Question n. 1

Discuss the different issues encountered when operating a MEMS gyroscope in “mode-matched” conditions.

Discuss, hence, at least one different operation mode where such issues can be solved or at least minimized, highlighting, however, which consequences and challenges may arise with the new proposed solution.

Mode matched operation implies that the sense mode of the gyro precisely matches the resonance frequency of the drive mode. In such conditions, the energy exchange between the modes is maximized as the Coriolis force, always occurring at the drive mode frequency, excites the sense mode at its natural frequency as well, maximizing the sense mode displacement through its Q_S factor. Such a maximization is positive in that it boosts the sensitivity (or scale-factor), reducing the input-referred noise of the following electronic stages.

There are, however, several troubles in effectively operating a gyroscope in mode-matched conditions.

1) Bandwidth-resolution trade-off

The ultimate noise performance of the gyro (NERD) depends on the sense mode damping coefficient, b_s.

$$NERD \propto \sqrt{\boldsymbol{b_s}}$$

However, the maximum sensing bandwidth depends on the sense mode Q factor, which is itself a function of the damping coefficient b_s.

$$\Delta f_{BW} = \frac{f_s}{2\,Q_S} = \frac{\boldsymbol{b_s}}{4\,\pi\,m_S}$$

In turn, any trial to reduce the intrinsic gyroscope noise (e.g. to reduce b_s by decreasing the package pressure) implies a reduction in the maximum useful bandwidth.

2) Fabrication imperfections and temperature dependence

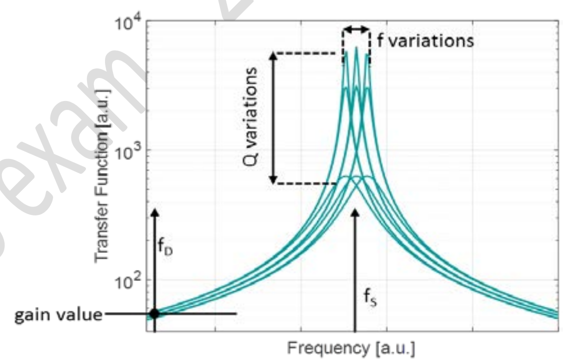
It is easy to match the drive and sense mode frequencies by design… however in the reality the actual frequencies will typically suffer from etching nonuniformities, process height nonuniformities, masks geometry differing from the ideal layout masks… and several other factors which make it impossible to obtain the ideal condition where $f_s = f_d$.

As the gyroscopes Q factor is usually high, a small deviation of the Coriolis force frequency f_d from the sense mode frequency f_S generates a large lowering in sensitivity. To bypass this issue, one could think to use tuning electrodes and to exploit a tunable equivalent electrostatic stiffness. In practice, the designer will set the sense mode frequency at a value larger than the drive mode, and then it will tune the sense mode accurately down to the drive mode.

Even using this tuning strategy, there are still issues which are really difficult to solve. These issues are related to the fact that the frequencies and quality factors in a MEMS will suffer a temperature dependence.

On one side, the frequency is a function of the stiffness, which is itself a function of the Young modulus. This material property is a function of temperature in Silicon, and sets a relative change by about – 30 ppm/K in the modes frequency. Therefore, if the two frequencies are not matched, their absolute changes under the same temperature variation will be different. As a consequence, even if we compensate the frequency difference via tuning electrodes at a certain temperature T, this compensation will not be effective at all temperatures.

On the other side, the Q factor of the modes is also a function of the temperature. Because of thermal agitation of gas particles inside the package, and due to the gas law (relating temperature and pressure at constant volume), the Q factor relative changes are approximately one half of the relative temperature changes. While the drive mode Q factor changes are compensated using an AGC circuit, the sensitivity in mode-matched conditions directly depends on the sense-mode Q factor, and thus changes with temperature.

