Question n. 1

Write and compare the sensitivity (capacitance variation per unit acceleration) of an accelerometer based on comb-finger readout with an accelerometer based on parallel-plate readout.

Assuming that you have no severe area and power consumption constraints, discuss which solution, among the two above, you would choose for (1) ultra-low-noise applications, (2) high-dynamic-range applications (where the dynamic range is here the ratio of the linear full-scale-range over the resolution), (3) wide-bandwidth applications.

The sensitivity can be easily derived if we split it into two terms, (i) the displacement per unit acceleration, and the capacitance change per unit displacement:

$$\frac{\Delta C}{\Delta a} = \frac{\delta x}{\delta a} \cdot \frac{\delta \mathrm{C}}{\delta x}$$

The first term is common to both the configurations, and equates k_{tot}/m, i.e. $1/\omega_0^2$., where ω_0 is the in-operation resonance frequency and the stiffness k_{tot} includes possible electrostatic induced terms. The second term depends on the specific topology.

For gap varying (parallel-plate, PP) accelerometers, the complete and linearized expressions in a differential configuration under a biasing voltage V_{DD} are therefore:

$$\frac{\Delta C}{\Delta a}_{PP} = \frac{1}{\omega_0^2} \cdot \frac{\delta \mathrm{C}}{\delta x} = \frac{1}{\omega_0^2} \cdot \frac{\delta \left(\frac{\epsilon_0 A}{g - x} - \frac{\epsilon_0 A}{g + x} \right) N_{PP}}{\delta x} \approx \frac{2}{\omega_0^2} \cdot \frac{\epsilon_0 A \, N_{PP}}{g^2} = \frac{2}{\omega_0^2} \cdot \frac{C_0}{g} = 2\, \frac{C_0}{g}\, \frac{m}{\left(k - 2 V_{DD}^2 \frac{C_0}{g^2} \right)}$$

where ω_0 takes into account the electrostatic softening effect expressed through the electrostatic stiffness.

For comb-finger (CF) sensing, the equation loses the electrostatic term dependence and can be written as:

$$\frac{\Delta C}{\Delta a}_{CF} = \frac{1}{\omega_0^2} \cdot \frac{\delta \mathrm{C}}{\delta x} = \frac{1}{\omega_0^2} \cdot \frac{\delta \left(\frac{\epsilon_0 h (L_0 + x)}{g} - \frac{\epsilon_0 h (L_0 - x)}{g} \right) 2\, N_{CF}}{\delta x} = \frac{4}{\omega_0^2} \cdot \frac{\epsilon_0 h \, N_{CF}}{g} = \frac{2}{\omega_0^2} \cdot \frac{C_0}{L_0} = 2\, \frac{C_0}{L_0}\, \frac{m}{k}$$

The comparison can be summarized through the following three points:

- the sensitivity of the CF configuration is independent of the displacement. The capacitance variation per unit acceleration is therefore very linear. For the PP configuration, this is true only under the small-displacement (linearized) approximation;
- further, the sensitivity of a CF configuration is independent of the used biasing voltage, while the PP solution has a dependence on the biasing which may lead to instability (pull-in) issues at large accelerations (or at small gaps, or at small mechanical stiffness);
- the sensitivity of the PP configuration is, for a given area, generally larger than the CF solutions. This can be readily seen by looking at the expressions and by considering that (1) g is usually much smaller

than L_0, and that, (2) due to the geometrical implementation, the rest capacitance C_0 for a given area is much larger in a PP solution than in a CF solution.

In light of the comments above, we discuss which solution to choose in the three cases proposed by the exercise:

1. ultra-low-noise

As we have no significant power constraints, noise contributions from the electronics can be minimized by using a large current to bias the operational amplifier. Further, as we have no severe area constraints, the sensor sensitivity can be made large enough to minimize anyway electronic noise. As a consequence, dominant noise contributions will come from the sensor thermomechanical noise. Recalling the NEAD expression:

$$NEAD = \sqrt{\frac{4\, k_B T\, b}{m^2}} = \sqrt{\frac{4\, k_B T\, \omega_0}{m\, Q}}$$

we note that the CF configuration usually gives a lower damping coefficient (and thus a larger quality factor) than a PP solution due to the absence of squeezed-film damping. We therefore choose the CF solution.

2. high-dynamic-range

The dynamic range is the ratio of the maximum measurable acceleration, limited by linearity issues, and the minimum measurable signal. The PP option has linearity limits given by the combination of the nonlinearity in the sensing principle, and the nonlinearity induced by the electrostatic softening.

As the CF solution has, in principle, no linearity constraints, and as the minimum obtainable noise is, in principle, lower than in the PP solution (see point above), we choose the CF option.

