Question n. 1

You are a designer of in-plane MEMS accelerometers, with the specific task of improving the sensitivity to accelerations. Due to budget limitations, you can ask to process engineers only one of the following possible process improvements:

1 - decrease of the process gap

2 - increase of the process thickness

3 – decrease of the minimum spring width

Which one would you choose? Motivate in details your choice. You can help yourself with formulas or graphs.

The **sensitivity** of a **MEMS accelerometer** can be first seen as the ratio between the suspended mass displacement *x* and the occurring acceleration a_{ext}. One easily notes that the unique parameter that appears in the formula is the resonance frequency:

$$\frac{x}{a_{ext}} = \frac{1}{\omega_0^2}$$

At a deeper level however, **one should consider the way the MEMS is coupled to the circuit**, and the arising **electrostatic force**. In the simplest situation of a charge amplifier readout, one can develop the sensitivity **equation** to write the **output voltage vs input acceleration**:

$$\frac{x}{a_{ext}} = \frac{m}{\left(k - 2V_{DD}^2 \frac{C_0}{g^2}\right)} \rightarrow \frac{\Delta V_{out}}{a_{ext}} = 2\, V_{DD} \frac{C_0}{C_f}\, \frac{1}{g} \frac{m}{\left(k - 2V_{DD}^2 \frac{C_0}{g^2}\right)}$$

In view of the derived formulas we can make the following comments:

1- decrease of the **process gap**

As in all parallel-plate sensing configuration, the sensitivity **at first order improves with the inverse of the squared gap**, (C_0/g). This seems thus to be a quite advantageous option.

However, for devices with **relatively low stiffness like accelerometers**, there is a second-order effect, highlighted at the denominator by the formula above, which is the **change in the stiffness and resonance frequency induced by electrostatic forces**. This effect is as well **a function of the gap**. A lower gap may thus become **critical for pull-in issues** in the considered *voltage-controlled* readout.

Further, **a decrease in the gap also reduces the displacement linearity range**. So, in order to remain within maximum allowed displacement one would need to increase the stiffness, in turn reducing the sensitivity…

2 – increase of the **process thickness**

An increase in the thickness *h* **directly implies an increase in the rest capacitance C_0**. This turns into a **linear increase in the sensitivity**. It appears thus as a good option.

Let us verify the **dependence of the pull-in voltage** on this parameter for an in-plane accelerometer. The expression is given below:

$$V_{DD,PI} = \sqrt{\frac{g^2\ k}{2\ C_0}} = \sqrt{\frac{g^3\ \ E\ \alpha\ h\ w^3/L^3}{2\ \varepsilon_0\ L_{PP}\ h\ N_{PP}}}$$

One can note that the **pull-in voltage includes itself a term related to C_0**; however, it also **includes the elastic stiffness *k***. Developing the formula for the case of **in-plane motion springs**, one will note the linear dependence of *k* on *h*. Therefore, the **pull-in voltage turns out to be independent on the process thickness**!

Linearity is as well independent on this parameter. So, there is **no drawback in increasing *h***, with the mentioned positive effect of increasing linearly the sensitivity.

3 – decrease in the **springs width**

A decrease in the springs width **directly determines a decrease in the spring stiffness**. This is apparently advantageous in terms of sensitivity. However, once more, **the pull-in voltage is strongly dependent on *w*** and more in general on the spring stiffness (see the formula above).

We can **therefore conclude that the best choice is likely to ask for an increase in the process height**. There are **a few final comments/exceptions that deserve to be given**, for alternative accelerometers configurations:

A – in case we adopted **a comb finger solution**, we would be void of pull-in and linearity issues. In this situation a choice of gap decrease would not be subject to those issues. However, the sensitivity gain would go with the inverse of the gap, and not with the inverse of the squared gap! In turn, the gain would be (at first order) linear with the gap decrease, exactly as it is linear with the thickness increase. So, there would be no specific advantage in choosing either of the two process changes.

B – if we adopted an **equivalent *charge-controlled* readout** (e.g. based on switched capacitors), the **sensitivity formula** would become:

$$\frac{\Delta V_{out}}{a_{ext}} = \frac{V_{DD}}{\omega_0^2\ \ g} = \frac{V_{DD}\ m}{k\ \ g}$$

Which clearly states that in this case – **contrarily to the discussion above** – we would have **advantages in changing the gap *g*** rather than in changing the height (*h* does not appear in the formula as both *m* and *k* are linear with this term)! No pull-in and linearity issues exist in this situation. This is a very nice example of how – when you design a MEMS and/or you develop a process – you should **always take into account the co-interaction between device and chose electronic readout scheme**!

