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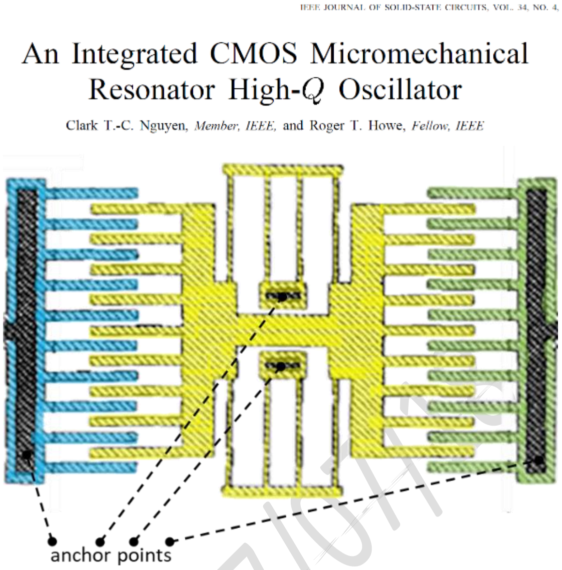
Question n. 1

The structure shown aside was conceived by Prof. Nguyen and Prof. Howe at UC Berkeley to demonstrate one of the first, high-Q, MEMS-based oscillators in 1999.

Describe, with detailed justifications, the structural parts that form it and the choices adopted for its design, the proper biasing and driving conditions at the three ports, its electrical equivalent circuit with possible parasitics, and the conditions that the sustaining circuit needs to satisfy for proper resonant operation.

You can assist yourself with graphs, block schemes, or circuit schematics, where useful.



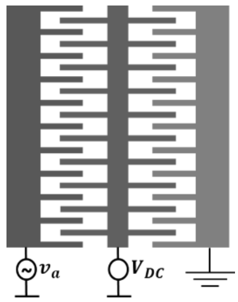
Structural parts and design choices:

The resonator is designed in the so-called 3-port configuration, the first port being an anchored comb actuator, the second being the suspended mass of the resonator, the third port being an anchored comb sensor.

The mass is suspended, in this specific implementation, by using folded springs with identical fold length to best reject temperature effects.

The actuation and driving ports are configured with comb fingers to maximize linearity at large motion amplitudes (this is e.g. required in applications where large motion is inherently desired, like in drive modes of gyroscopes), and to minimize damping losses, typical of parallel-plate configurations, which would lower the quality factor.

Biasing conditions:



We know that the applied electrostatic force on a MEMS capacitor is quadratic with the applied voltage. By just applying a sinusoidal stimulus at the resonance frequency to the drive port, we would obtain a force at twice that frequency, because of the square rectification. This would not work within a self-sustaining loop…

A simple solution consists in biasing the proof mass at a voltage much larger than the amplitude of the sinusoidal stimulus, with the sensor port kept at a (virtual) ground for current sensing, so to obtain a linearization as given by the expression below:

$$|F_{elec}| = \frac{\varepsilon_0 h\, N_{CF}}{g}\left[(v_a \sin(\omega_0 t))^2 + 2V_{DC} v_a \sin(\omega_0 t) + V_{DC}^2 - V_{DC}^2\right] \sim \frac{\varepsilon_0 h\, N_{CF}}{g} 2V_{DC} v_a \sin(\omega_0 t)$$

(N_{CF} is the number of comb fingers per port, h the process height, g the fingers gap and ω_0 is the resonance frequency).

Electrical equivalent circuit:

Seen as a black box, the resonator receives a voltage at its input and outputs a current. It can be therefore represented by an equivalent electrical admittance, just by taking the ratio of the output current to the input *voltage. For the sake of simplicity, the factor* $\frac{\varepsilon_0 h\, N_{CF}}{g} 2V_{DC}$ *is named electrostatic transduction factor* η *and represents both the coefficient between applied voltage and corresponding (linearized) electrostatic force, and the coefficient between the suspended mass velocity and the corresponding output current. If we now just add the relationship between a generic force applied to a MEMS suspended part, and its corresponding motion:*

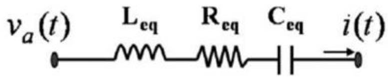
$$\frac{F_{elec}(s)}{V_a\,(s)} = \eta, \qquad \frac{i_m(s)}{sX(s)} = \eta \qquad \frac{X(s)}{F_{elec}(s)} = \frac{1/m}{\left(s^2+\frac{b}{m}s+\frac{k}{m}\right)}$$

we can obtain the final expression of the equivalent admittance:

