Last Name: _Del Voto_ Given Name: _Felice__ ID number: __20160203_

Question n. 1

"In a MEMS gyroscope, as the Coriolis force is proportional to the drive velocity, it is fundamental to keep the AC drive velocity amplitude constant during operation".

Comment in details the statement above, addressing in particular advantages and drawbacks of using this suggestion, referred (i) to the oscillator implementation, (ii) to possible process nonuniformities and (iii) to in-operation temperature variations.

You can make use of and refer to equations or graphs when needed.

In a MEMS gyroscope, though the Coriolis force is proportional to the drive velocity, if one develops the sensitivity equation, it turns out that this term is linear with the drive displacement x_d.

$$|y_s| = \left|\frac{F_{Cor} Q_S}{k_s}\right| = 2 m_s \dot{x}_d \Omega \frac{Q_s}{k_s} = 2 \frac{m_s}{k_s}\, \omega_d\, x_d\, \Omega\, Q_s = \frac{2Q_s}{\omega_s} \frac{\omega_d}{\omega_s}\, x_d\, \Omega \approx \frac{x_d}{\Delta\omega}\, \Omega \quad \rightarrow \quad \frac{|y_s|}{\Omega} = \frac{x_d}{\Delta\omega}$$

In the formulas above, Q_s can be assumed as the quality factor of the sense mode (for mode-matched operation), or the effective quality factor (if in off-resonance operation mode).

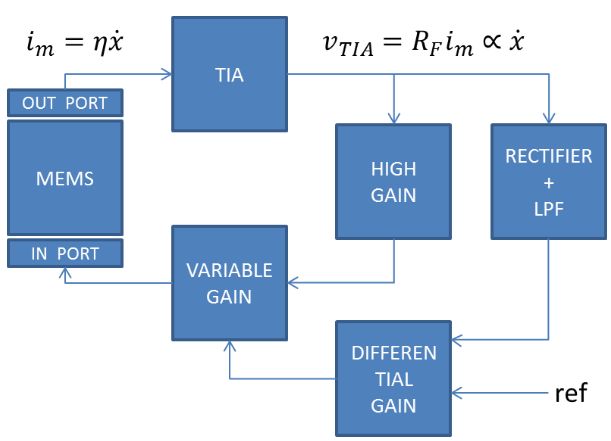
Noting that for AC signals the following relationship holds:

$$\dot{x_d} = \omega_d\, x_d$$

we observe that, if we control the velocity and if the drive frequency is kept stable, then we are indirectly controlling the displacement, which thus apparently keeps the sensitivity term $\frac{|y_s|}{\Omega}$ stable.

More detailed comments can be made as follows.

(i) on the drive oscillator topology: as the current at the output port of a MEMS resonator is proportional (through the electromechanical transduction factor η) to the resonator velocity, one can use a trans-resistance amplifier (TIA) as a front-end circuit. In this way its output voltage will be linear with the drive velocity. We can rectify and low-pass this signal, compare it with a reference voltage corresponding to the desired velocity, and use the difference to drive a variable gain in the primary loop (this is similar to what we have seen for the displacement control, where however a charge amplifier should be used as a front-end stage).



We note here that the implementation of an oscillator based on a TIA front-end stage avoids that need for a 90° shifter. It is however generally less performing in terms of noise because the value of the feedback resistance, to be repeatable, should be typically kept low. A schematic example of such an oscillator is reported in the figure above.

(ii) on process nonuniformities: unavoidable fabrication imperfections will cause the drive frequency of two nominally identical gyroscopes to be different. If the oscillator circuit controls the velocity, but the frequency of the two samples is different, the two gyroscopes will show a different oscillating displacement and thus a different sensitivity, according to the equations above.

This can be quite problematic: as an example, just consider that the frequencies can vary from part to part typically up to 500 Hz - 1 kHz, around drive frequencies of about 20 kHz.

This implies directly a sensitivity variation from part to part in the order of 2.5% to 5% even in mode-split operation, which is unacceptable.