3. wide bandwidth

The bandwidth of an accelerometer can be assumed to be roughly equal to the resonance frequency if the Q factor is chosen to be around 0.5. Assuming that we match this target, we note that, in order to extend the bandwidth, we would need a larger resonance frequency ω_0. This would induce a consistent sensitivity worsening (as this goes with $1/\omega_0^2$, whatever the chosen solution). Therefore, in order to recover the loss in sensitivity, a PP solution would be preferable in this case. Note that PP squeezed-film damping would be useful in this case to lower the Q factor to the required value (around about 0.5, as mentioned above).

NOTE: other comments or discussions, even partially differing from those above, will be positively evaluated if correctly developed and substantiated

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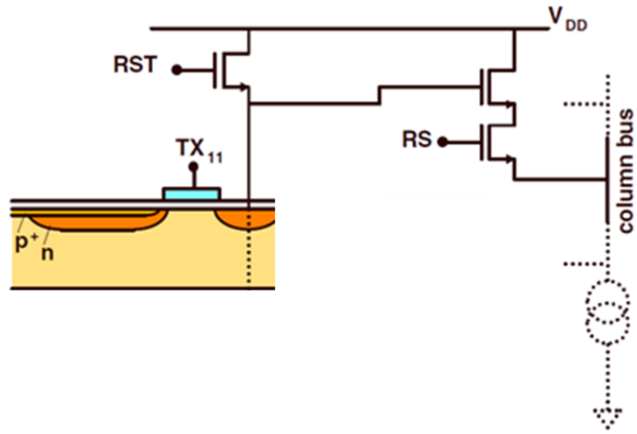
Question n. 2

You are developing a 10-MP 4T CMOS image sensor, whose overall width and height are 4.9 mm and 3.67 mm, respectively.

The sensor operates at 1.8 V supply voltage. The whole in-pixel electronic circuitry (Reset + Source Follower + Row Selection transistors, and inter-connections) occupies an area of 1100 nm x 1100 nm. The area occupied by both the Transmission Gate and the Floating Diffusion is negligible.

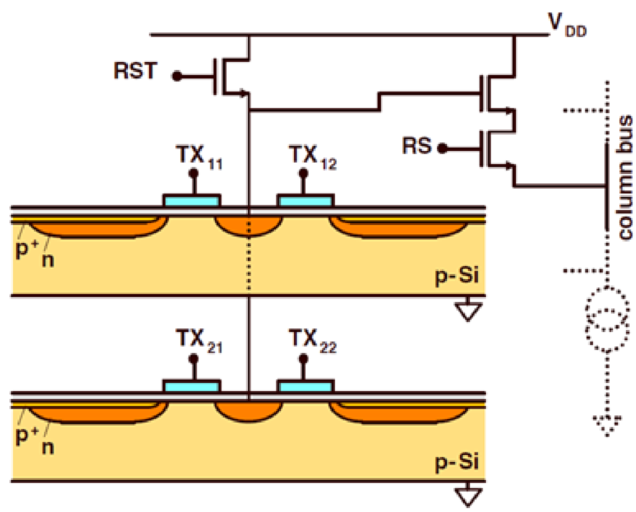


The quantum efficiency is 0.5. The dark current density is $3.4 \cdot 10^{-4}$ A/m^2. The input capacitance of the Source Follower is 0.5 fF.

1) Calculate the photocurrent generated in a pixel with an impinging photon flux of $0.5 \cdot 10^{18}$ ph/s/m^2.

2) Calculate the signal-to-noise ratio of the considered pixel, assuming an integration time of 8 ms.

Consider now another sensor, designed with the same technology and with the same overall area as above, now employing a 7/4T readout architecture, as shown here aside.



3) Evaluate the SNR improvement/worsening with respect to the first sensor.

4) Comment on which of the following parameters get improved/worsened in case CDS is used or not:

- SNR with same integration time;
- DR with same integration time.

Draw and comment a possible circuit to implement CDS.