For both situations A and B, decreasing the minimum width *w* – though advantageous as no pull-in/linearity issues are present – will likely face **process repeatability issues**.

Question n. 2

You are taking a picture with a camera equipped with a 3T CMOS image sensor. The sensor works with a 1.8 V supply voltage. Each pixel, 1.5 µm wide, has a 50% fill factor and an overall quantum efficiency of 0.5. The dark current is 0.3 fA. The average photodiode capacitance is 1 fF, while the input capacitance of the source follower is 0.5 fF.

1) The brightest pixel of the scene features an input photon flux of $2 \cdot 10^{18}$ ph/m^2. Which integration time would you choose in order to maximize the signal-to-noise ratio of the acquisition?

2) How much is the dynamic range at the chosen integration time?

You are equipped with another camera, which features a 4T CMOS sensor. Each pixel (again with a 1.5 µm size) is based on a pinned photodiode, a transfer gate (of negligible area), and the same in-pixel electronics, and implements correlated double sampling. The capacitance of the floating diffusion can be neglected.

3) Which integration time would you now choose in order to maximize the signal-to-noise ratio of the acquisition?

4) With the chosen integration time, evaluate the improvement/worsening of the dynamic range (expressed in dB) with respect to the 3T topology.

Physical Constants
$q = 1.6 \; 10^{-19}$ C
$k_b = 1.38 \; 10^{-23}$ J/K
T = 300 K (if not specified)
$\varepsilon_0 = 8.85 \; 10^{-12}$ F/m
$\varepsilon_{r,Si} = 11.7$

The photocurrent of the brightest pixel can be calculated as

$$i_{ph} = q \cdot \eta \cdot FF \cdot l_{pix}^2 \cdot \phi_{ph} = 180 \text{ fA}$$

The maximum charge that can be integrated is

$$Q_{max,3T} = V_{DD} \cdot C_{int} = V_{DD} \cdot (C_{PD} + C_{SF}) = 2.7 \text{ fC}$$

which corresponds to 16875 electrons. Both the photodiode capacitance and the source follower capacitance should be considered to determine the overall integration capacitance.

One can chose the maximum integration time as the one that sets the brighter pixel close to saturation:

$$i_{ph} t_{int,3T} = Q_{max,3T} \rightarrow t_{int,3T} = \frac{Q_{max,3T}}{i_{ph}} = 15 \text{ ms}$$

With this integration time, the dynamic range, expressed in terms of number of electrons, is

$$DR = 20\log_{10}\frac{\frac{i_{ph}t_{int,3T}}{q}}{\sqrt{\frac{kTC_{int}}{q^2}+\frac{i_d t_{int}}{q}}} = 20\log_{10}\frac{16875}{\sqrt{15.6^2+5.3^2}} = 20\log_{10}\frac{16875}{16.4} = 60\ \mathrm{dB}$$

As the 4T transistor features same in-pixel electronics and negligible area transfer gate, the pinned diode area is the same and the fill factor is 50%. Hence, the photocurrent is the same as before.

The maximum charge that can be integrated is

$$Q_{max,4T} = V_{DD}\cdot C_{int} = V_{DD}\cdot C_{SF} = 0.9\ \mathrm{fC}$$

which corresponds to 5625 electrons. Floating diffusion capacitance was neglected, as suggested in the text.

Again, one can chose the maximum integration time as the one that sets the brighter pixel close to saturation:

$$i_{ph}t_{int,4T} = Q_{max,4T} \rightarrow t_{int,4T} = \frac{Q_{max,4T}}{i_{ph}} = 5\ \mathrm{ms}$$

With this integration time, considering that the sensor implements correlated double sampling (i.e. no kTC noise), the dynamic range, expressed in terms of number of electrons, is

