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Question n. 1

Digital imaging sensors are characterized by a phenomenon known as fixed pattern noise (FPN). Describe **(i)** possible FPN sources and **(ii)** the impact that FPN has on the sensor performance as a function of the signal intensity. Describe **(iii)** compensation techniques that can be adopted to mitigate FPN, and **(iv)** the effects of Correlated Double Sampling on FPN.

Fixed pattern noise (FPN) is a kind of spatial noise (as opposed to typical temporal noise) that arises due to differences in gain and offset among different pixels of a CMOS image sensor, in particular of the "active" type. As a direct consequence, a uniform image will be rendered at the output as a noisy representation, even in the ideal case of absence of temporal noise.

To understand the sources for FPN, we need to look at the sub-elements inside an active pixel that characterize offset and gain. Further, it is convenient to split between FPN sources occurring from light collection to charge generation (basically optical or physical sources) and sources occurring from charge collection to pixel output voltage generation (basically electrical or electronic sources).

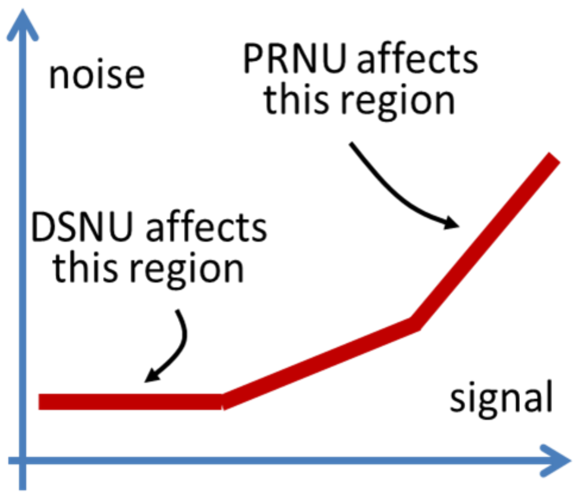
For what concerns gain differences, it is easy to understand that any variations from pixel to pixel in the optical path will cause a different responsivity and thus different gain: so, nonuniformities in microlenses, in color filters, in the stack of transparent dielectric materials and so on… Similarly, generated photo-carriers will be collected differently and will produce different voltage changes in the pixel when differences in the conversion gain appear: these can be due e.g. to geometrical differences, doping differences (together determining a different depletion capacitance value), differences in the oxide thickness of the source follower (determining a change in the seen gate capacitance) and so on... Once signal is converted into a voltage, any gain difference in the electronic circuit will produce as well FPN (e.g. all sources affecting the transistor g_m*, to a lower extent, can have some effects on FPN).*

Switching now to sources of offset differences, we can identify variations in the dark current from pixel to pixel (either due to dark current density differences, so to presence of defects, or due to geometrical differences from pixel to pixel given by process tolerances), and differences in the electronic offset (threshold voltage, biasing current…).

All these FPN sources can be conveniently grouped into two parameters. Those sources that determine offset differences lead to the appearance of the so called Dark Signal Nonuniformity, or DSNU, a spatial noise which manifests mostly at low signal intensity, as it is signal independent.

Those sources that determine gain differences lead to the appearance of the so called Photo Response Nonuniformity, or PRNU, a spatial noise which manifests mostly at large signal intensities, as it is a contribution directly proportional to the signal itself.

Indeed, in a typical Photon Transfer Curve (PTC) graph, we will find DSNU in the flat region (signal independent) and PRNU in the steepest region (signal proportional).



The peculiar characteristic of FPN is that it is (rather) constant over time. As a consequence, one can think to calibrate and compensate it in an initial characterization phase of the image sensor. The key point is to take advantage of the fact that, if we repeat identical measurements several times and combine them, temporal (white) noise will be reduced by the averaging process, while the remaining noise will become more and more of the spatial type only.

The idea is thus to make a first of such averaged characterizations in dark, to recover the values of the original DSNU and then to compensate it. As DSNU has a fixed (electronic) contribution and a time-dependent (dark-current) contribution, two measurements in dark at two different integration times are enough to recover all the information to compensate.

