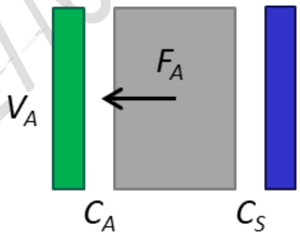
Last Name _Piatoles_ **Given Name** __Americo___ **ID Number** __20170908__

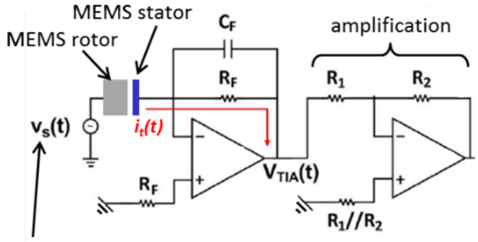
Question n. 1

The "C-V curve" method can be used to test a MEMS in the electromechanical characterization phase. Describe how this procedure is implemented (with applied signals at the different electrodes), how it can be related to a force-displacement graph, and which relevant electromechanical parameters can be extracted through this procedure. You can help yourself with equations and graphs, where useful.

The C-V curve is part of the so-called electromechanical characterizations, i.e. those characterization whose aim is the experimental validation of the electromechanical parameters of the design phase. In particular, the C-V curve is a quasi-stationary characterization in that it gives information on the low-frequency (DC) behavior of the MEMS. As a consequence, it can validate only those parameters which can be measured in a low-frequency range: typically, the stiffness, the DC offset and the pull-in voltage, while e.g. the resonance frequency and damping coefficient (Q factor) cannot be estimated with this method.

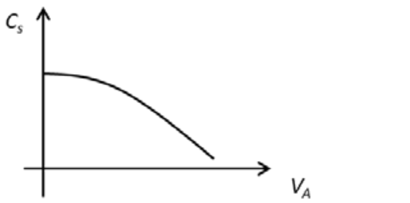


To implement a C-V curve one can use the scheme depicted aside, with an actuation voltage V_A applied at one port and the other port used as a capacitive sensor. A readout scheme that allows to measure DC capacitance variation caused by V_A (and by the corresponding force F_A) is reported in the second figure, and makes use of a high-frequency test signal $v_s(t)$, applied at the rotor, so that the current through the sense port $i_t(t)$ is dominated by the contribution $C_S\, dV_S/dt$ (linear with the sense capacitance) and not to $V_S(t)\, dC_S/dt$ (which is small as the capacitance change is quasi-stationary in time).



As the electrostatic force is quadratic with the applied voltage, and as the capacitance variation is (at a 1st-order approximation) linear with the displacement, the obtained curve is parabolic, with a negative slope (the rotor is attracted towards the actuator, ang goes away from the sense stator). In terms of equation one can derive the expression reported below:

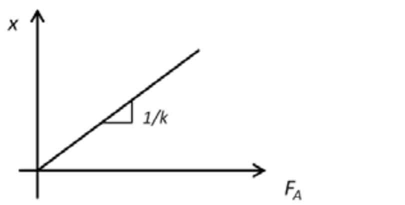
$$\Delta C_S = -\frac{C_S^2}{g^2\,k}\frac{V_A^2}{2} = -\frac{(\epsilon_0 A_{PP}\, N_{PP})^2}{g^4\,k}\frac{V_A^2}{2}$$



From the measured ΔC_S and from the two simple equations below:

$$|F_A| = \frac{V_A^2}{2}\frac{dC_A}{dx} \qquad x = \frac{\Delta C_S}{\frac{dC_S}{dx}}$$