(iii) on the temperature behavior: even considering a single gyroscope, we can observe that its frequency changes with changing operating temperature. Therefore if we control the velocity, and the frequency is changing, the motion amplitude is changing itself, and so does the sensitivity. Let us try to quantify whether this effect is relevant or not.

We know that resonance frequency changes in MEMS devices are mostly related to the temperature behavior of the Young's modulus of polysilicon, which leads to frequency variations in the order of -30 ppm/K.

Assuming a maximum temperature range of 120 K (e.g. -40°C to 80 °C), we have a maximum frequency change of -30 ppm/K * 120 K = 3600 ppm = 0.36% which can be considered negligible.

We can thus conclude that the proposed statement is not wholly correct: indeed, by controlling the velocity amplitude, we obtain sensitivity differences from sample to sample. Besides, though the implementation of the oscillator is simplified, a TIA solution will result more noisy.

A better approach would be to control the displacement amplitude using a low-noise charge amplifier front-end. The control of the displacement would keep in this case the sensitivity stable also for devices with different drive frequencies.

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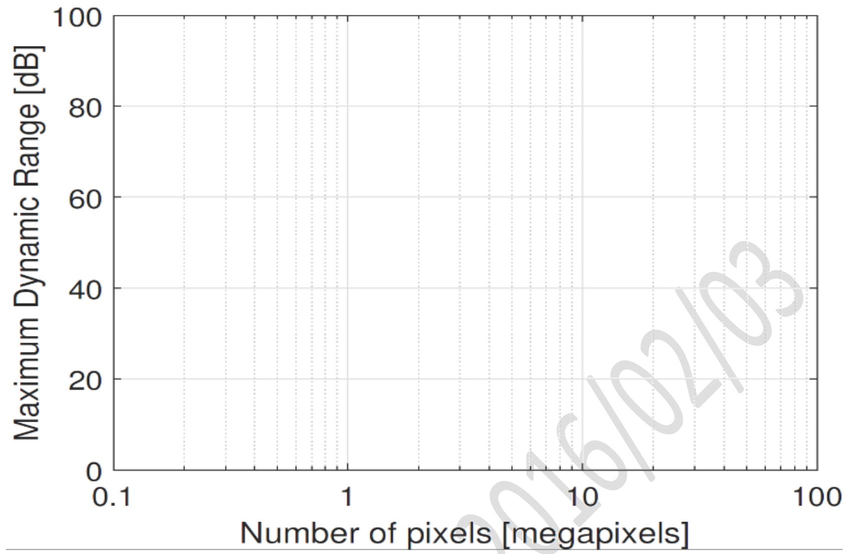
Question n. 2

A CMOS image sensor for smartphones features a (4.9 x 3.67) mm^2 3T APS sensor. Each photodiode has a depletion width of 1 µm. The technology used to produce the sensor allows a supply voltage of 3.3 V, and a minimum dimension of 150 nm. All in-pixel transistors and interconnections occupy a fixed overall area of $(820\ nm)^2$.



1. You have to choose the number of pixels.

a) Draw a quoted graph where the maximum dynamic range, expressed in dB, is plotted as a function of the number of pixels (from 0.1-megapixel to 100-megapixel). Remember the minimum dimension issue.

b) Choose the maximum number of megapixels that guarantees a maximum dynamic range higher than 56.3 dB and evaluate the corresponding fill factor.

2. The just-dimensioned sensor is used to register a slo-mo video at 120 fps, with Full-HD resolution (1920x1080), employing windowing. The output resistance of each pixel is 500 Ω, while the load capacitance of each pixel connected to the readout electronics is 1 pF. Used pixels are readout sequentially.

a) Assuming a global shutter scheme (all pixels integrate light simultaneously), determine the maximum integration time you can use when shooting your slo-mo video. Allow proper settling of transient signals, and neglect any post-processing time.

b) Consider a certain pixel that features a quantum efficiency of 0.7, including CFA effects, subject to an impinging photon flux of $7.6\ 10^{15}$ ph/s/m^2. Assuming to use the just-calculated integration time, determine whether the acquisition is reset-noise or shot-noise limited and evaluate the signal-to-noise ratio of the acquisition.