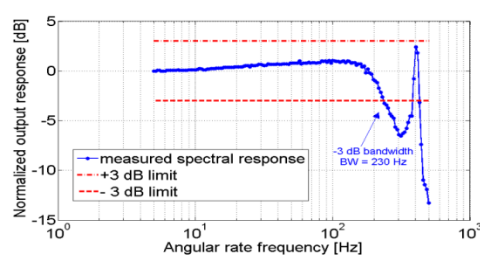


Mentioned trade-offs and changes make it unpractical to operate a gyroscope in mode-matched conditions. One well consolidated alternative consists in using the so-called mode-split operation. In this configuration, the two frequencies are on purpose designed with a mismatch:

$$\Delta f_{MS} = f_S - f_D$$

(usually, the drive frequency is set at a lower value than the sense one, but the opposite works the same way). The result of this choice is that, even for varying frequencies and Q factors, the gain of the sense mode transfer function at the drive frequency (which we named effective quality factor Q_{eff}) will remain rather stable (with a value of about $f_D/2\Delta f_{MS}$), as shown in the sample picture above. Therefore, all effects of temperature dependencies, which influence the modes and Q factor, are minimized. In the same way, effects of fabrication differences from part to part are made much less relevant than in mode-matched conditions.

Finally, the bandwidth resolution trade-off is solved, because the expression of the NERD is still dependent on the damping coefficient b_s, while the sensing bandwidth is now set by the modes distance and usually filtered by an electronic low-pass stage at about 1/2 to 1/3 of the split value. An example of resulting sensing bandwidth in mode split operation is given aside.



The drawback to pay in mode-split operation is that the signal (e.g. in terms of sense mode displacement, or capacitance variation, and so on…) is now reduced by a factor:

$$\frac{\Delta f_{MS}}{\Delta f_{BW}}$$

Therefore, noise of all the following stages in the sense chain (in particular the analog front-end amplifier) will have a larger impact. No change is on the contrary observed for the intrinsic (thermomechanical) noise.

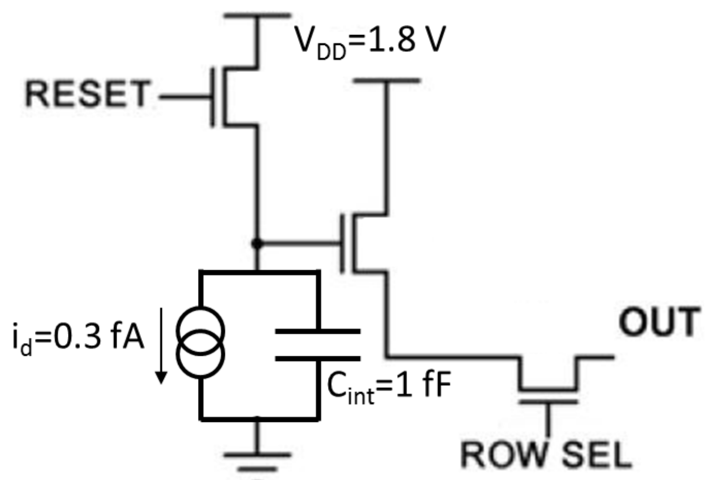
Question n. 2

You are taking a picture of a scenery with a digital camera. The light spectrum of the framed scenery can be considered white, with good approximation. You can consider an average wavelength of 500 nm, if needed.

You are equipped with two different cameras. They both feature a CMOS sensor with the shown pixel topology. Each active pixel has a 1.25 µm side, with a fill factor of 0.5. You can assume a quantum efficiency of 1 all along the spectrum.

The photon flux impinging onto the sensor is $0.3 \cdot 10^{18}$ photons/s/m^2. You take a picture with both cameras, using the same integration time (1 ms).

The sole difference between the cameras is that the first one is for black-and-white photography (no CFA), while the second one features a standard Bayer CFA on top of the pixels.



1. Evaluate the number of collected electrons in a single pixel of the B&W camera
2. Evaluate the signal-to-noise ratio for one pixel of the picture taken with the B&W camera
3. Evaluate and comment the signal-to-noise ratio for a green pixel of the picture taken with the CFA camera
4. Evaluate and comment the difference in spatial resolution between the two sensors

Physical Constants
$q = 1.6\ 10^{-19}$ C
$k_b = 1.38\ 10^{-23}$ J/K
T = 300 K (if not specified)
$\varepsilon_0 = 8.85\ 10^{-12}$ F/m
$\varepsilon_{r,Si} = 11.7$

1.

