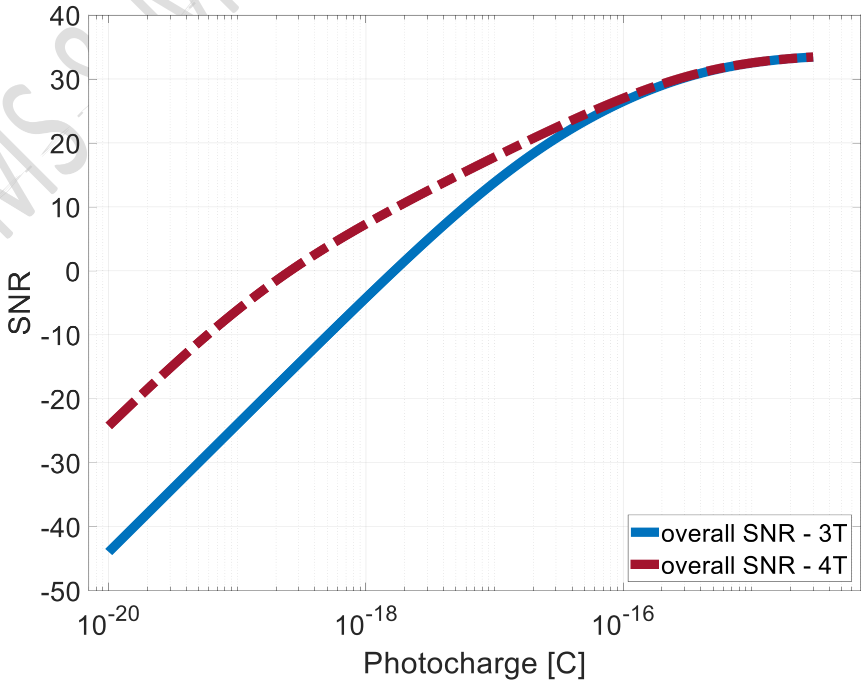
At a charge value corresponding to the maximum charge that can be integrated in the photodiode (the value can be approximated to the product of the biasing voltage times the sum of the capacitances affecting the anode node), the signal saturates and the SNR looses meaning (dropping essentially to 0…).

The DR can be quantified by looking at two points on the x-axis of this graph: the first point is the just mentioned situation where the pixel saturates. The second point is where the SNR equals 0 dB (unity). The ratio of the photocharges in these two situations gives the DR, as shown in the graph. In this example, the DR can be calculated as $20\log_{10}(3\ 10^{-15}/2\ 10^{-18}) = 62$ dB.

When using a 4T topology, we can expect

(i) a reduction in the dark current due to the pinning effect of the pinned photodiode, reducing collected dark current from dirty interface regions;
(ii) a consistent reduction of kTC noise, if correlated double sampling is used (this technique is indeed only effective in 4T topologies).

As a consequence, we can expect a better SNR in the signal-independent region of the SNR. In the figure below, a factor 10 of reduction in dark current and in kTC noise is assumed as a qualitative example. The SNR grows by 20 dB in this region.



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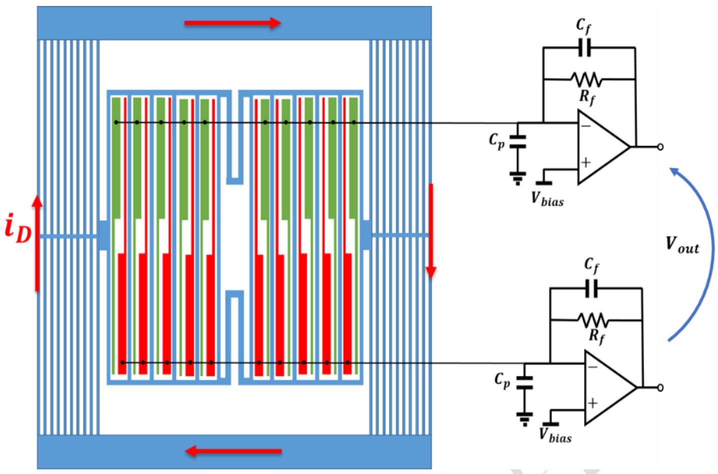
Question n. 2

You have to design a Lorentz-force Z-axis MEMS magnetometer, like the one depicted in the figure. The frequency of the current flowing in the multi-loop recirculation path is slightly mismatched with respect to the resonant frequency of the structure.