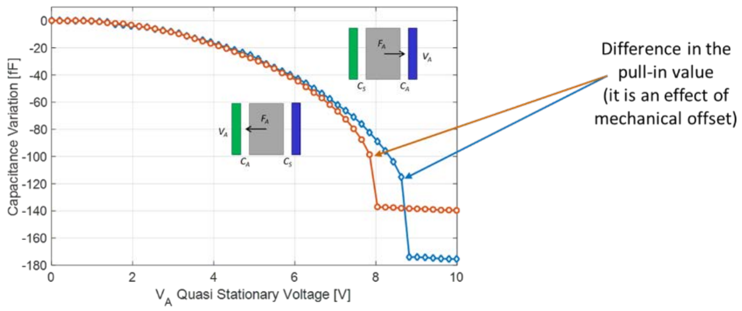
one can easily transform the measured C-V curve into a force-displacement plot, which will be a linear graph whose slope represents the device stiffness.



Note that in presence of mechanical offsets, the C-V curve can be also used to give an estimated of such offsets, simply repeating the measurement described above with inverted actuator and sensor ports.

A second parameter that can be easily estimated through a C-V curve is the pull-in voltage in parallel-plate configurations. Indeed, increasing the applied voltage, the structure will pass, for a specific voltage value, the critical condition of instability and the rotor will collapse towards the actuator.

This can be evidenced first by a deviation from the parabolic behavior (due to the no longer valid small-displacement approximation) and then by a sudden capacitance change (in particular, a decrease of the capacitance used as the sensing port).



Again, the difference in the value of the measured pull-in voltage when exchanging sensor and actuator ports can be used as an indication to estimate the mechanical offset.

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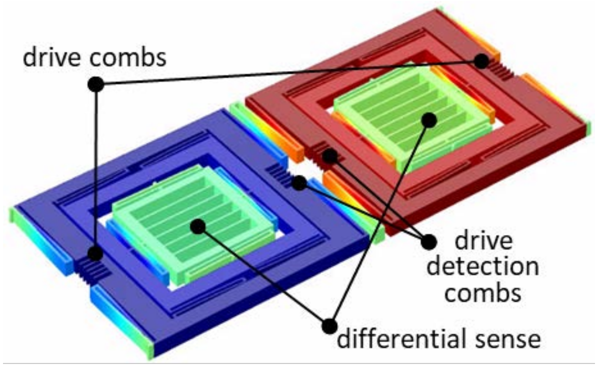
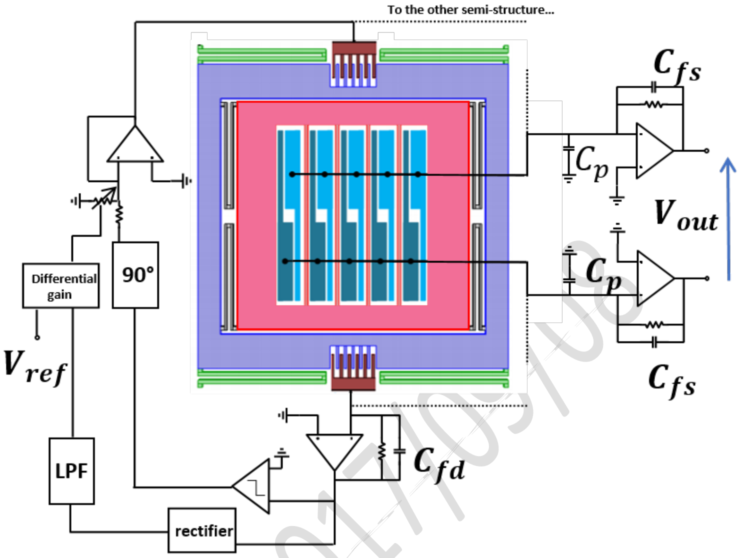
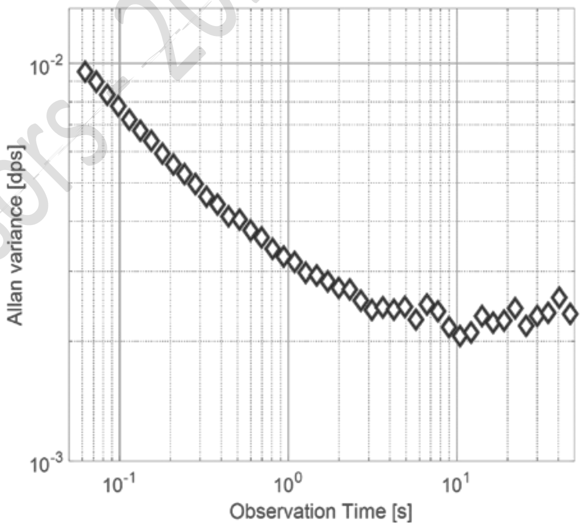
Question n. 2