The photocurrent can be easily calculated as the flux impinging on the pixel surface is known, and we have all information about pixel size and fill factor. Assuming a unitary quantum efficiency, we have:

$$\mathrm{i_{ph1}} = \mathrm{q} \cdot \mathrm{FF} \cdot \mathrm{l_{pix}^2} \cdot \phi = 37.5\ \mathrm{fA}$$

In these conditions, the number of collected electrons is calculated from the charge integrated during the exposure time:

$$\mathrm{N_1} = \frac{\mathrm{i_{ph} t_{int}}}{\mathrm{q}} = 234\ e^-$$

2.

Reset and dark and signal shot noise can be calculated as 12.7, 1.4, and 15.6 electrons rms, respectively, using the formulas below:

$$\sigma_{reset} = \frac{\sqrt{k_b T C_{int}}}{q} = 12.7\ e^-$$

$$\sigma_{dark} = \frac{\sqrt{q i_d t_{int}}}{q} = 1.4\ e^-$$

$$\sigma_{signal} = \frac{\sqrt{q i_{ph} t_{int}}}{q} = 15.6\ e^-$$

Hence, total noise can be calculated to be about 20.2 electrons rms through the power sum of the contributions above.

We now have all the information to calculate the signal-to-noise ratio, whose dB expression is:

$$\mathrm{SNR_1} = 20 \log_{10} \frac{\mathrm{N_1}}{\sigma_1} = 21.3\ \mathrm{dB}$$

3.

For the green pixel of the CFA-based sensor, one can assume that roughly 1/3 of the impinging photons pass through the filter (while the remaining 2/3 are absorbed).

Hence the signal should be divided by 3 (78 electrons), and the signal shot noise power should be divided by 3 too (from 15.6 electrons rms we now have 9 electrons rms):

$$\mathrm{SNR_2} = 20 \log_{10} \frac{\frac{\mathrm{N_1}}{3}}{\sqrt{13^2 + 1^2 + 9^2}} = 20 \log_{10} \frac{78}{15} = 14\ \mathrm{dB}$$

As expected, SNR is lower with the CFA-based camera because of the loss in signal.

The lowering will be effectively perceived by the human eye as we are discussing about SNR well below the 30 dB limit (beyond which the eye does not distinguish consistent differences for improving SNR).

4.

Regarding resolution, as the F-number is not given, we can first check the value of the Airy diameter for the best resolution conditions given by the optics. This usually occurs at F# between 4 and 5.6. Using e.g. the value of 4 we obtain at an average wavelength of 500 nm:

$$d_{Airy} = 2.44 \cdot \lambda \cdot F_{\#} = 4.88\ \mu m$$

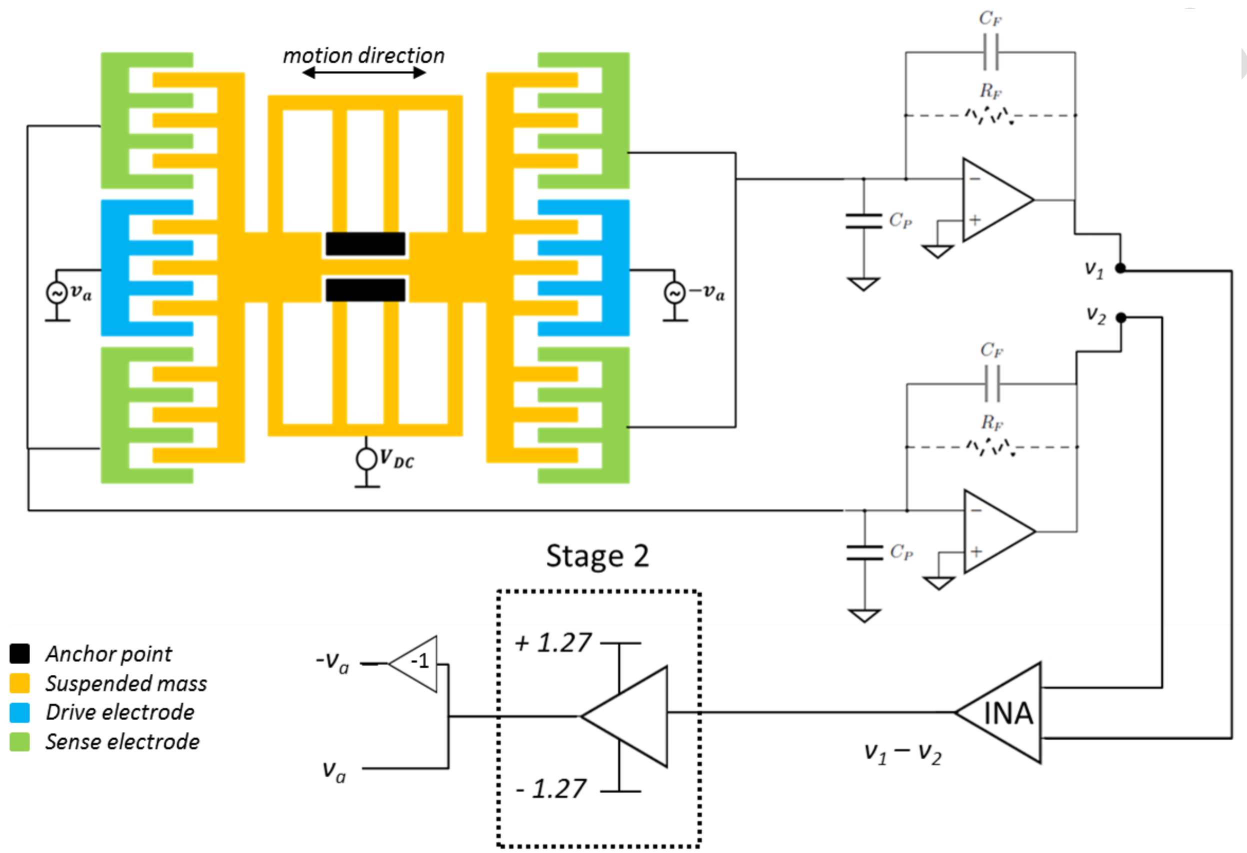
The Airy diameter is larger than the pixel side. Hence, the spatial resolution is the same for both the sensors, as it is diffraction limited.