A second averaged characterization should be done under high signals to put in evidence the PRNU. As the PRNU is the result of the integration of the product of a more or less complex light spectrum times a variable pixel spectral response, there is a need to reproduce PRNU effects under "typical" capturing condition. This is usually accomplished by adopting a color chart composed by reflectance spectra which are well representative of typical spectra that one captures in everyday-life acquisitions. The average result obtained by capturing several shots (to average temporal noise) on several spectra (to make the calibration as much as possible realistic) can be used as the basis for compensation.

For both DSNU and PRNU, after calibration one needs just to store in a look-up table suitable correction values (additive and multiplicative) to compensate in the digital domain, after acquisition, the captured images. Such stored values bring offset and gain of all pixels to be all similar, to the largest possible extent, to the mean value across the entire matrix.

Finally, we know that some pixel topologies, in particular the 4T scheme with pinned photodiode, usually rely on Correlated Double Sampling (CDS) to compensate reset noise. Reminding that CDS samples a first point before the transfer gate (TG) is activated and a second point after TG transfers the charge, we now try to understand whether this operation has any effects on FPN.

We recognize that DSNU contributions given by electronic offsets will be compensated, as they are sampled in both phases and thus subtracted in the following CDS operation. However, DSNU due to dark current differences is not corrected by CDS, as dark current effects appear only in the second sample of a CDS operation. Similarly, PRNU is not compensated by CDS, as signal appears only in the second sample of a CDS operation.

We conclude thus that the effects of CDS on FPN are very moderate, and CDS alone cannot be exploited to significantly compensate FPN.

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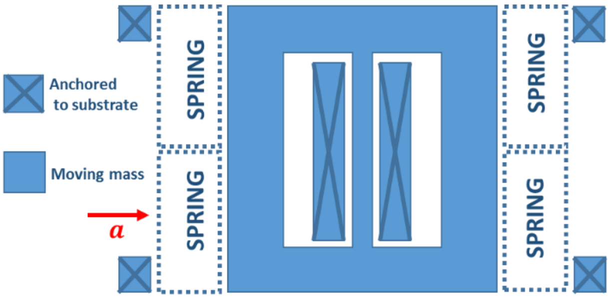
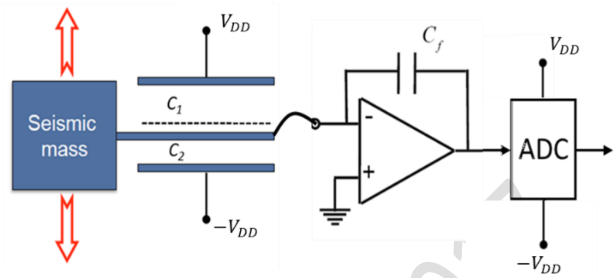
Question n. 2

You are asked to design an in-plane, single-axis, parallel-plate MEMS accelerometer (as schematically represented below) for consumer applications, with a technology that features a 2 μm minimum gap.





(i) starting from the project specifications given in the Table, calculate the accelerometer sensitivity in terms of differential capacitance variation per unit acceleration (in gravity units);

(ii) calculate the stiffness of each spring;

(iii) properly dimension the feedback capacitance for the readout configuration given above;

(iv) assume now that, after a technological improvement, you can decrease the gap size by a factor 2. Assuming also that the pull-in voltage and the area cannot be modified, would you switch to the new gap size, and why? Use equations and considerations to assist your choice.