(i) Evaluate the sensitivity in terms of output voltage V_{OUT} per unit magnetic field (in [V/T]), making reasonable assumptions for the unknown parameters.

(ii) Choose the input referred voltage noise density (S_V, in [V/√Hz]) of the readout operational amplifier, in order to obtain a well-balanced system in terms of noise.

(iii) Choose the optimum number of bits of the ADC that you will place at the end of the electronic readout chain.

(iv) After this first design stage, you notice that your resolution is not sufficient for the target application, but you are allowed to consume 50 µA more. Do you prefer to assign this additional current to the driving current or to the front-end amplifier input transistors?

Driving current	100 µArms
Parallel plates length	300 µm
# of parallel plates cells (whole structure)	10
Process height	26 µm
Parallel plates gap	1.5 µm
Sensing bandwidth	50 Hz
Stiffness (half structure)	60 N/m
Resonant frequency	20 kHz
Stator bias voltage	6 V
# of current loops	10
Spring length	800 µm
Quality factor	3000
Feedback capacitance	1 pF
Parasitic capacitance at the sense node	15 pF
Required full scale range	±5 mT

Physical Constants

$k_b = 1.38 \cdot 10^{-23}$ J/K

T = 300 k

(i) The expression of the sensitivity of a Lorentz force multi-loop magnetometer driven with a current slightly mismatched from the device resonance is:

$$S = \frac{\Delta V_{out}}{\Delta B} = i_{peak} N_{loop} L \cdot \frac{Q_{eff}}{2k_{1/2}} \cdot \frac{2C_0}{g} \cdot \frac{V_{bias}}{C_f} = 2.3\,\frac{V}{T}$$

Where $i_{peak} = \sqrt{2} \cdot i_{rms}$ is the peak value of the driving current, $C_0 = \frac{\epsilon_0 L_{pp} H N_{pp}}{g}$ is the single- ended rest capacitance of the whole structure, $Q_{eff} = \frac{f_0}{2\Delta f}$ is the effective quality factor computed considering a reasonable mismatch value of $BW \cdot 3 = 150\,Hz$, in order to satisfy the sensing bandwidth requirements.

(ii) A system can be assumed "well-balanced" in terms of noise if the two main noise contributions to the total noise (thermo-mechanical and electronics) are comparable. For what concerns the device intrinsic noise, we know its expression:

$$NEMD_{intr} = \frac{4}{iLN_{loop}}\sqrt{k_B T b} = \frac{4}{iLN_{loop}}\sqrt{k_B T \cdot \frac{2k_{1/2}}{\omega_0 Q}}$$

On the other hand, the input-referred front-end electronics contribution, considering negligible the current noise of the opamp and the feedback resistance thermal noise, is equal to:

$$NEMD_{eln} = \sqrt{2S_v} \cdot \frac{\left(1 + \frac{C_p}{C_f}\right)}{S}$$

(the factor 2 is due to the presence of two amplifiers). Equating the two contributions, we obtain $S_v = 13\,\frac{nV}{\sqrt{Hz}}$.

(iii) In order to calculate in a straightforward way the number of bits to correctly quantize our signal, we just consider resolution and full-scale-range in terms of magnetic field.

The required full-scale-range is given in the data, and it is equal to ±5 mT. The resolution can be obtained from the previously calculated noise density and from the bandwidth specification:

$$B_{min} = \sqrt{(NEMD_{eln}^2 + NEMD_{intr}^2) \cdot BW} = \sqrt{2} \cdot NEMD_{intr} \cdot \sqrt{BW} = 1.28\ \mu T$$

And thus:

$$2^{N_{bit}} = \frac{FSR}{B_{min}} = \frac{10\ mT}{1.28\ \mu T} \rightarrow N_{bit} = 13$$

(iv) The most convenient choice is to assign the additional current to the MEMS driving. Indeed, we can note that both the $NEMD_{intr}$ (proportional to $\frac{1}{i_{peak}}$) and $NEMD_{eln}$ (proportional to $\frac{1}{S}$ and thus to $\frac{1}{i_{peak}}$) benefits from a drive current increasing.

On the other hand, increasing the current in the electronics front-end would decrease the S_v and consequently only the electronics contribution, leaving untouched the MEMS intrinsic noise.