Whatever the choice of the F#, the resolution can only worsen (for larger F# due to increasing diffraction, for lower F# due to increasing aberrations).

As a consequence, we can say that the sensors resolution will be always limited by the optics in both cases, and will not differ among the BW and CFA sensors.

Question n. 3

A MEMS resonator is coupled to an electronic circuit to generate a reference frequency signal at 145 kHz. The schematic view shown in the figure indicates a push-pull driving configuration, with a differential sensing, an instrumentation amplifier to turn the differential signal into a single-ended one, and a further gain Stage 2.



Parameter	Symbol	Value
Process thickness	h	10 μm
Process Gap	g	1 μm
Permittivity of vacuum	ε_0	8.85 10^{-12} F/m
Drive fingers (half structure)	$N_{cf,D}$	15
Sense fingers (half structure)	$N_{cf,S}$	30
Elastic stiffness	k	112 N/m
Quality factor	Q	10000
Resonance frequency	f_0	145 kHz
Rest overlap length of fingers	L_0	20 μm
Rotor DC voltage	V_{DC}	5 V
AC drive voltage (1st harmonic)	v_a	1 V
Feedback capacitance	C_F	0.5 pF
Feedback resistance	R_F	1 GΩ

Parasitic capacitance	C_P	3 pF
1st amplifier noise	$S_{v,n}$	$(20\ nV/\sqrt{Hz})^2$

1) At the driving parameters given in the Table, calculate the resonator motion and the single-ended capacitance variation at one of the sense ports.

2) Calculate the signal to noise ratio (in dB) at the output of the INA differential stage ($v_1 - v_2$).

3) Define the gain and phase shift needed by the "Stage 2" in the figure, to satisfy the oscillation conditions (assume a minimum loop gain of 10 is required). Why is this stage biased at a supply of ±1.27 V?

4) Draw a qualitative plot of the resonator loop gain in presence of a feedthrough capacitance (hint: pay attention to the assumption about the "Stage 2" topology).

1.