Parameter	Symbol	Value
Full Scale Range	FSR	$\pm 8g$
Linearity error @ FSR	$\epsilon_{lin,FSR}$	2%
Total parallel plates area (single-ended)	A_{PP}	$75000\mu m^2$
Parallel plates gap	g_{PP}	$2\mu m$
Mass	m	$4.5nKg$
Bias voltage	V_{DD}	$3.3V$

Physical Constants

q = 1.6 10^{-19} C

k_b = 1.38 10^{-23} J/K

T = 300 K (if not specified)

ε_0 = 8.85 10^{-12} F/m

First of all, starting from the linearity error specification, the maximum displacement can be fixed:

$$x_{FSR} = g_{PP} \cdot \sqrt{\frac{\epsilon_{lin,FSR}}{100}} = 283nm$$

Hence, if an external acceleration equal to the FSR occurs, the differential capacitance variation results:

$$\Delta C_{FSR} = 2 \cdot \frac{C_0}{g} \cdot \frac{x_{FSR}}{\left(1 - \left(\frac{x_{FSR}}{g_{pp}}\right)^2\right)} \sim 2 \cdot \frac{C_0}{g} \cdot x_{FSR} = 2 \cdot \frac{\epsilon_0 A_{pp}}{g^2} \cdot x_{FSR} = 94\ fF$$

The mechanical sensitivity is readily obtained:

$$S_{mech} = \frac{\Delta C_{FSR}}{FSR} = 12\frac{fF}{g}$$

The resonant frequency of the accelerometer can be written as:

$$\omega_0 = \sqrt{\frac{a_{FSR}}{x_{FSR}}} = 16.6\ krad/s$$

Using this result, the total stiffness can be derived:

$$k_{tot} = m\omega_0^2 = 1.25\frac{N}{m}$$

This stiffness is obtained summing the mechanical and electrostatic contributions. The latter is given by:

$$k_{el} = -2\frac{C_0 V_{DD}^2}{g_{PP}^2} = -1{,}81\frac{N}{m}$$

The stiffness of each spring is equal to the total mechanical stiffness divided by 4, since, from the figure, we can notice that there are 4 springs in a parallel configuration:

$$k_{spring} = \frac{k_{mech}}{4} = \frac{k_{tot} - k_{el}}{4} = 0.76\ N/m$$

The feedback capacitance can be sized in order to have, at FSR, the maximum allowed voltage at the ADC input (e.g. to minimize quantization error):

$$V_{in,ADC} = FSR \cdot S_{mech} \cdot \frac{V_{DD}}{C_f} = V_{DD} \rightarrow C_f = FSR \cdot S_{mech} = \Delta C_{FSR} = 94fF$$

Writing down the pull-in voltage expression:

$$V_{PI} = \sqrt{\frac{g_{PP}^2 k_{mech}}{2C_0}} = \sqrt{\frac{g_{PP}^3 k_{mech}}{2\epsilon_0 A_{PP}}} = 4.3\ V$$

So, since I can't modify the device's area, I can only act on the mechanical stiffness in order to keep the pull-in voltage constant. Thus, if $g_{PP,new} = \frac{g_{PP}}{2}$:

$$k_{mech,new} = 8 \cdot k_{mech}$$

Considering that also the electrostatic stiffness goes with $1/g_{PP}^3$:

$$k_{tot,new} = 8 \cdot k_{tot}$$

Again, I can't modify the area, so the device's mass remains constant. Thus:

$$\omega_{0,new}^2 = 8 \cdot \omega_0^2$$

Hence:

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$$S_{mech,new} = \frac{1}{\omega_{0,new}^2} \frac{\epsilon_0 A_{PP}}{g_{pp,new}^2} = \frac{S_{mech}}{2}$$

With respect to the previous case, the sensitivity is halved. This is detrimental for our system: for example, the input-referred electronics noise will be higher, since it will be divided by a lower sensitivity.

For what concerns the intrinsic NEAD, we can say that it doesn't change or it is slightly worse for a lower gap, due to squeezed-film damping.

At the end of the analysis, it can be affirmed that lowering the gap is not a convenient choice in our case.