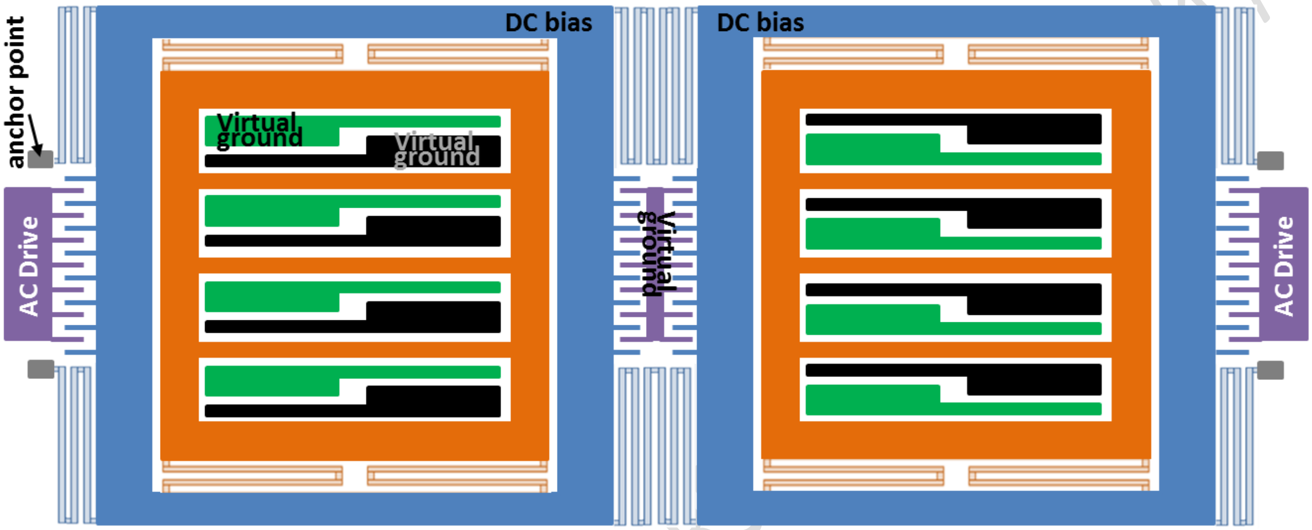
$$DR = 20\log_{10}\frac{\frac{i_{ph}t_{int,4T}}{q}}{\sqrt{\frac{i_d t_{int}}{q}}} = 20\log_{10}\frac{5625}{3} = 65\ \mathrm{dB}$$

This means that the 4T sensor enables a 5 dB increase of the dynamic range.

One should note that, in reality, the increase will be even higher, since pinned diode structures usually enable lower dark currents. With a typical 10x reduction of the dark current, an additional 10 dB increase of the dynamic range is obtained.

Question n. 3

Consider the Z-axis, dual-mass, tuning-fork gyroscope structure shown in the figure below. The structure is actuated via comb fingers along the drive mode, and senses Coriolis induced displacements through differential parallel plates along the sense mode. The gyroscope is biased with a DC voltage applied to the rotor (suspended mass) and an AC sine voltage applied to the driving stators. The drive-detection stators and the sense stators are kept at 0 V through virtual grounds. Relevant parameters are given in the Table below.



1) Choose the values of the DC and AC voltage to apply, in order to obtain a drive mode displacement of 5 μm.

2) Calculate the sensitivity in terms of sense mode displacement y per unit angular rate Ω; then in terms of differential capacitance variation ΔC_{diff} per unit angular rate; finally in terms of output voltage change ΔV_{out} per unit rate, assuming a differential sense front-end based on charge amplifiers (feedback capacitance C_F), a further differential amplifier with a gain G_{INA}, and a demodulation followed by a low-pass filter.

3) Evaluate the maximum measurable angular rate that guarantees a linearity error < 0.2%. Estimate then the supply voltage $\pm V_{DD}$ required by the amplifiers of the sense chain to match the maximum rate.

4) Choose (with motivations) the frequency of the 2nd order low-pass filter, so to filter out the undesired peak corresponding to the mode-split frequency.

Parameter	Symbol	Value
Process thickness	h	20 μm
Process Gap	g	2 μm
Young's modulus	E	168 GPa
Permittivity of vacuum	ε_0	8.85 10^{-12} F/m
Drive mode		
Drive frame mass	m_D	2.7 10^{-9} kg
Number of comb fingers	N_{CF}	60
Spring fold length	L_F	149 μm
Spring fold width	w_F	3.4 μm
Quality factor	Q_D	5000
Sense mode		
Sense frame mass	m_S	3.2 10^{-9} kg
Elastic stiffness	k_S	33 N/m
Parallel-plate length	L_{PP}	198 μm
Parallel-plate cell number	N_{PP}	10
Quality factor	Q_S	200
Electronics		
Feedback capacitance	C_F	0.5 pF
Amplifier gain	G_{INA}	20
Low-pass-filter slope	LPF	-40 dB/dec

5) Finally, evaluate the percentage variation in the sensitivity under temperature changes of ±60°C, with respect to the value calculated at point 3) above.