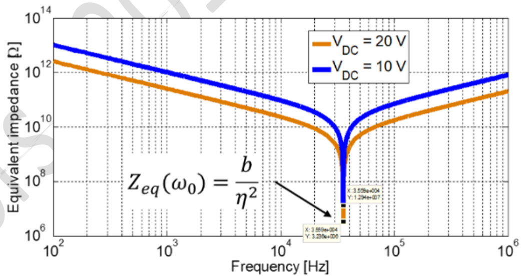
$$\frac{i_m(s)}{V_a\,(s)} = \eta^2 \frac{s}{(ms^2+bs+k)}$$

We note at this point that this expression is in the same form of a series RLC resonator. We can therefore model our 3-port MEMS resonator as a series equivalent RLC circuit with parameters given by:

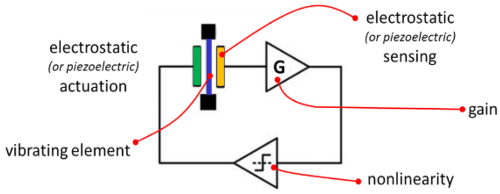
$$\frac{i_m(s)}{V_a\,(s)} = \frac{s}{\left(L_{eq}s^2+R_{eq}s+\frac{1}{C_{eq}}\right)}$$



$$C_{eq} = \frac{\eta^2}{k} \qquad R_{eq} = \frac{b}{\eta^2} \qquad L_{eq} = \frac{m}{\eta^2}$$



The admittance is maximized at resonance, which means that the equivalent resistance is minimized, as in the shown graph. Additionally, the series circuit representation seen aside is only apparently a 2-port system, indeed the third port (i.e. the rotor DC voltage) determines the value of η *and thus of the electrical equivalent parameters.*



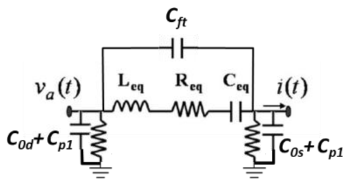
Oscillator conditions:

In order to have a self-sustained oscillation, a circuit must be used to provide a compensation of the residual resistive losses at resonance (the equivalent impedance is minimized, but not null at ω_0*!). There are different possible implementations, depending on which front-end topology is used (e.g. a trans-resistance or a trans-capacitance amplifier): whatever the choice, the conditions to satisfy for stable oscillation are the so-called Barkhausen criteria:*

$$\left|G_{loop}(j\omega_0)\right| = 1 = 0\;dB \qquad \measuredangle\left(G_{loop}(j\omega_0)\right) = 0°$$

stating that a self-sustained signal should have a unitary gain and no phase lag after a complete turn-around across the oscillator loop (which is somewhat obvious). Note that this condition is ideally satisfied for just one specific, ideal, value of the circuit gain (corresponding to R_{eq}*). With lower circuit gains the oscillation would never start; for larger circuit gains the oscillation would diverge. The solution adopted to obtain a stable oscillation is, in general, to have a circuit gain which is larger than needed for the start-up, and which is then adjusted by a nonlinearity in the loop (in the simplest case, the saturation of the amplifiers), so to match the ideal condition of unitary gain.*

A more realistic electrical model, in presence of parasitics, is represented aside. Among the added parasitics, the most critical one is the capacitance feeding directly through from the drive port to the sense port. Indeed, it adds a direct current contribution which can be critical for the sustaining circuitry, as it may add other undesired frequencies that satisfy the resonant condition.



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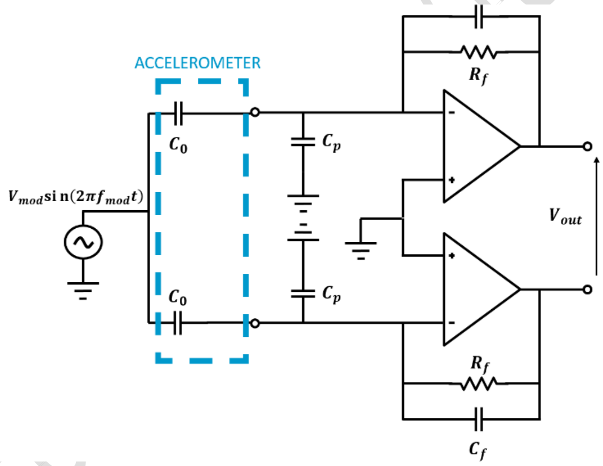
Question n. 2

You need to finalize the design of an accelerometer targeting the detection of input signals up to 500 Hz. Using the readout scheme depicted in the figure and the parameters listed in the table, you are asked to:

- choose a proper Q factor, in order to reach a $NEAD \leq 50\mu g/\sqrt{Hz}$;
- knowing that electronic noise is dominated by the voltage noise of the differential input pair of the two amplifiers, estimate the required current consumption of the front-end stage, in order to have an input-referred electronic noise density contribution equal to $\frac{NEAD}{4}$;
- choose the best among two possible technological improvements: (i) *halving the minimum gap* or (ii) *doubling the process height*. Discuss in deep details the motivations of your choice as a function of relevant parameters of MEMS accelerometers.