Physical Constants

$q = 1.6\ 10^{-19}$ C

$k_b = 1.38\ 10^{-23}$ J/K

T = 300 K (if not specified)

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$\varepsilon_{r,Si} = 11.7$

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1. The area of the sensor is calculated as:

$$A_{sensor} = W \cdot H$$

and the following relationship holds:

$$A_{sensor} = (A_{PD} + A_{eln}) \cdot N_{pix}$$

Hence:

$$A_{PD} = \frac{A_{sensor}}{N_{pix}} - A_{eln} = (767\ \mathrm{nm})^2$$

The photocurrent i_{ph} under a photon flux ϕ at the given quantum efficiency η is thus:

$$i_{ph} = q\,\eta\,\phi\,A_{PD} = 23.5\ \mathrm{fA}$$

2. The number of collected electrons within the integration time can be calculated:

$$N_{e^-} = \frac{i_{ph} \cdot t_{int}}{q} = 1176\ e^-$$

At this point, one can calculate the reset noise, the dark current shot noise and the signal shot noise, in terms of electrons rms:

$$\sigma_{reset} = \frac{\sqrt{kTC_{SF}}}{q} = 9\ e^-_{rms}$$

$$\sigma_{dark} = \frac{\sqrt{q i_{dark} t_{int}}}{q} = 3.2\ e^-_{rms}$$

$$\sigma_{shot} = \frac{\sqrt{q i_{ph} t_{int}}}{q} = 34.3\ e^-_{rms}$$

Note that reset noise may be cancelled, thanks to the possibility of applying a CDS technique in the 4T architecture.

In any case, shot noise dominates and the SNR is:

$$SNR_{dB} = 20\log_{10}\frac{N_{e^-}}{\sigma_{tot}} \simeq 10\log_{10}\frac{N_{ph}}{\sigma_{shot}} = 30.6\ \mathrm{dB}$$

3. With the 7/4T architecture, the area of the single photodiode within the pixel can be made bigger, as large part of the electronics is shared:

$$(4A_{PD} + A_{eln})\frac{N_{pix}}{4} = A_{sensor}$$

Hence:

$$A_{PD} = \frac{1}{4}\left(\frac{A_{sensor}}{\frac{N_{pix}}{4}} - A_{eln}\right) = (1.22\ \mu\text{m})^2$$

Note that the photosensitive area increases by more than a factor 2.5. The new photocurrent and number of collected electrons are calculated with the same expression as above:

$$i_{ph} = 60\ \text{fA}$$

$$N_{e^-} = 2991\ e^-$$

Reset noise is the same, dark current shot noise may be a little bit higher due to the increased area, but signal shot noise (increased by the area increase too) will dominate, again:

$$\sigma_{shot} = 54.87\ e^-_{rms}$$

The SNR becomes

$$SNR_{dB} \simeq 20\log_{10}\frac{N_{ph}}{\sigma_{shot}} = 34.73\ \text{dB}$$

which is 4.6 dB higher than in a standard 4T transistor.

Alternatively, this result could be anticipated without making all calculations above, according to the fact that SNR increases with the square root of the active area when shot noise dominates. As the active area increases by a factor 2.53, SNR increases by a factor:

$$20\log_{10}\sqrt{2.53} = 4.03\ dB$$

In case CDS is used, SNR with same integration time does not change, as shot noise dominates, and the signal is not affected by CDS.

On the other hand, DR improves, as dark current shot noise becomes dominant (5 e^-_{rms}) since reset noise contribution (9 e^-_{rms}) is now eliminated.

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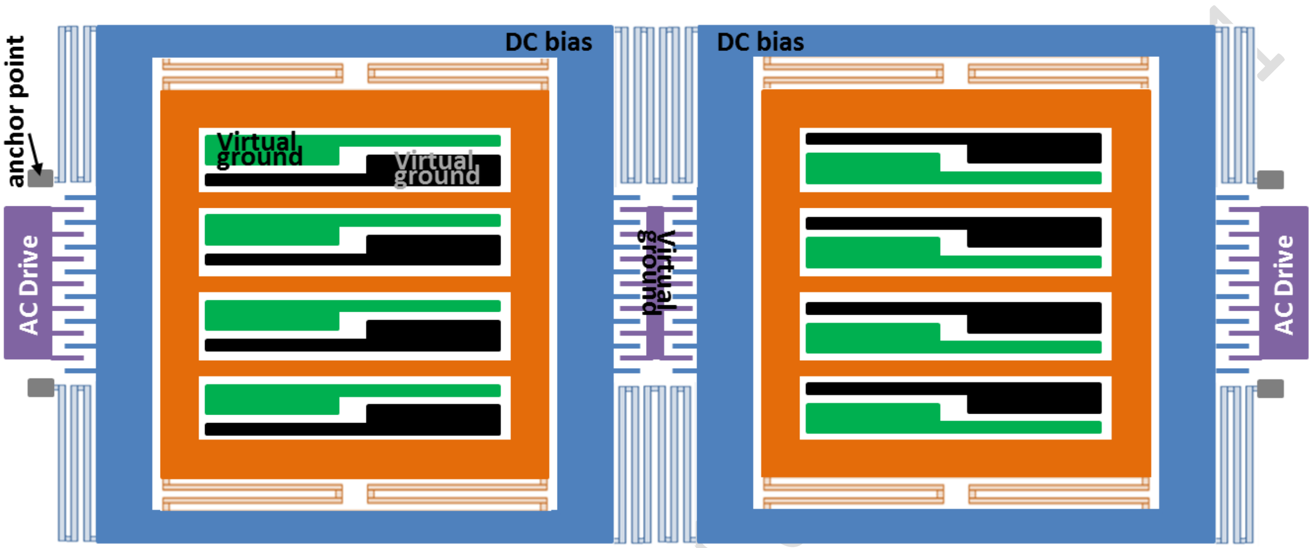
Question n. 3