A tuning-fork Z-axis gyroscope operates in mode-split conditions with the parameters indicated in the Table.





Rotor bias voltage	V_{dc}	10	V
Drive detection fingers (½ structure)	N_{cf}	32	-
Process thickness	h	25	µm
Process gap	g	2.1	µm
Drive circuit feedback capacitance	C_{fd}	350	fF
Sense plate length	L_{pp}	350	µm
Sense circuit feedback capacitance	C_{fs}	1	pF
Differential PP cells (½ structure)	N_{PP}	5	-
Sense mode Q factor	Q_s	100	-
Sense mass (half structure)	m_s	2	nkg
Opamp voltage noise density	S_{Vn}	10	nV/√Hz
Parasitic capacitance	C_P	10	pF
Sense mode resonance frequency	$f_{0,s}$	20	kHz



1) evaluate the voltage V_{ref} in the figure, in order to obtain a controlled drive displacement of ±5 µm;
2) evaluate the overall sensitivity (in [V/dps]), making reasonable design assumptions to guarantee a flat bandwidth of about 200 Hz;
3) evaluate the overall, input-referred white noise density (in [dps/√Hz]);
4) verify the white noise density measured through the Allan curve given above, and evaluate the 1/f noise coefficient.

Physical Constants

q = 1.6 10^{-19} C

k_b = 1.38 10^{-23} J/K

T = 300 K (if not specified)

ε_0 = 8.85 10^{-12} F/m

We know that the AGC, through a negative feedback loop, forces the LPF output voltage to be equal to the external V_{ref}. This voltage can be properly set in order to obtain the desired oscillation amplitude: in this case, we want $x_D = 5\mu m$. Such a displacement generates a motional current equal to:

$$i_m = \eta_{DD} \cdot v_D = \eta_{DD} \cdot \dot{x}_D = 2 \cdot \frac{2\epsilon_0 H N_{cf}}{g} V_{DC} \cdot \omega_D \cdot x_D$$

This current will flow in the drive-detection TCA feedback capacitance, causing an output voltage equal to:

$$V_{out,TCA} = i_m \cdot \frac{1}{\omega_D C_{fd}} = 2 \cdot \frac{2\epsilon_0 H N_{cf}}{g} V_{DC} \cdot x_D \cdot \frac{1}{C_{fd}} = 1.92\, V$$

This voltage is rectified and then its mean value is extracted through a low-pass filter:

$$V_{ref} \sim V_{out,LPF} = \frac{2}{\pi} \cdot V_{out,TCA} = 1.22\, V$$

We know that, in mode-split operation with a fixed Δf_{MS}, the gyroscope bandwidth is flat for roughly $\frac{\Delta f_{MS}}{2} \div \frac{\Delta f_{MS}}{3}$. We can thus choose a $\Delta f_{MS} = 600\ Hz$ to guarantee a $200\ Hz$ bandwidth.

