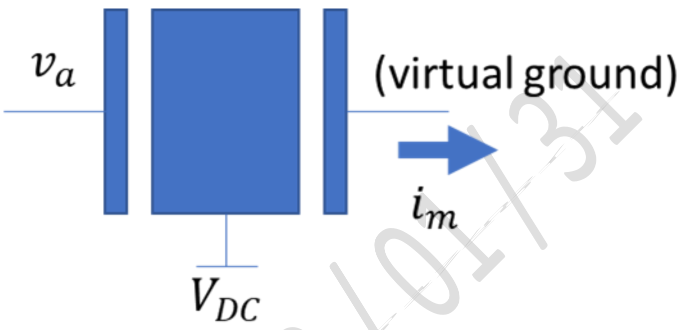
Last Name __Minotti__ **Given Name** __Paolo__ **ID Number** __20180131__

Question n. 1

Draw and describe the simplest electrical equivalent model of a 3-port MEMS resonator, and its frequency behavior. Introduce possible nonidealities and indicate which effects they have on the behavior of the overall oscillator. Indicate how the impact of these effects changes as a function of the Q factor value. You can assist your discussion with circuit schematics, graphs and block schemes, when helpful.



A three-port MEMS resonator is formed by a capacitive actuation port, the rotor port and a sensing port. In a typical in-operation configuration, the system is biased as in the image, with a high DC voltage on the rotor (>> v_a) so to linearize the applied force with respect to the applied voltage and null the DC components seen at both ports. After developing equations that relate 1) the applied force to the applied driving voltage v_a, and 2) the motion velocity to the motional current i_m at the sense output, one can write the ratio between i_m and v_a which thus represents the admittance between the drive and sense ports:

$$i_m(s) = V_{DC}\dot{C}_s = V_{DC}\frac{dC_s}{dx}\dot{x} = V_{DC}\frac{dC_s}{dx}sx = V_{DC}\frac{dC_s}{dx}s\frac{F_d}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)} =$$

$$= V_{DC}\frac{dC_s}{dx}\frac{s}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)}\frac{(V_{DC}-v_a)^2}{2}\frac{dC_a}{dx} \approx V_{DC}\frac{dC_s}{dx}\frac{s}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)}\frac{1}{2}2V_{DC}v_a\frac{dC_a}{dx} =$$

$$= V_{DC}\frac{dC_s}{dx}\frac{s}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)}V_{DC}v_a\frac{dC_a}{dx} = \left(V_{DC}\frac{dC}{dx}\right)^2\frac{s}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)}v_a =$$

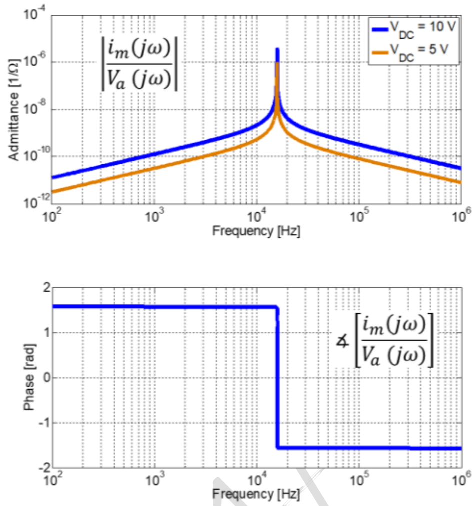
$$= \frac{\eta^2 s}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)}v_a \quad \rightarrow \quad \boldsymbol{\frac{i_m}{v_a} = \frac{\eta^2 s}{m\left(s^2+\frac{s\omega_0}{Q}+\omega_0^2\right)}}$$

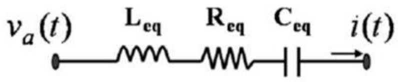
The obtained result indicates that a 3-port resonator shows an admittance between the drive and sense port that:

- at low frequencies ($\omega \ll \omega_0$) equals a capacitive behavior, $\frac{i_m}{v_a} = \frac{\eta^2 s}{k} = C_{eq}s$
- at high frequencies ($\omega \gg \omega_0$) equals an inductive behavior, $\frac{i_m}{v_a} = \frac{\eta^2}{ms} = \frac{1}{L_{eq}s}$
- at resonance ($\omega = \omega_0$) equals a resistive behavior, $\frac{i_m}{v_a} = \frac{\eta^2}{b} = \frac{1}{R_{eq}}$

Indeed, the same equation above is matched by an RLC equivalent electrical model, whose spectral behavior indicates an admittance increase up to a maximum value obtained at resonance, and then a decrease towards higher frequencies. The phase correspondingly passes from +90° to -90° with a 0°-shift at resonance.