Consider the Z-axis, dual-mass, tuning-fork gyroscope structure shown in the figure below, used for aerospace applications. The structure is actuated via comb fingers along the drive mode, and senses Coriolis induced displacements through differential parallel plates along the sense mode. Relevant parameters are given in the Table below. During take-off and landing operations, the gyroscope can be subject to large accelerations.



1) For each half-mass of the sense mode, evaluate the single-ended capacitance variation per unit displacement. Then, evaluate the overall nominal differential capacitive sensitivity of the gyroscope, $S_{nom} = \Delta C_{diff}/\Delta\Omega$, and the maximum displacement corresponding to the FSR.

2) Assume that a 170 g (g=9.8 m/s^2) acceleration signal (with a frequency content << f_d) acts on the gyroscope along the direction of the sense mode. Evaluate the displacement caused by this acceleration on each half-mass of the sense mode.

3) Evaluate again the sensitivity of the gyroscope, this time under the acceleration considered at point 2). Discuss the obtained result in light of the requirements on maximum acceptable sensitivity and offset variations (assume quadrature as the dominant offset source).

Parameter	Symbol	Value
General parameters		
Process Gap	g	2 μm
Permittivity of vacuum	ε_0	8.85 10^{-12} F/m
Target full-scale-range	FSR	1000 dps
Drive mode		
Drive frequency	f_D	15 kHz
Controlled motion amplitude	x_D	6 μm
Sense mode		
Sense frequency (in operation)	f_S	16 kHz
Single-ended capacitance of ½ mass	C_{se}	200 fF
Quadrature error	B	10 dps
Required tolerance		
Maximum offset variation	OS_{max}	1% FSR*
Maximum sensitivity variation	ΔS_{max}	1% S_{nom}**

** this specification indicates that offset can vary, with respect to the initial absolute offset value, by maximum 1% of the full-scale-range*

*** this specification indicates that the sensitivity can change by maximum 1% of its nominal value*

4) The gyroscope is readout through a pair of charge amplifiers. Assuming all other noise sources as negligible, determine the required voltage noise spectral density of the operational amplifiers to guarantee a 10 mdps/√Hz resolution when operating with the rotor biased at 7 V, and assuming 5 pF parasitic capacitance.

1. We begin by assuming that displacements induced by the Coriolis force are small, so we use the linearized approximation for the capacitance variation. This will be given (e.g. for the left, black electrodes) by:

$$\left.\frac{\Delta C}{\Delta y}\right|_{\frac{1}{2},SE} = \frac{C_0}{g} = \frac{200\,fF}{2\,\mu m} = 100\,\frac{fF}{\mu m}$$

The gyroscope features a differential readout (green electrodes as well), and one has to consider also the second half of the device. Therefore, the overall differential capacitive variation per unit displacement is four times the value above, i.e. $\left.\frac{\Delta C}{\Delta y}\right|_{diff} = \mathbf{400\,fF/\mu m}$. To evaluate the overall capacitive sensitivity per unit angular rate, we just need to write the dependence of the displacement *y* in the sensing direction per unit angular rate, and to combine this with the found capacitance change per unit *y* in mode-split conditions:

$$\frac{\Delta y}{\Delta \Omega} = \frac{x_d}{\Delta \omega} = \frac{6\mu m}{2\,\pi\,1\,kHz} = 0.95\,\frac{nm}{rad/s} \quad \rightarrow \quad \frac{\Delta C_{diff}}{\Delta \Omega} = 400\,\frac{fF}{\mu m} \cdot 0.95\,\frac{nm}{\frac{rad}{s}} = 380\,\frac{aF}{\frac{rad}{s}} = \mathbf{6.6\,\frac{aF}{dps}}$$

To conclude point n. 1, the maximum displacement is readily found by multiplying 0.95 nm/(rad/s) by the FSR (1000 dps = 17.45 rad/s). One obtains a value of y_{max} = **16.6 nm**.