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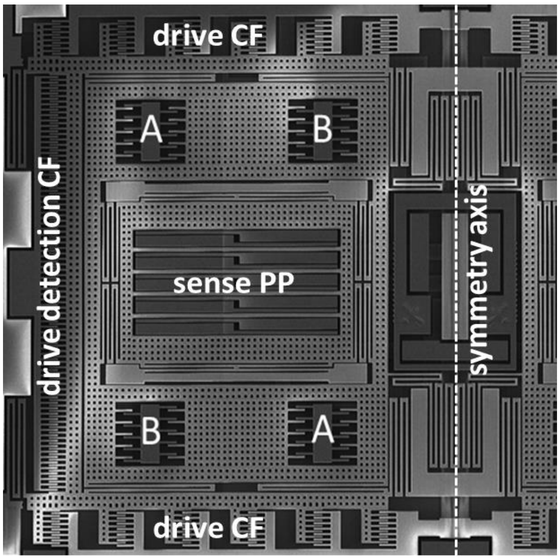
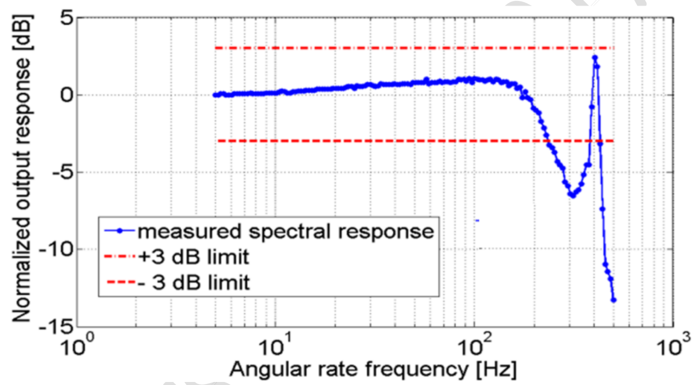
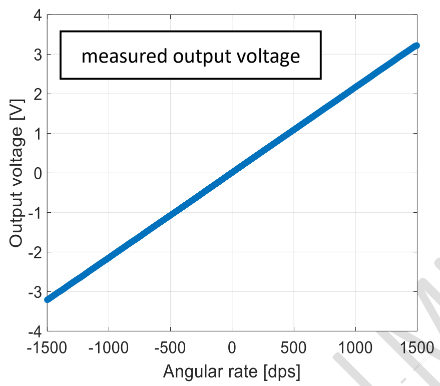
Question n. 3

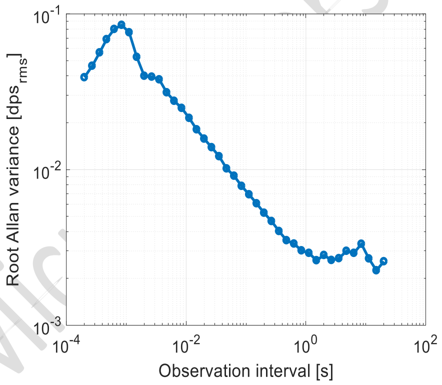
The doubly-decoupled tuning fork gyroscope represented in the figure (half device) is fabricated with a novel technology, and it is thus undergoing a deep characterization phase, with comparison of measured results with theoretical predictions. With the parameters given in the Table (valid for half device),

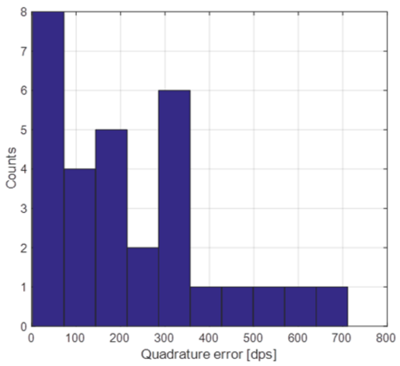


drive motion	7 µm	feedback resistance	1 GΩ
drive frequency	21 kHz	rotor voltage	10 V
sense frequency	21.5 kHz	op-amp noise density	(30 nV/√Hz)^2
sense mass	3.9·10^{-9} kg	parasitic capacitance	10 pF
sense capacitance	250 fF (single-ended)	electronic filter	200 Hz (-3 dB)
gap	2.5 µm	$F_{q,max}$ max quad comp force	100 nN









(i) compare the measured bandwidth graph with the expected one;

(ii) compare the predicted overall differential sensitivity with the measured one; comment on the matching between the obtained and predicted gap value;

(iii) compare the predicted white noise rate density with the measured one;

(iv) verify whether all measured quadrature errors can be compensated with the designed compensation electrodes, capable to apply a maximum force $F_{q,max}$.