Accelerometer total stiffness	k_{tot}	1.4	N/m
Accelerometer total mass	m	8.8	NKg
Differential capacitive sensitivity	S_{mech}	7	fF/g
Readout feedback capacitance	C_f	0.5	pF
Parasitic input capacitance	C_p	3	pF
Rotor modulation voltage amplitude	V_{mod}	1	V
Input-pair MOS overdrive voltage	V_{ov}	0.1	V
MOS transistor γ coefficient	γ	2/3	-



Physical Constants
q = 1.6 10^{-19} C
k_b = 1.38 10^{-23} J/K
T = 300 K (if not specified)
ε_0 = 8.85 10^{-12} F/m

Starting from the provided data, the resonant frequency value can be derived:

$$\omega_0 = \sqrt{\frac{k_{tot}}{m}} = 12.6\ krad/s$$

The noise equivalent acceleration density can be written as:

$$NEAD = \sqrt{\frac{4k_bT\omega_0}{Qm}}$$

So, the minimum value of the quality factor in order to satisfy noise requirements is equal to:

$$Q_{min} = \frac{4k_bT\ \omega_0}{(NEAD)^2 m} = 0.1$$

Such a low value is not suitable for large-bandwidth applications. We know this from theory, but we can estimate the lower pole position starting from the transfer function between force and displacement:

$$T_{XF} = \frac{1}{m} \cdot \frac{1}{s^2 + s \cdot \frac{\omega_0}{Q} + \omega_0}$$

Using a dominant pole approximation, the first pole has $\tau = \frac{1}{Q\omega_0}$ *and thus* $f_{pole} \sim 200Hz$.

We have input signals up to 500Hz: a reasonable solution is to set a slightly higher quality factor, e.g. Q = 0.5. In this case:

$$NEAD = 22 \frac{\mu g}{\sqrt{Hz}}$$

The sensitivity of our system can be evaluated as:

$$S = S_{mech} \frac{V_{mod}}{C_f} = 14 \frac{mV}{g}$$

Knowing this parameter, we can write down the expression of the input-referred voltage noise of the operational amplifiers and impose it equal to $\frac{NEAD}{4}$.

$$\sqrt{S_{v,in}} = \frac{\sqrt{2S_v}\left(1 + \frac{C_p}{C_f}\right)}{S} = \frac{NEAD}{4}$$

In an operational amplifier, the input-referred voltage noise generated by the input differential pair is equal to:

$$S_v = 2 \cdot \frac{4k_b T\gamma}{g_m}$$

Where:

$$g_m = \frac{2I_D}{V_{ov}}$$

Where I_D is the current flowing in the single MOS transistor of the differential pair. We can now finally evaluate the total current consumption of the front-end electronics:

$$I_{front-end} = 4 \cdot I_D = 4 \cdot \frac{4k_b T\gamma V_{ov}}{\frac{1}{2}\left(\frac{\frac{NEAD}{4} S}{1 + \frac{C_p}{Cf}}\right)^2} = 71\mu A$$

The choice between the two technological improvements can be motivated by the analysis of their influence on the most important parameters in a MEMS accelerometer: sensitivity, pull-in voltage and noise.

For what concerns the sensitivity:

$$S = \frac{1}{\omega_0^2} \frac{C_0}{g} = \frac{1}{\omega_0^2} \frac{e_0 H L_{pp}}{g^2}$$

Both solutions improve sensitivity, raising H by a factor 2 and halving the minimum gap by a factor 4.

Let’s take a look to the pull-in voltage expression:

$$V_{PI} = \sqrt{\frac{kg^3}{2\epsilon_0 H L_{pp} N_{PP}}}$$

Remembering that the mechanical stiffness is directly proportional to the process height, we can conclude that the pull-in voltage is constant with H. On the other hand, V_{PI} goes with $g^{\frac{3}{2}}$: a lower gap get the system closer to instability.

At last, we can evaluate the effect on the thermomechanical contribution to the resolution:

$$NEAD = \sqrt{\frac{4k_bT\omega_0}{Qm}} = \sqrt{\frac{4k_bTb}{m^2}}$$

Due to squeezed-film damping, a gap reduction will slightly increase the b coefficient, and consequently, the NEAD. Conversely, a doubled H will increase damping coefficient by a factor 2 and the mass by a factor 4, improving the NEAD by a factor $\frac{1}{\sqrt{2}}$.