2. Ideally, the low-frequency acceleration will shift both the half-masses by the same quantity, given (as in an accelerometer) by:

$$\Delta y_{acc} = \frac{F_{acc}}{k_y} = \frac{m_y a}{k_y} = \frac{a}{\omega_y^2} = \frac{170 \cdot 9.8\,\frac{m^2}{s}}{(2\,\pi\,16\,kHz)^2} = \mathbf{165\,nm}$$

We note that this value is 10 times larger than the maximum displacement induced by the Coriolis force at the gyroscope FSR. We also note that this displacement is about 1/10 of the gap, which suggests that we are close to the point where a linear approximation may fail.

3. Following the last consideration of point 2, we write the non-linearized expression of the capacitance variation per unit displacement when considering the low-frequency (say DC with respect to the operation frequency of 15 kHz) displacement given by the acceleration.

We stress again that the differential configuration rejects the largest acceleration effect, i.e. the change in ΔC that would be obtained by looking just at a single half of the device. However, there is a residual effect which can be seen as a rigid change in the gap between parallel plates. Consider the left half of the device, and write the sensitivity while the device is displaced by Δy_{acc}, then double the found value as also the right part needs to be considered:

$$\left.\frac{\Delta C}{\Delta y}\right|_{SE,acc} = \frac{C_0\,g}{(g-\Delta y_{acc})^2} + \frac{C_0\,g}{(g+\Delta y_{acc})} = 204.1\,\frac{fF}{\mu m} \quad \rightarrow \quad \left.\frac{\Delta C}{\Delta y}\right|_{diff,acc} = 2\,\left.\frac{\Delta C}{\Delta y}\right|_{SE,acc} = 408.2\,\frac{fF}{\mu m}$$

The percentage sensitivity change with respect to the nominal condition can be thus evaluated as:

$$\Delta S_{\%} = \frac{(408.2 - 400)}{400} * 100 = \mathbf{2.06\%}$$

from which we can conclude that the requirements on maximum sensitivity variations are not satisfied during operation at such large accelerations.

If the capacitive transduction factor changes by 2.06%, the offset generated by quadrature will correspondingly change. Indeed quadrature motion is sensed exactly by the same electrodes used for the sensitivity above. Therefore, the quadrature offset will experience itself a 2.06% variation of the nominal value (10 dps). In absolute value, this offset changes thus by 0.206 dps. This corresponds to much less than 1% of the FSR (1000 dps), which guarantees that the tolerance on offset drift is satisfied by a rather large amount, as **0.206 dps is << 10 dps**.

4. Assuming the operational amplifier noise is the dominant contribution, amplified through the parasitic capacitance C_P, the expression of the input-referred noise density in mode-split conditions becomes, after a few simplifications:

$$\sqrt{S_{\Omega n}} = \frac{\sqrt{2 \cdot S_{n,op}}\left(1 + \frac{C_P}{C_F}\right)}{\frac{\Delta V_{out}}{\Omega}} = \sqrt{\frac{S_{n,op}}{2}}\left(\cancel{1+}\frac{C_P}{C_F}\right)\frac{C_F}{C_{SE}}\frac{g}{V_{DC}}\frac{\Delta\omega}{x_D}\frac{180}{\pi}$$

$$\sqrt{S_{\Omega n}} = \frac{180}{\pi}\frac{1}{x_D}\sqrt{\frac{S_{n,op}}{2}}\frac{C_P}{C_{SE}}\frac{g\,\Delta\omega}{V_{DC}}$$

The expression is valid for a feedback capacitance C_F of the charge amplifiers much lower than the parasitic value of 5 pF, and becomes independent of C_F. This is a reasonable approximation. In these conditions, by inverting the expression above, we find the amplifier noise that guarantees the required noise density:

$$\sqrt{S_{n,op}} = \sqrt{2\, S_{\Omega n}}\frac{\pi}{180}\, x_D \frac{C_{SE}}{C_P}\frac{V_{DC}}{g\,\Delta\omega_{MS}} = \sqrt{2}\, 10\frac{mdps}{\sqrt{Hz}}\frac{\pi}{180}\; 6\;\mu m\frac{400\, fF}{5\, pF}\frac{7\; V}{2\;\mu m\; 2\;\pi\;\; 1\;\text{kHz}} = \mathbf{65\; nV/\sqrt{Hz}}$$

Intuitively, to target a given input-referred rate density $\sqrt{S_{\Omega n}}$, we can accept a larger operational amplifier noise $\sqrt{S_{n,op}}$ if either the gyroscope displacement x_D is larger, or the biasing voltage V_{DC} is larger, or the parasitic C_P is lower, or the gap is lower… and so on.