Physical Constants

$q = 1.6\ 10^{-19}$ C

$k_b = 1.38\ 10^{-23}$ J/K

T = 300 K (if not specified)

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

The expected bandwidth of the gyro is 200 Hz (from Table). The measured one is a little bit higher, around 220 Hz, probably due to the rising of the MEMS transfer function due to the sense-axis resonant peak.

We can correctly see the mismatch appearing at 400 Hz, which deviates slightly from the prediction of 500 Hz.

The measured sensitivity can be extrapolated as

$$S_{meas} = \frac{3.2\ \mathrm{V}}{1500\ \mathrm{dps}} = 2.1\ \mathrm{mV/dps}$$

To predict the sensitivity, we lack the value of the feedback capacitance of the front-end's charge amplifier. As the operating frequency is 21000 Hz, and the feedback resistance is 1 GΩ, the feedback capacitance can be set so that the low-frequency pole of the charge amplifier is one decade below the operating frequency:

$$C_F = \frac{1}{2\pi \frac{f_d}{10} R_F} = 75\ \mathrm{fF}$$

The expected one can be calculated as

$$S_{theo} = \frac{x_a}{\omega_\Delta} \frac{2C_{0,1/2}}{g} \frac{V_{DC}}{C_F} 2 \frac{\pi}{180} = 2.05\ \mathrm{mV/dps}$$

Theoretically-predicted and measured sensitivities are perfectly in line.

As the sensitivity can be rewritten as

$$S_{theo} = \frac{x_a}{\omega_\Delta} \frac{2N_{PP,1/2}A_{PP}}{g^2} \frac{V_{DC}}{C_F} 2 \frac{\pi}{180}$$

and since the measured frequency mismatch is lower than expected, this means that, in order to obtain the same sensitivity, the gap turned out to be higher than the designed one.

From Allan graph, we see that the -1/2 slope asymptote crosses the 1 s axis around 2 mdps$_{rms}$. Hence, the measured white noise rate density is

$$NERD_{meas} = \sqrt{2} * 2\ \mathrm{mdps} * \mathrm{Hz}^{-\frac{1}{2}} \simeq 3\ \mathrm{mdps}/\sqrt{\mathrm{Hz}}$$

As the gyroscope is mode-split operated, we can expect that its noise performance is electronics-noise-limited, rather than thermomechanical-noise-limited. We can further assume that the main noise contribution is due to the two op-amps of the differential front-end. Hence,

$$NERD_{theo} = \sqrt{2}\sqrt{S_{n,OA,v}} \frac{\left(1 + \frac{C_P}{C_F}\right)}{S_{theo}} = 2.77\ \mathrm{mdps}/\sqrt{\mathrm{Hz}}$$

We could then calculate the NERD due to the two feedback resistors: we would find that their contribution is negligible with respect to op-amps' one.

The uncertainty can be due to wrong parasitics wrong estimation or due to non-negligible thermomechanical noise.

Maximum quadrature error is 700 dps.

Hence, the maximum quadrature-induced displacement amplitude is

$$y_{aq} = \Omega_q \frac{x_a}{\omega_\Delta} \frac{\pi}{180} = 27.2 \text{ nm}$$

The effective Q-factor of the gyro is

$$Q_{eff} = \frac{f_s}{2f_\Delta} = 21.5$$

Sense-axis spring constant (1/2 device) is

$$k_s = m_s(2\pi f_s)^2 = 71 \text{ N/m}$$

As the maximum quadrature-compensation force is 100 nN, the obtained quadrature-compensation-induced displacement amplitude is

$$y_{aQC} = F_{q,max} \frac{Q_{eff}}{k_s} = 30 \text{ nm}$$

As this is higher than the maximum quadrature-induced displacement, all quadrature errors can be compensated.