Knowing that the limit to the system resolution is the thermomechanical noise of the MEMS device, a process height improvement is the smarter solution.

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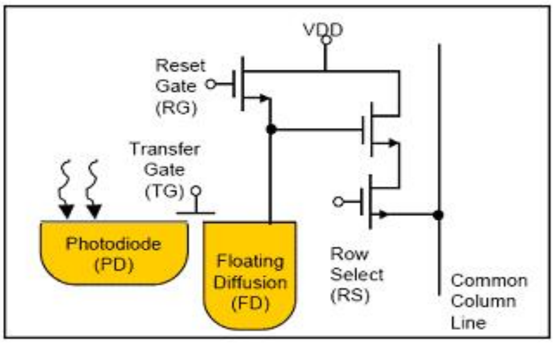
Question n. 3

You are working in a company developing innovative CMOS sensors with the aim of reaching single-photon counting capabilities. The technology is based on standard 4T topology with pinned photodiode and correlated double sampling (CDS) circuit. The pixel parameters are listed in the table.

- calculate the required reset-noise-rejection factor for the CDS circuit to have identical contributions from reset noise and dark current shot noise at an integration time of 1 ms;
- assuming no signal, calculate the minimum number of electrons that can be measured for the integration time above;
- calculate the pixel dynamic range for the integration time above;
- discuss about the fundamental limit in measuring charges that can be accomplished with this pixel topology (*suggestion to start: calculate the SNR for the case of a single collected photon*).



Pixel side	3	µm
Pinned photodiode depletion region	3	µm
Floating diffusion side	0.5	µm
Floating diffusion depletion region	0.5	µm
Fill factor	0.5	-
Source follower gate capacitance	0.1	fF
Bulk dark current density	2	aA/µm^2
Bias voltage	2	V

Physical Constants

$q = 1.6 \cdot 10^{-19}$ C

$k_b = 1.38 \cdot 10^{-23}$ J/K

T = 300 K (if not specified)

$\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m

$\varepsilon_{Si} = 11.7\ \varepsilon_0$

The dark current of the detector can be evaluated as

$$i_{dark} = j_{dark} A_{PD} = j_{dark} \cdot FF \cdot l_{pix}^2 = 9\ \text{aA}$$

The floating diffusion capacitance can be evaluated as

$$C_{FD} = \varepsilon_{Si} \frac{l_{FD}^2}{x_{dep,FD}} = 51.7\ \text{aF}$$

comparable with the one of the source follower (100 aF). The total integration capacitance can be thus evaluated as

$$C_{int} = C_{FD} + C_{G,SF} = 152\ \text{aF}$$

Intrinsic reset noise (i.e., without CDS) can be estimated as

$$\sigma_{reset} = \frac{\sqrt{k_B T C_{int}}}{q} = 4.9\ \text{e}$$

Dark current shot noise can be estimated as

$$\sigma_{dark} = \sqrt{\frac{i_{dark} t_{int}}{q}} = 0.23\ \text{e}$$

Last Name __Miritiro__ **Given Name** ____Serena___ **ID Number** __20170718__

The required reset-noise-rejection factor for the CDS to have identical contributions for reset noise and dark current shot noise can be thus evaluated as

$$RF = \frac{\sigma_{reset}}{\sigma_{dark}} = 20$$

Assuming no signal, the minimum number of electrons that can be measured is

$$N_{min} = \sigma_{read} = \sqrt{\frac{\sigma_{reset}^2}{RF^2} + \sigma_{dark}^2} = \sqrt{2}\sigma_{dark} = 0.32 \text{ e}$$

The dynamic range can be estimated as

$$DR = \frac{N_{max}}{N_{min}} = \frac{\frac{V_{DD}C_{int}}{q}}{0.32 \text{ e}} = \frac{1897 \text{ e}}{0.32 \text{ e}} = 5656$$

i.e., 75 dB.

The minimum detectable signal due to readout electronic noise only is 0.32 electrons.

However, if we consider the signal with its associated shot noise, we know that, due to Poisson-like statistics of photon flux, the variance of the number of collected photons is equal to the number of photons. Hence the standard deviation, with unity signal, is unity as well:

$$\sigma_{shot} = \sqrt{\sigma_{shot}^2} = \sqrt{N} = \sqrt{1} = 1$$

The signal-to-noise ratio can be thus roughly described as

$$SNR = \frac{N}{\sigma_{tot}} = \frac{N}{\sqrt{\sigma_{shot}^2 + \sigma_{read}^2}} \simeq \frac{1}{1+\varepsilon} \simeq 1$$

The pixel is thus conceptually capable to reach single-photon counting capabilities.