The graph aside shows the obtained spectral behavior. The electrical equivalent model is reported below.

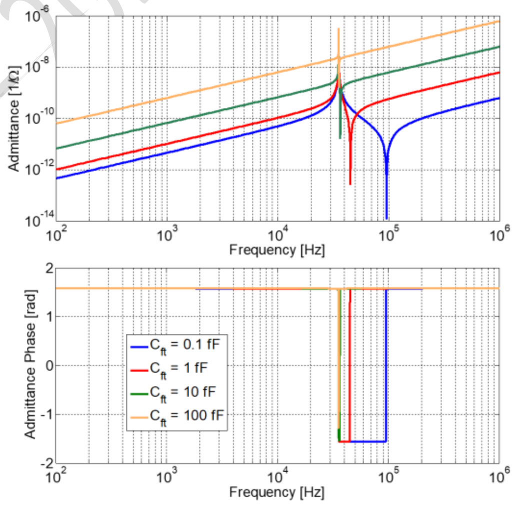


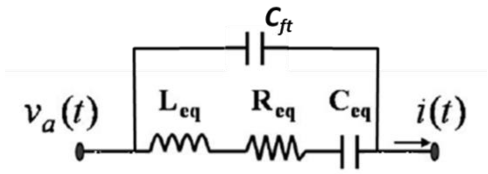


In more realistic situations, parasitic electrical elements affect the ideal resonator behavior. Among possible nonidealities, the most important one is represented by a parasitic capacitive coupling between the actuation and sensing port, as an actuation AC signal can be fed directly through this capacitance to the sensing port (this parasitic is thus known as feedthrough capacitance C_{ft}).

To understand the relevance of a feedthrough term, we need to remember that the resonator will be typically coupled to a circuit that compensates the losses, synthesizing an equivalent negative resistance $-R_{eq}$, once in steady state conditions, and thus giving a unitary loop gain at resonance. In presence of a feedthrough term, the electrical admittance equation becomes:

$$\frac{i_m}{v_a} = \frac{\eta^2 s}{m\left(s^2 + \frac{s\omega_0}{Q} + \omega_0^2\right)} + sC_{ft}$$





The equation (and the corresponding graph aside) clearly shows that at large frequencies the admittance is dominated by feedthrough effects, and may become even larger than the value at resonance. In presence of poles introduced by the circuit, the Barkhausen criterion can be thus satisfied at frequencies other than resonance, implying undesired oscillating signals in the loop. This clearly impairs the correct operation of the circuit.

Though circuital solutions can be found to mitigate this effect (e.g. compensation with poles, or feedthrough-compensation circuits), the easiest way to make feedthrough negligible is to rise the Q factor value by lowering the damping coefficient. In this case indeed the admittance value at resonance increases, while the feedthrough contribution remains unchanged. A high Q is thus beneficial against feedthrough effects.