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Question n. 3

Drive frame mass	$3\cdot10^{-9}$ kg
Sense frame mass	$10\cdot10^{-9}$ kg
Drive CF capacitance	300 fF
CF overlap length	13 µm
Sense PP capacitance (s.e.)	180 fF
PP gap	2 µm
Reference drive voltage V_{REF}	1.48 V
In operation rotor voltage V_{DC}	10 V
Drive feedback capacitance C_F	1 pF
Sense feedback capacitance C_{FS}	1 pF
Maximum quadrature	1200 dps

A MEMS gyroscope is designed with the parameters listed in the Table. Once fabricated, the gyroscope is subject to electromechanical characterization tests.

- **(i)** From the first test, the simple ringdown curves shown in Fig. 1, evaluate the natural resonance frequency and the Q factor of the two modes.
- **(ii)** The gyroscope is designed to operate with the circuit shown in Fig. 2. Calculate the gyroscope drive displacement.
- **(iii)** Estimate the gyroscope sensitivity (amplitude of the voltage V_{OUT} per unit rate, in [V/dps]).
- **(iv)** As a final characterization test, you adopt the measurement scheme indicated in Fig. 3, where the amplitude v_a is set identical to the one obtained in operation. Draw a quoted graph of the amplitude of the voltage V_{OUT} as a function of the frequency f_a of the small actuation signal v_a.