Physical Constants

$q = 1.6\ 10^{-19}$ C

$k_b = 1.38\ 10^{-23}$ J/K

T = 300 K (if not specified)

$\varepsilon_0 = 8.85\ 10^{-12}$ F/m

$\varepsilon_{r,Si} = 11.7$

1)

The maximum dynamic range can be expressed as the ratio between the full-well charge and reset noise. Its expression is

$$DR_{max,dB} = 20\log_{10}\frac{V_{DD}}{\sqrt{\frac{kT}{C_{PD}}}} = 20\log_{10}\frac{V_{DD}\sqrt{\varepsilon_o\varepsilon_{r,Si}A_{PD}}}{\sqrt{kT}}$$

The area of the photodiode can be expressed as a function of the number of pixels:

$$A_{PD} = \frac{A_{sensor}}{N_{pix}} - A_{eln}$$

Due to dimension limits, the maximum number of pixels that can be implemented is given by

$$A_{PD} = \frac{A_{sensor}}{N_{pix}} - A_{eln} > (150\text{ nm})^2$$

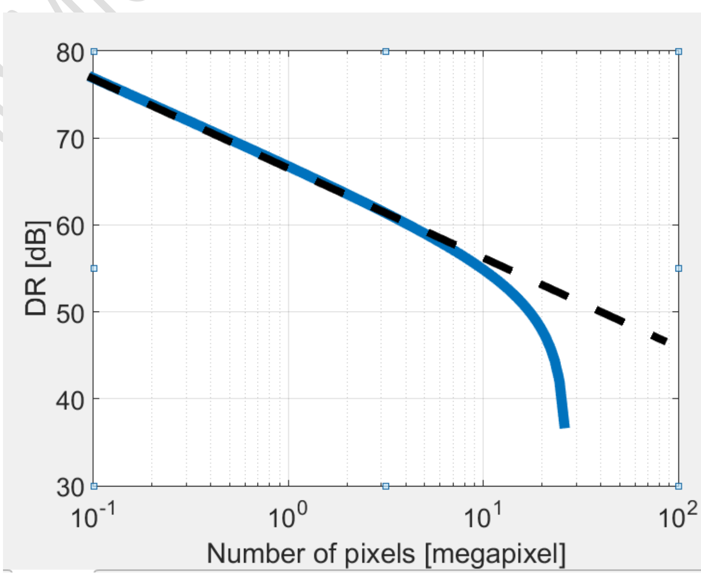
Hence

$$N_{pix,max} = \frac{4.9\text{ mm}\times 3.67\text{ mm}}{(150\text{ nm})^2 + (820\text{ nm})^2} = 26\text{ megapixel}$$

With this in mind, we can rewrite the DR as

$$DR_{max,dB} = 20\log_{10}\frac{V_{DD}\sqrt{\varepsilon_o\varepsilon_{r,Si}\left(\frac{A_{sensor}}{N_{pix}} - A_{eln}\right)}}{\sqrt{kT}}$$

and we would obtain a graph as the one shown



For low number of pixels, i.e. large photodiodes with fill-factors close to 1, DR is a straight line with – 10 dB/dec slope. Then it starts to drop, due to FF reduction. There is no DR after 26 megapixels as no sensor can be realized with the given technology.

By equating the DR expression with the required DR

$$20\log_{10}\frac{V_{DD}\sqrt{\varepsilon_o\varepsilon_{r,Si}\left(\frac{A_{sensor}}{N_{pix}}-A_{eln}\right)}}{\sqrt{kT}}=56.3\text{ dB}$$

one can obtain the maximum number of pixel: 8 megapixel.

With this dimensioning, each pixel has a side of 1.5 um, and the fill factor is 0.7.

2)

The frame time, equal to 8.3 ms, can be divided in three sections: reset, integration and readout. Reset time is negligible (~ 1 ps). The maximum integration time can thus be estimated by subtracting from the frame time the time needed to readout all the pixels.

Each pixel has a readout time constant equal to

$$\tau=R_{OUT}C_L=500\text{ ps}$$

Considering transients, i.e. 5 τ, the readout time for each pixel is 2.5 ns. As all the pixels are readout sequentuially, the total readout time is

$$t_{readout}=t_{pixel}*1920*1080=5\text{ ms}$$

Hence, at maximum 3.3 ms are left for the integration.