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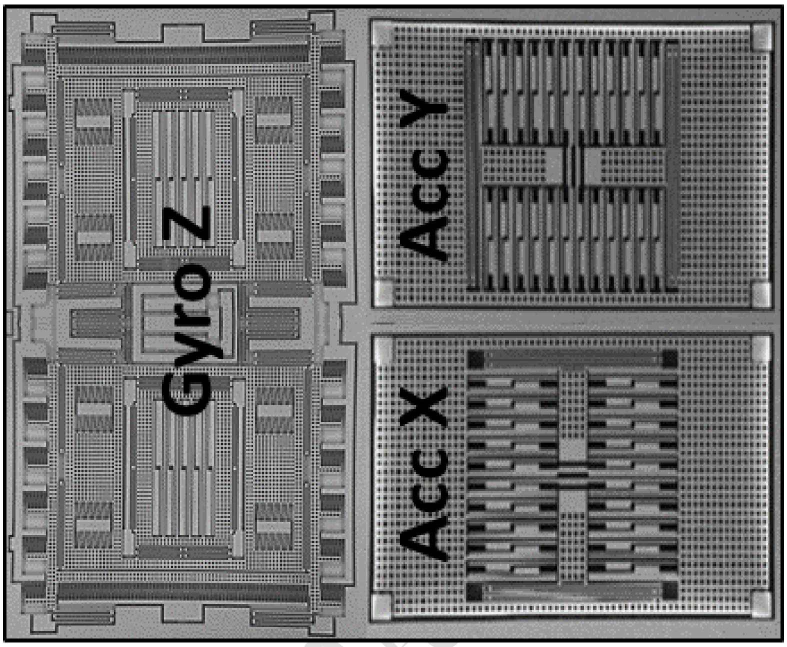
Question n. 2



A module for Electronic Stability Control (ESC) in automotive applications is formed by 2 in-plane accelerometers for X- and Y- axis acceleration detection, and a yaw gyroscope for Z-axis rotation detection (see the figure). The sensors are integrated on the same die at the same pressure. Given the parameters listed in the table:

(i) evaluate the maximum accelerometer sensitivity (in [fF/g]) and the gyroscope sensitivity (in [fF/dps]), such that the bandwidth copes with the specified values;

(ii) evaluate the damping coefficients and the gyroscope resonance frequency to cope with noise specifications (assume that damping is dominated by squeezed film effects of sensing parallel plates);

(iii) assuming typical readout circuits and biasing schemes for the accelerometers and gyroscope, evaluate the needed feedback capacitance value to best exploit, in both cases, the ±1.8 V supply range of the operational amplifiers, without any additional gain stage.

(iv) in your opinion, is the system well dimensioned for the proposed application? Can you propose a technological solution that ensures a better optimization of the system?

Physical Constants

$k_b = 1.38\ 10^{-23}$ J/K

$g = 9.81$ m/s^2

Process gap	1.5 µm
Operating temperature range	-45 – +125 °C
Electronic amplifiers supply voltage	±1.8 V
Spurious vibrations range	<30 kHz
Accelerometer parameters	
Mass	$3\cdot10^{-9}$ kg
Sensing bandwidth	300 Hz
Noise density	10 µg/√Hz
Rest capacitance (single ended)	300 fF
Stator bias voltage	±3 V
Required FSR	±16 g
Maximum input-referred mechanical offset	±5 g
Gyroscope parameters	
Sense mass (half structure)	$1.5\cdot10^{-9}$ kg
Maximum drive displacement	10 µm
Sensing bandwidth	300 Hz
Noise density	5 mdps/√Hz
Rest capacitance (all structure, single-ended)	300 fF
Rotor bias voltage	10 V
Required FSR	±2000 dps
Maximum quadrature (non compensated)	±1000 dps

(i)

We have to set the devices' sensitivities in order to cope with bandwidth specifications. For what concerns the accelerometers, we want a flat bandwidth of 300 Hz: we can thus choose a resonant frequency $f_{0,acc}$ about one order of magnitude larger (e.g. of 3kHz), with a safety margin. The sensitivity of the accelerometers turns out to be:

$$S_{mech,acc} = \frac{2C_0}{g} \cdot \frac{1}{(2\pi f_{0,acc})^2} = 11\, fF/g$$

On the other hand, for the gyroscope, we can infer from the 300-Hz bandwidth specification that the mode-split operation is more convenient with respect to the mode-matched one.

Thus, in order to obtain such a bandwidth, we can design the drive and the sense mode with a frequency mismatch of about 2 or 3 times the BW specification. For instance:

$$\Delta\omega = 2\pi \cdot (3 \cdot 300\, Hz)$$

Choosing – obviously – the largest allowed drive displacement, we obtain in this way a mechanical sensitivity of:

$$S_{mech,gyro} = \frac{2C_0}{g} \cdot \frac{x_d}{\Delta\omega} = 0.012\, fF/dps$$

(ii)

Starting from the accelerometer noise specification we can quickly derive its required damping coefficient:

$$NEAD = \frac{\sqrt{4k_b T_{max} b_{acc}}}{m_{acc}} \rightarrow b_{acc} = \frac{(m_{acc} NEAD)^2}{4kT_{max}} = 3.9 \frac{\mu N}{m/s} = 3.9\; 10^{-6}\; kg/s$$

Since the accelerometers and the gyroscope are sealed at the same pressure (and since the sense mode of a gyro features parallel plates like accelerometers), their damping coefficient per unit area is identical. Furthermore, being their rest capacitance (and consequently their squeezed film area) equal, also their overall damping coefficient is identical, so $b_{gyro} = b_{acc}$.