In the used push-pull configuration, a perfect cancellation of the 2nd order term occurs and allows to use an AC voltage which is not << of the DC value. The exact expression of the drive force exerted from the two electrodes is thus linear with the applied AC voltage v_a:

$$F_{el} = F_{el,1} - F_{el,2} = \frac{2\epsilon_0 h N_{CF,D}}{g}\frac{(V_{DC}+v_A)^2-(V_{DC}-v_A)^2}{2} = \frac{2\epsilon_0 h N_{CF,D}}{g}\frac{V_{DC}^2+v_a^2-V_{DC}^2-v_a^2+4V_{DC}v_a}{2}$$
$$= \frac{4\epsilon_0 h N_{CF,D}}{g} V_{DC} v_a = \eta_D\, V_{DC}\, v_a = 26.5\ \mathrm{nN}$$

This force is applied at resonance (once the resonator is embedded in the oscillator) to a suspended mass having a known stiffness, so that the displacement is easily found through the Hooke's law, amplified by the Q factor:

$$x = F_{el}\frac{Q}{k} = \frac{4\epsilon_0 h N_{CF,D}}{g} V_{DC} v_a \frac{Q}{k} = 2.37\ \mu\mathrm{m}$$

The corresponding single-ended capacitance variation is readily obtained:

$$\Delta C_S = \frac{dC_S}{dx}x = \frac{2\epsilon_0 h N_{CF,S}}{g}\, x = \frac{\eta_s}{2}\, x\ =\ 2.6\ \mathrm{fF}$$

2.

The single-ended voltage signal is easily calculated after verifying that the amplifier is used in a trans-capacitance (CA) configuration:

$$\frac{1}{2\,\pi\, R_F C_F} = 318\ \mathrm{Hz}$$

The pole falls at a frequencies much smaller than the resonance (145 kHz), so the front-end is effectively based on a CA configuration. In this condition, the capacitance to voltage gain is simply the ratio between the DC rotor voltage and the feedback capacitance, leading to a single-ended output voltage and to a differential voltage at the INA output respectively of:

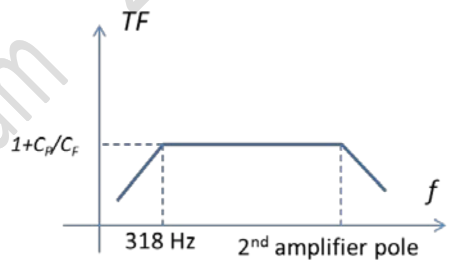
$$\Delta V_{SE} = \frac{V_{DC}}{C_F}\,\Delta C_S = 126\ \text{mV}$$

$$\Delta V_{INA} = v_1 - v_2 = 252\ \text{mV}$$

This is our signal. In order to evaluate the SNR at the INA output, we need to recover the information on noise. The value of the amplifier noise density is given, and it is the dominant noise source (you can verify that the other noise components are filtered through the feedback pole of the CA configuration):

$$\sigma_{v,opamp} = \sqrt{2\,S_{v,n}\left(1+\left(\frac{C_P+C_0}{C_F}\right)\right)^2} \approx \sqrt{2\,S_{v,n}\left(1+\frac{C_P}{C_F}\right)^2} = 200\,\frac{\text{nV}}{\sqrt{\text{Hz}}}$$

Which bandwidth should be used for the integration of the amplifier noise density into an rms value? Note that it is neither the MEMS mechanical bandwidth (amplifier noise occurs after the MEMS filtering action), nor the value of the pole calculated above. Indeed, the amplifier behaves like a constant gain (1+Cp/Cf) for a voltage signal applied at the positive input (like in our example) between the pole calculated above (318 Hz) and the second pole of the amplifier. This second pole will be set by the GBWP of the amplifier, and we can assume that - for a well-designed amplifier - this value will be set to about 1.45 MHz for a signal operating at 145 kHz.



The final noise value is thus calculated as:

$$v_n = \sigma_{v,opamp}\cdot\sqrt{1.45\text{MHz}} = 260\ \mu\text{V}$$

And the SNR can be finally evaluated:

$$\text{SNR} = 20\log_{10}\frac{252\ \text{mV}}{260\ \mu\text{V}} \approx 20\log_{10}10^3 = 60\ \text{dB}$$

3.