We can now easily evaluate the sensitivity in term of [V/dps]:

$$S = \frac{dy}{d\Omega} \cdot \frac{dC}{dy} \cdot \frac{dV_{out}}{dC} = \frac{x_D}{\Delta\omega_{MS}} \cdot 2 \cdot \frac{2\epsilon_0 N_{pp} L_{pp} H}{g^2} \cdot \frac{V_{DC}}{C_{fs}} \cdot \frac{\pi}{180} = 81\mu V/dps$$

First of all, the device intrinsic noise contribution can be evaluated. The damping coefficient is a missing data, but we can derive its value from the sense quality factor:

$$b_s = \frac{\omega_s (2 \cdot m_s)}{Q_s} = 5 \cdot 10^{-6}\, N/(m/s)$$

So, we can easily evaluate the Noise Equivalent Rate Density:

$$NERD = \frac{180}{\pi} \cdot \frac{1}{x_D \omega_S m_S} \cdot \sqrt{K_b T b_s} = 3.3 \frac{mdps}{\sqrt{Hz}}$$

The contribution of electronic noise (supposing a sufficiently high value R_f with negligible noise) is given by:

$$\sqrt{S_{in,opamp}} = \frac{\sqrt{2 \cdot S_{vn}} \cdot (1 + C_p/C_f)}{S^2} = 1.9 \frac{mdps}{\sqrt{Hz}}$$

Thus, the overall input-referred white noise density is equal to:

$$\sqrt{S_{in,tot}} = \sqrt{NERD^2 + S_{in,opamp}} = 3.8 \frac{mdps}{\sqrt{Hz}}$$

We know that, in an Allan Variance plot, the graph portion with -1/2 slope is associated with the white noise density, while the flat portion is determined by the flicker noise. We can thus graphically evaluate the two contributions:

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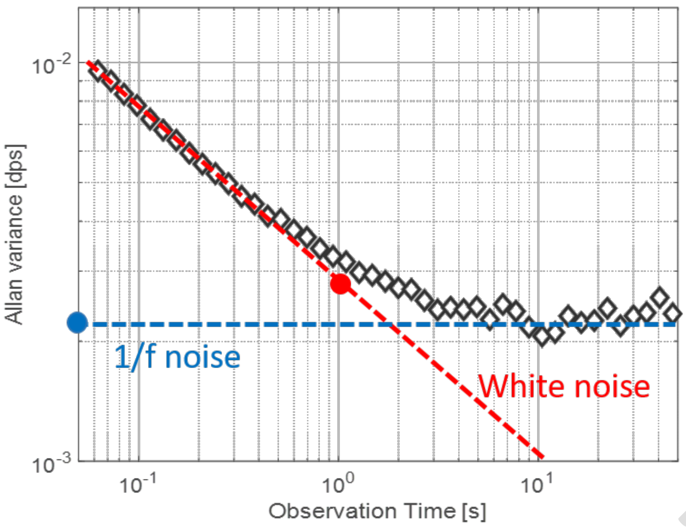

For what concerns the white noise:

$$\sqrt{S_{in,tot}} = \sqrt{2} \cdot \sigma_{AV}(\tau = 0) = \sqrt{2} \cdot 2.7 \frac{mdps}{\sqrt{Hz}} = 3.85 \frac{mpds}{\sqrt{Hz}}$$

The result is very similar to the theoretical calculation. Let's finally compute the 1/f noise (with power spectral density $\frac{\alpha_n}{f}$) coefficient:

$$\alpha_n = \frac{\sigma_{AV}^2}{2log2} = 3.18 \cdot 10^{-6} \frac{dps^2}{Hz}$$

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Question n. 3

An imaging system is used in an industrial environment to monitor the quality of a product.

For this reason, the product, assumed to be an isotropic reflector with a 0.2 reflectance coefficient, square-shaped of 10-cm side, is illuminated with a light source (with average wavelength of 500 nm) so that the optical power impinging on the product is 107 mW.

The imaging system, composed of a lens and a 3T CMOS sensor, is placed 50 cm far from the product, orthogonal to the plane of the object.

The camera takes pictures of the object with a 10-ms integration time and an F number of 4.

Some sensor and pixel parameters are reported in the Table.

1) Evaluate the magnification factor and the required focal length in order to fit the object within the sensor.