Thus, in order to obtain the desired gyroscope resolution, we can design its resonance frequency (which is the only undetermined parameter so far) in the following way:

$$NERD = \frac{\sqrt{4k_b T_{max} b_{gyro}}}{m_s \omega_D x_d} \;\rightarrow\; \omega_D = \frac{\sqrt{4k_b T_{max}}}{NERD \cdot m_s x_D} = 2\pi \cdot (17.9\, kHz)$$

(iii)

We want that the amplitude of the differential signal at the front-end stage output is equal to the full rail-to-rail voltage span of the operational amplifier. Offset and quadrature may degrade our output dynamics, so we have to correctly take into account their effect:

$$V_{out,acc} = V_{supply} = S_{mech,acc} \frac{V_{bias}}{C_{f,acc}} \cdot (FSR_{acc} + offset) \rightarrow C_{f,acc} = 380\ fF$$

$$V_{out,gyro} = V_{supply} = S_{mech,gyro} \frac{V_{bias}}{C_{f,gyro}} \cdot \sqrt{\left(FSR_{gyro}^2 + QE^2\right)} \rightarrow C_{f,gyro} = 153\ fF$$

Note how offset in accelerometers should be directly summed to the desired FSR to prevent unwanted saturation, while the gyroscope quadrature should be summed quadratically due the 90° phase shift between signals and quadrature spurious terms.

(iv)

Generally, it is always better to work with "decoupled" pressures in multi-degree-of-freedom unit. Indeed, this option gives much more freedom in sizing the parameters of the different sensors – though at the cost of a slightly increased area. This situation can be obtained realizing two separate cavities in our die, with two different pressure values.

In our design, the accelerometer quality factor is relatively high (~14) and the ring down time constant (5·τ = 5·Q/(πf_0) = 7.5 ms) somewhat degrades the bandwidth of our device in presence of shocks and following ringdown.

Furthermore, the gyroscope resonant frequency is relatively low, and thus sensitive to spurious vibrations. In harsh environment, like a car chassis, it would be better to set the frequency at values larger than the maximum vibration frequency (>30 kHz from the given specifications in the table).

NOTE: our design has some other weaknesses, and consequently other clever comments are very well accepted!

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Question n. 3

You are in front of the spectacular scene shown aside, and thus you would like to capture it with your phone camera, which features a 3T-topology CMOS image active pixel sensor with microlenses.

(i) The brightest pixel in the image sees an impinging photon flux of 10^{18} ph/s/m^2. Choose the integration time in order to be at the saturation limit for this pixel.

(ii) In the conditions above, calculate the SNR for the pixels in the darkest portion of the scene, which receive the smallest photon flux of $2.5\cdot10^{14}$ ph/s/m^2.

(iii) Unsatisfied by the obtained noisy image, try to quickly adjust the F# number, so to reach a SNR>1 also for the darkest pixels.

(iv) Which is the challenge we are experiencing with this specific scene (qualitatively, and quantitatively)? Can you propose a solution, which does not require to change the sensor?