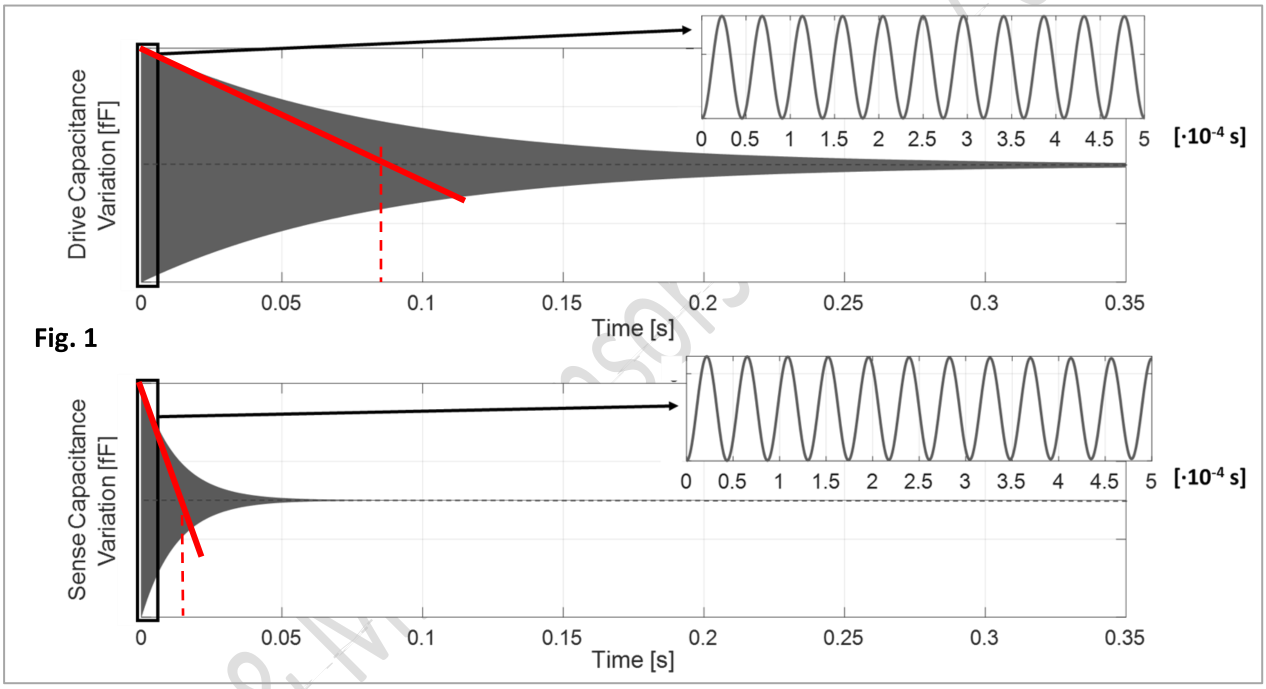


Fig. 1

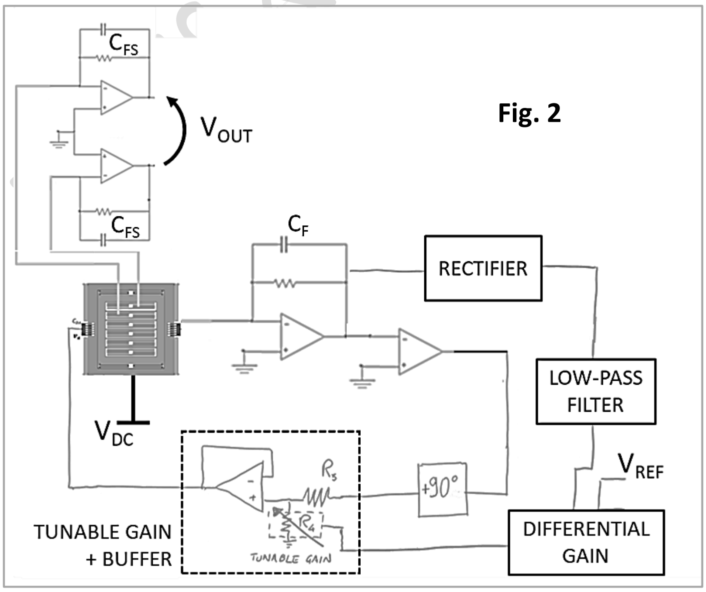


Fig. 2

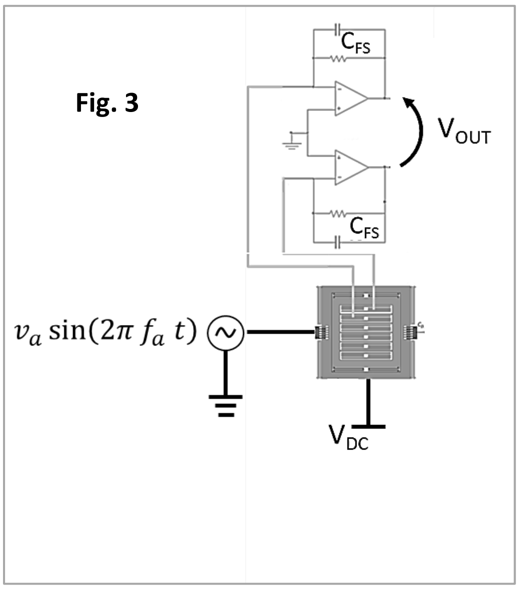


Fig. 3

(i)

From the ringdown graph of each mode we can derive the resonance frequency and the quality factor. For the frequency, the relevant information is that appearing in the zoomed part of the image, where we find 11 periods in 500 μs for the drive mode, and 11.5 periods in the same time for the sense mode. This gives us a resonant frequency of

$$f_d = \frac{11}{500\,\mu s} = 22\,kHz$$

$$f_s = \frac{11.5}{500\,\mu s} = 23\,kHz$$

And in turn a native mode split of 1 kHz. The Q factors are easily identified from the ringdown time constant. Indeed we know that for a given mode at a resonance f_0, the quality factor and the time constant are related through:

$$\tau = \frac{Q}{\pi\, f_0}$$

which yields (using the time constant values approximately found in the graphs) a value of

$$Q_d = 0.085\,s \cdot \pi \cdot 22\,kHz \approx 6000$$

$$Q_s = 0.015\,s \cdot \pi \cdot 23\,kHz \approx 1000$$

(ii)

We see that the circuit includes an AGC. Therefore, we know that the drive motion is regulated through the value of the reference voltage V_{REF}. As we are using a charge amplifier configuration (it is the only configuration that satisfies the Barkhausen conditions with the shown building blocks of the drive loop), we know that the motion is transformed into a voltage in the secondary loop through the equation below:

$$V_{out,LPF} = i_m \frac{1}{\omega_d C_F}\frac{2}{\pi} = \eta x \omega_d \frac{1}{\omega_d C_F}\frac{2}{\pi} = \eta x \frac{1}{C_F}\frac{2}{\pi}$$

where i_m is the motional current and η is the transduction factor of the comb-finger capacitance of the drive mode. The secondary loop is a negative feedback that forces this voltage to match V_{REF}. Therefore, we find:

$$x_d = \frac{V_{REF} C_F}{\eta}\frac{\pi}{2} = \frac{V_{REF} C_F}{V_{ROT}\; C_{0d}/x_{ol}}\frac{\pi}{2} = 10\,\mu m$$

(iii)

The gyroscope sensitivity in mode-split operation is found as:

$$\frac{\Delta V_{out}}{\Omega} = 2 \cdot \frac{C_{0s}}{g} \cdot \frac{V_{ROT}}{C_{FS}} \cdot \frac{x_d}{\Delta\omega}$$