The photocurrent of the considered pixel can be evaluated as

$$i_{ph}=q\Phi\eta A_{PD}=1.3\text{ fA}$$

where the area of the photodiode is (1.24 um)^2, calculated from results of point 1.

The number of collected electrons is thus

$$N_{ph}=\frac{i_{ph}t_{int}}{q}=26\text{ electrons}$$

Reset noise can be evaluated as

$$\sigma_{reset} = \frac{\sqrt{kTC_{PD}}}{q} = 5.1 \text{ electrons}$$

Shot noise can be evaluated as

$$\sigma_{shot} = \sqrt{N_{ph}} = 5.1 \text{ electrons}$$

Reset and shot noise contribution are thus comparable.

Total noise is

$$\sigma_{tot} = \sqrt{\sigma_{reset}^2 + \sigma_{shot}^2} = 7.2 \text{ electrons}$$

and the signal to noise ratio is thus

$$SNR_{dB} = 20 \log_{10} \frac{N_{ph}}{\sigma_{tot}} = 11 \text{ dB}$$

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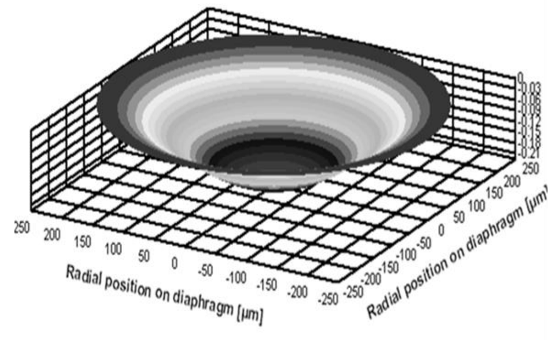
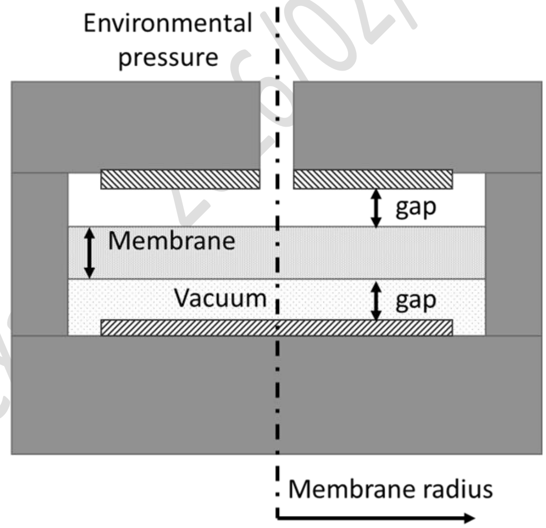
Question n. 3

You are asked to develop a MEMS capacitive pressure sensor to measure altitude from atmospheric pressure changes.



The sensor is based on a novel technology that enables the fabrication of suspended circular membranes.

An example of a real membrane vertical displacement is shown aside, but, for sake of simplicity, you can assume rigid translations whose average displacement *y* is half of the displacement of the membrane center. You can therefore consider the shown simplified cross-section of the device, as reported in the figure below. Two identical parallel-plate electrodes of radius r_e equal to the membrane one are designed beneath and above the membrane for differential capacitive sensing.



Known that the altitude pressure gradient is P_g = - 12 Pa/m, and assuming a reference atmospheric pressure of 10^5 Pa at sea level:

1) find the maximum displacement *y* in order to guarantee a linearity error ε_{lin} = 1%;

2) for the shown mode, the average membrane displacement *y* can be calculated through the following stiffness:

$$k = 1.06 \cdot 10^{13} \frac{N}{m^2} \frac{t^3}{r_m^2}$$

Neglecting electrostatic and gravity forces, write the force balance equation in stationary conditions. Find the minimum membrane radius r_m to satisfy the linearity requirement, and the corresponding mechanical stiffness *k* of the membrane;

3) calculate the membrane displacement corresponding to the altitude resolution to be detected, σ_{AR}. Evaluate the maximum acceleration that causes a membrane displacement within the resolution limit.