1) The picture represents a differential gyroscope based on a tuning fork. The geometry relies on a doubly decoupled architecture. Start with the forces balance, in particular the balance of the elastic force and electrostatic force, at resonance:

$$k_d \cdot x = F_{elec,tot} \cdot Q_d$$

Then, find the missing parameter k_d. **Focus on the left half of the device**: note 2 springs (drive spring), with 3 folds. On the inner side, note 2 springs (tuning fork) with 3 folds each: the middle point of the entire tuning fork is "virtually fixed" thanks to the action-reaction principle. Thus, 4 springs (number of springs N_s=4) with 3 folds (number of folds N_f=3) are connected to the drive frame. So, the total elastic stiffness is

$$k_d = E\frac{N_s}{N_f}h\left(\frac{w_F}{L_F}\right)^3 = 53\ N/m$$

Two different contributions give the total electrostatic force: the former is by the drive-actuation comb finger stator, the latter is by the drive-detection comb finger stator.

$$F_{elec,tot} = \varepsilon_0 N_{CF}\frac{h}{g}(V_{DC} - v_{ac})^2 - \varepsilon_0 N_{CF}\frac{h}{g}{V_{DC}}^2 \xrightarrow{v_{ac} \ll 2V_{DC}} F_{elec,tot} = \varepsilon_0 N_{CF}\frac{h}{g}2V_{DC}v_{AC}$$

In general and as a rule of thumb, "much-greater" or "much-lower" means one order of magnitude (1 << 10), thus $v_{AC}=V_{DC}/5$. Then,

$$V_{DC}v_{AC} = 5v_{AC}^2 = x\frac{k_d}{Q_d}\frac{1}{2\varepsilon_0 N_{CF}\frac{h}{g}} \rightarrow v_{AC} = 1.0012\ V, \qquad V_{DC} = 5.0061\ V.$$

All the solutions that reasonably guarantee the condition $v_{ac} \ll 2V_{DC}$ are considered correct. Reasonably means that too large voltages (e.g. > 20 V) would be challenging to generate.

2) The sensitivity in terms of sense mode displacement y per unit angular rate Ω is given by

$$\frac{y}{\Omega} = \frac{x}{\Delta\omega}$$

We need first to check whether the device is operating at resonance or in mode-split conditions. So, we first find the two fundamental resonance frequencies. For the drive mode

$$\omega_d = \sqrt{\frac{k_d}{m_d + m_s}} = 95\frac{krad}{s} \qquad f_d = 15117Hz$$

For the sense mode, we need to take into account effects of electrostatic softening:

$$k_{el} = -2\varepsilon_0 N_{PP}\frac{hL_{PP}}{g^3}{V_{DC}}^2 = -2.2\frac{N}{m} \qquad \rightarrow \qquad k_{s,0} = k_s + k_{el} = 30.8\frac{N}{m}$$

$$\omega_s = \sqrt{\frac{k_{s,0}}{m_s}} = 98\frac{krad}{s}, \qquad f_s = 15615Hz$$

$$\Delta\omega = \omega_s - \omega_d = 3130\frac{rad}{s}, \quad \Delta f = f_s - f_d = 498\ Hz$$

$$\frac{\Delta y}{\Delta\Omega} = \frac{x}{\Delta\omega} = 1.6\frac{nm}{rad/s} = 28\frac{pm}{°/s}$$

The sensitivity in terms of differential capacitance variation ΔC_{diff} per unit angular rate is given by (a further factor 2 is due to the other "half" of the device):

$$\frac{\Delta C_{diff}}{\Omega} = 2 \cdot 2\varepsilon_0 N_{PP} \frac{hL_{PP}}{g^2} \frac{\Delta y}{\Delta\Omega} = 0.56 \frac{fF}{\frac{rad}{s}} = 9.8 \frac{aF}{°/s}$$

Finally, the sensitivity in terms of output voltage change ΔV_{out} per unit rate is given by

$$\frac{\Delta V_{out}}{\Omega} = \frac{\Delta C_{diff}}{\Omega} \frac{V_{DC}}{C_f} G_{INA} T_{dem} T_{LPF}$$

The output signal is modulated at f_d. The transfer function of an ideal demodulation, based on the multiplication by a harmonic sinewave at f_d (obtained from the drive loop) gives a factor ½ for the baseband signal. The signal component at twice f_d is filtered out, so:

$$T_{dem} = \frac{1}{2}$$

We further assume that the transfer function of the LPF is unitary for signals within the gyroscope bandwidth. Thus

$$\frac{\Delta V_{out}}{\Omega} = \frac{\Delta C_{diff}}{\Omega} \frac{V_{DC}}{C_f} G_{INA} \frac{1}{2} 1 = 56 \frac{mV}{rad/s} = 1 \frac{mV}{°/s}$$