Apparently, we have all the parameters to do this calculation. However, the exact split value needs to take into account not the natural frequencies, but the actual frequencies in operation. Due to the presence of the rotor voltage, the sense mode is subject to electrostatic softening, whose equivalent stiffness is given by:

$$k_{elec} = -2V_{rot}^2 \frac{C_{0s}}{g^2} = -9\ N/m$$

We thus need to add this value to the mechanical stiffness, given by:

$$\omega_s = \sqrt{\frac{k_s}{m_s}} \ \rightarrow \ k_s = \omega_s^2 m_s = 209 \frac{N}{m}$$

To obtain the actual mechanical stiffness of the sense mode of 200 N/m. This gives an in-operation frequency of 22.5 kHz, and thus a split value of 500 Hz. With this number, the differential sensitivity becomes

$$\frac{\Delta V_{out}}{\Omega} = 2 \cdot \frac{C_{0s}}{g} \cdot \frac{V_{ROT}}{C_{FS}} \cdot \frac{x_d}{\Delta\omega} = 2 \cdot \frac{180\ fF}{2\ \mu m} \cdot \frac{10\ V}{1\ pF} \cdot \frac{10\ \mu m}{2\ \pi\ 500\ \text{Hz}} = 5.8 \frac{mV}{rad/s}$$

Which is more conveniently given in units of dps as

$$\left.\frac{\Delta V_{out}}{\Omega}\right|_{dps} = \frac{\Delta V_{out}}{\Omega} \frac{\pi}{180} = 100 \frac{\mu V}{dps}$$

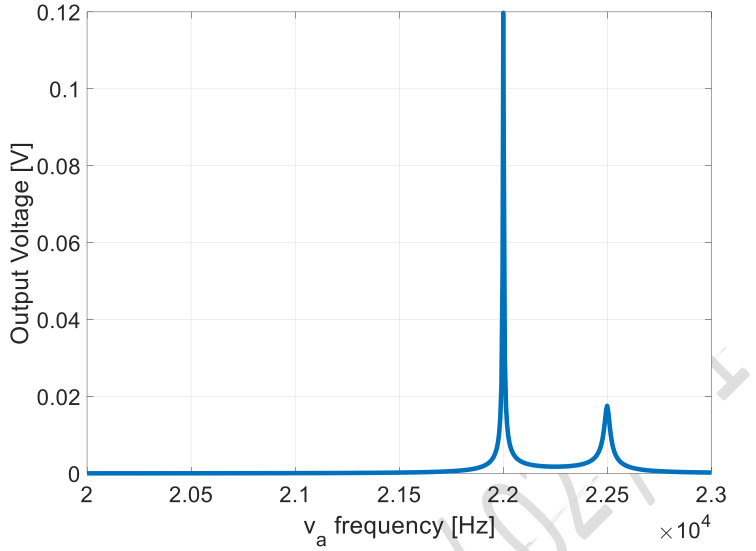
(iv)

We assume the amplitude of the voltage v_a to match the first harmonic of the square wave in operation. Further, we assume no angular rate applied during this characterization phase. The only contribution that links the drive and sense motion is quadrature.

For low frequencies, the drive mode will move much less that for resonant motion. When passing through the drive frequency, the drive mode will see its maximum displacement (which at resonance matches exactly the one in operation).

In absence of quadrature, nothing relates the drive and sense motion. In presence of quadrature, the output voltage for a signal v_a at the drive mode frequency will match the product of the sensitivity calculated above times the input-referred quadrature, so:

$$\Delta V_{out,\omega_d} = 1200\ dps \cdot 100 \frac{\mu V}{dps} = 0.12\ V$$



When moving forward in frequency the signal v_a, we pass also through the sense frequency. Here the drive motion will be lower with respect to the case above by a factor equal to Q_{eff}/Q_d. However, the quadrature force will be now amplified by the full Q factor of the sense mode, and thus increased by a factor Q_s/Q_{eff}. As a consequence, the output voltage can be easily quantified as the voltage calculated above, multiplied by a factor Q_s/Q_d.:

$$\Delta V_{out,\omega_s} = 0.12\ V \cdot \frac{Q_s}{Q_d} = 0.02\ V$$

(as an alternative solution, the full equations of the drive and sense modes could be used for this calculation, which explains the presence of the drive mass in the data of the exercise). The graph above reports thus the required plot.

This technique is commonly used to characterize quadrature in gyroscopes at wafer level.