The overall gain around the loop (in modulus) can be calculated assuming that at resonance our MEMS resonator is fully modeled by its equivalent resistance R_{eq}:

$$G_{loop} = \frac{1}{R_{eq}}\frac{1}{\omega_0 C_F}\,2\,G_2 = 10$$

The expression of the equivalent resistance is given through the transduction factors of the drive and sense ports:

$$R_{eq} = \frac{b}{\eta_D\, \eta_S} = \frac{k/(Q\, \omega_0)}{\eta_D\, \eta_S} = 8.7\ M\Omega$$

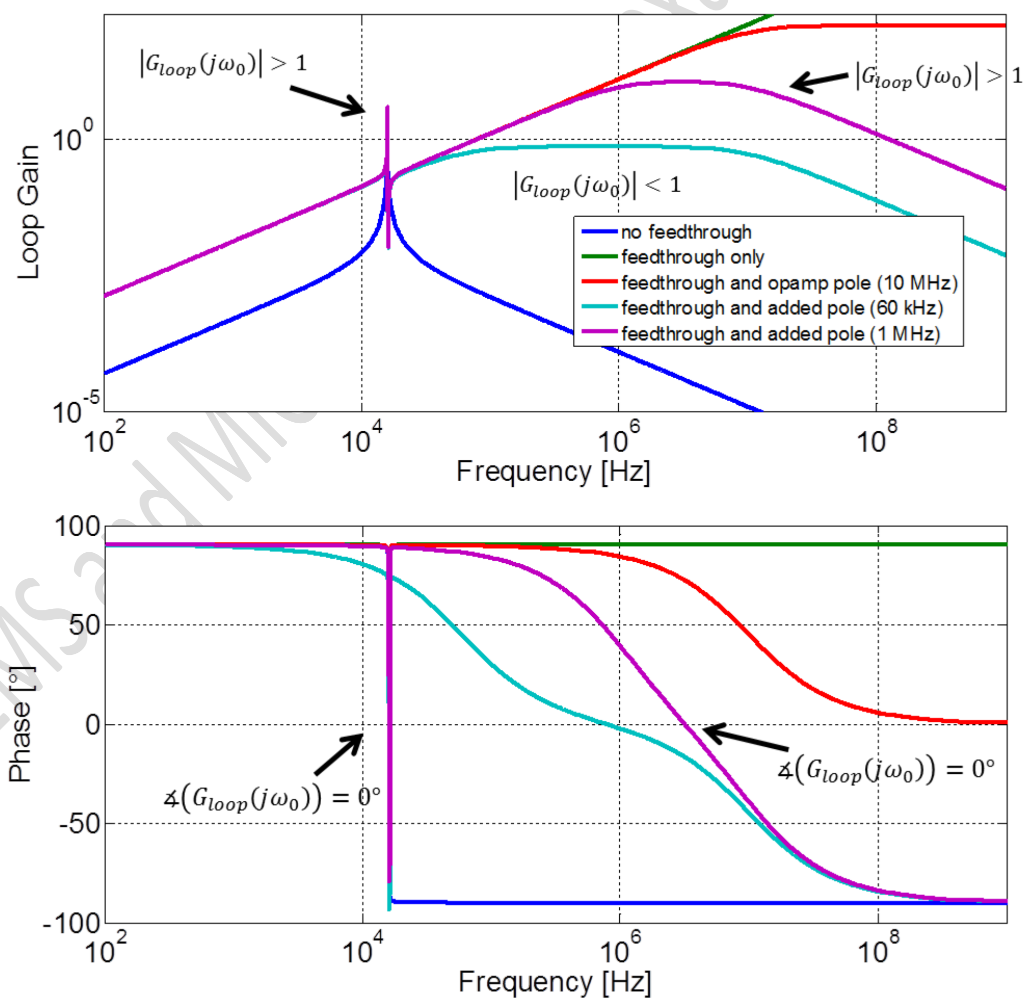
And thus a value of about 20 for the minimum gain needed by the second stage is obtained.

As the phase shift introduced by the CA stage is -270°, and as the MEMS at resonance (a resistance) does not provide any phase shift, the second stage should be thus designed to provide an additional phase shift of -90° at 145 kHz. A derivator stage can e.g. be used to this purpose.

4.

The feedthrough capacitance acts in parallel to the RLC MEMS equivalent net, modifying its transfer function.

As we use a combination of an integrator and a derivator stage, around the frequencies of interest the behavior of the loop gain does not change and we can assume that the loop gain will be itself modified like the MEMS transfer function.



The figure above shows sample plots of the changes occurring in the phase and gain of the loop (note: the figure is taken from the slides and refers to a frequency different from 145 kHz, so the indication of the frequency poles in the caption does not refer to the 145 kHz resonator case).

Suitable positioning of the poles should be pursued to avoid the circuit oscillation at frequencies other than resonance.