3) The linearity error is defined as

$$\epsilon_{lin} = \frac{(\Delta C_{real,FSR} - \Delta C_{lin,FSR})}{\Delta C_{real,FSR}} \cdot 100$$

Where

$$\Delta C_{real,FSR} = C_0 \left(\frac{2\frac{y_{FSR}}{g}}{1 - \left(\frac{y_{FSR}}{g}\right)^2} \right), \qquad \Delta C_{lin,FSR} = 2C_0 \frac{y_{FSR}}{g}$$

Thus

$$y_{FSR} = g\sqrt{\frac{\epsilon_{lin}}{100}} = 90\ nm$$

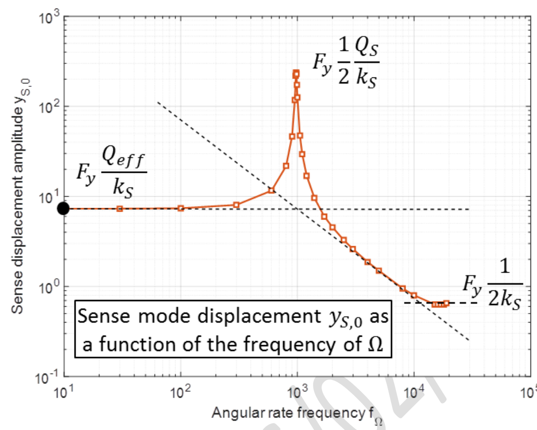
And then, using the expression of mechanical sensitivity, find the maximum measurable angular rate that guarantees a linearity error < 0.2%

$$\Omega_{FSR} = \frac{y_{FSR}}{\frac{\Delta y}{\Delta\Omega}} = 62 \frac{rad}{s} = 3210°/s$$

The supply voltage $\pm V_{DD}$ required by the amplifiers of the sense chain to match the maximum rate is given by

$$\pm V_{DD} = \pm \frac{\Delta V_{out}}{\Omega} \Omega_{FSR} = 3.21\ V$$

4) In the figure below it is reported a sample graph of the sensitivity as a function of the angular rate frequency (note that the graph shows the case of a mode-split value of 1 kHz, while here



we have 500 Hz). The frequency of the 2nd order low-pass filter is selected in order to filter out the undesired peak corresponding to the angular rates occurring close to the mode-split value.

The peak of the considered curve is $Q_s/2/Q_{eff}$ times larger than its DC response.

$$\frac{Peak\ Value}{DC\ Value} = \frac{\frac{Q_s}{2}}{\frac{f_s}{2\,\Delta f}} = \frac{100}{15.6} = 6.4$$

As a first approximation, we may force that this amplification is cancelled by the LPF in such a way that the gain at the mode-split value is brought back to the DC value. We thus have to reduce the gain by a factor 6.4 with a two-pole system (-40 dB/dec). The equation to find the cut-off value for the LPF can be thus written as:

$$\left|\left(\frac{1}{1+j\Delta\omega\tau_{LPF}}\right)^2\right| = \frac{1}{6.4} \to \left|\left(\frac{1}{j\Delta\omega\tau_{LPF}}\right)^2\right| = \frac{1}{6.4} \to \left(\frac{f_{LPF}}{\Delta f}\right)^2 = \frac{1}{6.4} \to f_{LPF} = \frac{\Delta f}{\sqrt{6.4}} = 197\ Hz$$

A good approximation is to say – without taking the calculations above, that the filter should be typically placed between $\Delta f/2$ (250 Hz) and $\Delta f/3$ (166 Hz).

5) In a first approximation, one can consider only the frequency variation of the resonance frequencies.

$$\partial f_d = TC_F \cdot \pm dT \cdot f_d = \pm 27.21\ Hz$$
$$\partial f_s = TC_F \cdot \pm dT \cdot f_s = \pm 28.11\ Hz$$
$$\partial \Delta f = \partial f_s - \partial f_d = \pm 0.9\ Hz$$
$$\partial Sens = \frac{\partial \Delta f}{\Delta f} = \pm 0.18\%$$

Indeed, in presence of an AGC, the temperature effects on the quality factor will be controlled. However, as the text says nothing about the presence of the AGC, we also check the effects caused by changes in the drive mode quality factor:

$$\partial Sens = \frac{dQ_D}{Q_D} = -\frac{1}{2}\frac{dT}{T} = \pm 10\%$$

AS the sensitivity is linear with the displacement, which is in turn linear with Q_D, this is the variability that would affect the sensitivity in absence of an AGC.