Square pixel side	3 µm
Fill factor	40 %
Bias voltage	3 V
Average quantum efficiency	0.7
Depletion region width	2 µm
Parasitic gate capacitance	0.3 fF
Dark current density	0.11 fA/µm^2
Initial F# number	8

Physical Constants

$q = 1.6\cdot10^{-19}$ C

$k_b = 1.38\cdot10^{-23}$ J/K

T = 300 K

$\varepsilon_{Si} = 8.85\cdot10^{-12}$ F/m $\cdot 11.7$

(i)

In a 3T topology saturation occurs when the sum of the photo and dark generated charges equals the maximum well capacity of the photodiode, approximately given by

$$Q_{max} \approx (C_g + C_{dep})V_{DD} = \left(C_g + \frac{\epsilon_0 \epsilon_{Si} A_{pd}}{x_{dep}}\right)V_{DD} = \left(C_g + \frac{\epsilon_0 \epsilon_{Si}\, FF\, A_{pix}}{x_{dep}}\right)V_{DD}$$

where $C_{dep} = 0.18\, fF$ *is the depletion region capacitance. The condition is thus:*

$$t_{int} = \frac{Q_{max}}{i_{ph} + i_d} = \frac{\left(C_g + \frac{\epsilon_0 \epsilon_{Si}\, FF\, A_{pix}}{x_{dep}}\right)V_{DD}}{\eta\, \phi_{max}\, q\, A_{pix} + j_d A_{pix}\, FF} = 1.4\, ms$$

Note how the FF term should be used when calculating the depletion region capacitance and the dark current. Yet, it should not be used in the calculation of the photocurrent as the presence of microlenses re-boosts light gathering by the pixel to almost 100%.

(ii)

The SNR expression for the 3T pixel is simply given by the signal term calculated for the minimum photon flux, and the noise terms.

$$i_{ph,min} = \eta\, \phi_{min}\, q\, A_{pix} = 252\ aA$$

$$SNR = 20 \cdot \log_{10}\left(\frac{i_{ph,min}}{\sqrt{q(i_{ph,min} + i_d)t_{int} + k_B T(C_g + C_{dep})}}\right) = 0.25 = -12\ dB$$

In this case we can consider only shot noise and reset noise as we have no information on other possible noise sources (e.g. quantization noise). We note that the SNR is well lower than unity, which indicates that the image appears rather noisy in the region of poorest illumination.

(iii)

One way to recover performance in terms of SNR is to boost the overall light amount impinging on the pixel. We know that this can be done by acting on either the integration time or the F#. We are asked to check how I can adjust the F# number to reach a SNR>1 for these pixels. We just need to remember that

- the photocurrent depends also on the parameters of the main lens. In particular, it goes with the inverse of the squared F#, $i_{ph} \propto \left(\frac{1}{F\#}\right)^2$
- if signal independent noise is dominant over shot noise (which is what we can expect in this configuration as we are operating at low integration times), the SNR is linear with the photocurrent (as a verification, reset noise squared can be calculated to be 13 times larger than shot noise squared in our situation).

Therefore, to recover a SNR = 1, starting from an initial SNR = 0.25 we should gain a factor 4 in terms of signal. This can be obtained by halving the F# from 8 to 4.

(iv)

The solution above however does not solve completely our problem. Indeed, it is true that we now have a (barely) acceptable SNR for the darkest pixels, but the brightest pixels will now saturate. It is a typical situation where the dynamic range of the scene:

$$DR_{scene} = 20 \cdot \log_{10}\left(\frac{\phi_{max}}{\phi_{min}}\right) = 72\ dB$$

is larger than the DR of the camera:

$$DR_{camera} = 20 \cdot \log_{10}\left(\frac{Q_{max}}{\sqrt{q i_d t_{int} + k_B T\left(C_g + C_{dep}\right)}}\right) = 60\ dB$$

Decreasing the integration time to improve the DR is not significantly effective in this case, as we are already in a situation where dark current shot noise is negligible compared to reset noise. Without any chance to act on the sensor parameters at pixel level, a possible option could be to act on the maximum biasing voltage. This option is however generally not suitable for low-voltage operation typical of mobile phones and portable devices. We can in the end conclude that with such a 3T topology, there is basically no "hardware" solution to capture a scene with a 72 dB DR with a good SNR on all the pixels and simultaneously without saturation.

The only chance one has to improve the DR is to exploit software processing of multiple images. We have indeed seen that

- in the first configuration we optimize the image for the brightest pixels, putting them at the saturation limit;
- in the second situation, we optimize the image for the darkest pixels, putting them at the SNR=1 limit.

The idea could be to merge two consecutive images captured in these different configurations via software processing. Something similar could be obtained at different integration times (instead of different F#) – and indeed this is what is usually called "HDR mode" in our mobile phone cameras.