2) Calculate the photon flux, expressed in photons/s/m^2, impinging on the sensor.

3) Calculate the required pixel size so that the signal-to-noise ratio of the pixel is 40 dB. Which is the dominant noise source?

4) Is the system diffraction limited?

Sensor width	15	mm
Sensor height	10	mm
Pixel supply voltage	1.8	V
Photodiode depletion region width	1	µm
Dark current density	0.1	aA/µm^2

Physical Constants
q = 1.6 10^{-19} C
k_b = 1.38 10^{-23} J/K
T = 300 K (if not specified)
ε_0 = 8.85 10^{-12} F/m
ε_{Si} = 11.7 ε_0

In order to fit the square-shaped object within the rectangular sensor, the required magnification factor can be evaluated as

$$m = \frac{H_{sens}}{H_{scene}} = 0.1$$

Hence, the focal length of the optics should be

$$f = d * m = 50 \text{ mm}$$

where d is the distance from the object to the lens (50 cm). The diameter of the lens, d_{lens}, is thus

$$d_{lens} = \frac{f}{F} = 12.5 \text{ mm}$$

The optical power reflected by the object is

$$P_R = R * P_I$$

where R is the reflectance coefficient, and P_I is the optical power impinging on the sensor. Since the reflection is isotropic, the power impinging on the lens, hence on the sensor, can be evaluated as

$$P_{sens} = P_R \frac{\Omega_{lens}}{2\pi} = RP_I \frac{\frac{A_{lens}}{d^2}}{2\pi} = RP_I \frac{\frac{\pi\left(\frac{d_{lens}}{2}\right)^2}{d^2}}{2\pi} = 1.67 \text{ µW}$$

The optical intensity (power / surface) can be found by dividing the power for the area of the sensor corresponding to the object, i.e., H_{sens}^2*. Hence,*

$$I_{sens} = \frac{P_{sens}}{H_{sens}^2} = 16\ \mathrm{mW/m^2}$$

As the average energy of one photon is

$$E_{ph} = \frac{hc}{\lambda} = 396\times10^{-21}\ \mathrm{J}$$

The photon flux can be evaluated as

$$\phi = \frac{I_{sens}}{E_{ph}} = 42\times10^{15}\ \frac{\mathrm{photons}}{\mathrm{s}\cdot\mathrm{m}^2}$$

Targeting a 40 dB SNR, i.e., a SNR of 100, assuming that the system is shot noise limited, the required number of photons is

$$N_{ph} = SNR^2 = 10\,000$$

with an associated shot noise of

$$\sigma_{shot} = \sqrt{N_{ph}} = 100$$

Given the integration time, the required photon rate, n'_{ph}*, is*

$$n'_{ph} = \frac{N_{ph}}{t_{int}} = 1\,000\,000\ \frac{\mathrm{photons}}{\mathrm{s}}$$

The required pixel size is then

$$l_{pix} = \sqrt{\frac{n'_{ph}}{\phi}} = 4.8\ \mathrm{\mu m}$$

Assuming a high fill factor (reasonable assumption with a $4.8\ \mathrm{\mu m}$*-sized pixel),* $F \simeq 1$*, the photodiode capacitance is*

$$C_{PD} = \frac{\varepsilon_{si} l_{pix}^2}{x_{dep}} = 2.4\ \mathrm{fF}$$

The associated reset noise is

$$\sigma_{reset} = \frac{\sqrt{kTC_{PD}}}{q} = 20$$

negligible with respect to shot noise (remember that comparison should be done with variances, not with standard deviations). Also dark current shot noise is negligible (< 1 electron rms). The dominant noise source is, as expected, signal shot noise.

The Airy diameter can be evaluated as $d_{Airy} = 2.44 \cdot \lambda \cdot F = 4.8\ \mathrm{\mu m}$*. Resolution limit due to diffraction is thus comparable with the resolution due to spatial sampling.*