Assume an *ideal* charge amplifier configuration (with infinite feedback resistance) for the readout electronics. The electrodes are biased at $\pm V_b$ = 5V with the membrane kept at the charge amplifier virtual ground:

4) calculate the amplifier noise contribution and the intrinsic device noise contribution at the charge amplifier output; verify whether the overall noise satisfies the required altitude resolution σ_{AR};

5) is it possible to use this circuit in a *real* implementation? If not, propose an improved topology, indicating possible values for the components.

Parameter	Symbol	Value
Membrane thickness	t	2 μm
Gap	g	2μm
Damping factor per unit area	b_{area}	7 10^3 kg/s/m^2
Membrane density	ρ	2390 kg/m^3
Equivalent noise bandwidth	BW	10 Hz
Feedback capacitance	C_f	0.5 pF
Parasitic capacitance	C_p	10 pF
Op-Amp voltage noise	S_{vn}	(4.5 nV/√Hz)^2
Altitude resolution	σ_{AR}	0.04 m_{rms}

Solution

1) Named *y* the average displacement of the membrane, you can assume the membrane motion as rigid, as indicated in the text. It is thus possible to write the linearity error equation as a function of *y*:

$$\epsilon_{lin}\% = \frac{\left(\Delta C_{real,FSR} - \Delta C_{lin,FSR}\right)}{\Delta C_{real,FSR}} \cdot 100$$

For the maximum displacement y_{max} that satisfies the maximum linearity error, $\Delta C_{real,FSR} = C_0\left(\frac{2\frac{y_{max}}{g}}{1-\left(\frac{y_{max}}{g}\right)^2}\right)$ the expression simplifies as:

$$\epsilon_{lin}\% = \frac{C_0\left(\frac{2\frac{y_{max}}{g}}{1-\left(\frac{y_{max}}{g}\right)^2}\right) - 2C_0\frac{y_{max}}{g}}{C_0\left(\frac{2\frac{y_{max}}{g}}{1-\left(\frac{y_{max}}{g}\right)^2}\right)} \cdot 100 = \frac{\left(\frac{1}{1-\left(\frac{y_{max}}{g}\right)^2} - 1\right)}{\frac{1}{1-\left(\frac{y_{max}}{g}\right)^2}} \cdot 100 = \left(\frac{y_{max}}{g}\right)^2 \cdot 100$$

The maximum membrane displacement can be thus evaluated:

$$y_{max} = g \cdot \sqrt{\frac{\epsilon_{lin}\%}{100}} = \boldsymbol{200\ nm}$$

2) The text suggests to neglect electrostatic forces and the gravity force. Further, in the force balance equation the terms dependent on the derivative and second derivative of the displacement does not have to be considered, as the problem indicates to focus on stationary conditions. The resulting force balance equation is therefore:

$$F_{ext} = k \cdot y$$

The relation between force and pressure is well known and it is:

$$P = \frac{F}{A}$$

where *A* is the surface where the force is applied. For this specific case we have $A = \pi r_m^2$. Substituting the expressions for *F* and *k*, the force balance equation can be re-written as:

$$P_{ext} \cdot \pi r_m^2 = 1.06 \cdot 10^{13}\frac{N}{m^2} \cdot \frac{t^3}{r_m^2} \cdot y$$

The maximum displacement of the membrane is obtained at sea level (indeed the pressure gradient is negative, and thus the pressure exerted on the membrane can only decrease with altitude). Therefore from this equation, evaluated for the sea-level pressure of 10^5 Pa and for the maximum displacement found above, it is possible to derive the membrane radius that satisfies the linearity requirement:

$$\frac{P_{ext} \cdot \pi r_m^2}{1.06 \cdot 10^{13} \frac{N}{m^2} \cdot \frac{t^3}{r_m^2}} < y_{max}$$

$$r_m < \sqrt[4]{\frac{1.06 \cdot 10^{13} \cdot t^3 \cdot y_{max}}{P_{amb} \cdot \pi}} = \mathbf{85.65\ \mu m}$$

NOTE: the text indicated the minimum radius, but it is actually a maximum radius!

Once the membrane radius is obtained, it is possible to calculate the mechanical stiffness k from the given formula:

$$k = 1.06 \cdot 10^{13} \cdot \frac{t^3}{r_m^2} = \mathbf{11.52 \cdot 10^3 \frac{N}{m}}$$

3) To solve this point, it is first necessary to evaluate the membrane displacement corresponding to the desired measurement resolution. This is given in terms of altitude (0.04 m_{rms}). Through the pressure gradient we can convert it into pressure resolution σ_P:

$$\sigma_P = |P_g \cdot \sigma_{AR}| = 0.48\ Pa$$

The corresponding resolution in terms of membrane displacement σ_y is therefore:

$$\sigma_y = \frac{\sigma_P \cdot A}{k} = \mathbf{0.96\ pm}$$

We now need to check which is the external acceleration that causes such a displacement. We make use of the known relationship between an external acceleration and the displacement of the MEMS suspended mass (like in accelerometers!):

$$\frac{\Delta y}{\Delta a} = \frac{m}{k} = \frac{1}{\omega_0^2}$$

The mass of the membrane can be evaluated through the membrane density and geometry, to find the angular resonance frequency:

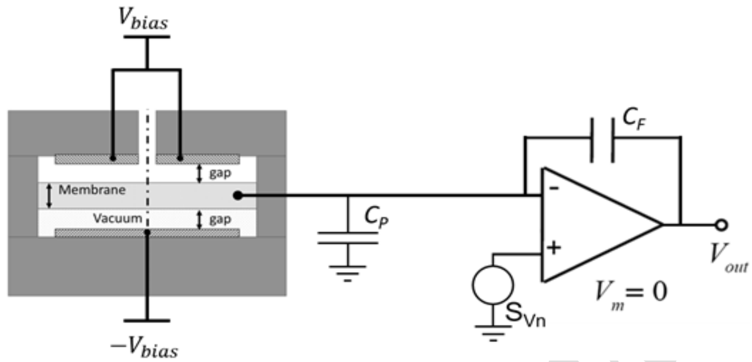
$$m = \rho\, A\, t = 0.11\ nKg$$

$$\omega_0 = \sqrt{\frac{k}{m}} = 10.23 \cdot 10^6 \frac{rad}{s}$$

This leads to a maximum acceptable acceleration:

$$a_{ext,max} = \sigma_y \cdot \omega_0^2 = \mathbf{100.4 \frac{m}{s^2}} = \mathbf{10.24\ \hat{g}}$$

4) The complete system is represented in the figure below. The membrane is directly connected to the virtual ground of a charge amplifier. A parasitic capacitance is placed with one terminal at ground and the other one at virtual ground. In this way it does not have any effect on the signal transfer.



Amplifier noise: the only noise to be considered is the voltage noise contribution. For the ideal configuration considered here, its contribution at the charge amplifier output is amplified through the parasitic:

$$S_{v,out} = S_v \cdot \left(1 + \frac{C_P}{C_F}\right)^2 == \left(94.5 \frac{nV}{\sqrt{Hz}}\right)^2$$

Device noise: the thermo-mechanical noise of the device, in terms of force, can be expressed as: $S_b = 4\, k_B T\, b$, where $b = b_{area} \cdot A = b_{area} \cdot \pi r_m^2$ (note that the area term should not be multiplied by 2, because in this structure on one side there is ideal vacuum, so no gas damping term). To compare the two noise contributions, intrinsic noise is written at the amplifier output:

$$S_{b,out} = S_b \cdot \frac{1}{k^2} \cdot \left(2 \frac{C_0}{g} \cdot \frac{V_{bias}}{C_F}\right)^2$$

Knowing from the text that the electrode radius is equal to the membrane one ($r_e = r_m$), the capacitance C_0 and the intrinsic noise contribution at the output of the charge amplifier stage turns out to be:

$$C_0 = \frac{\epsilon_0 A_{electrode}}{g} = \frac{\epsilon_0 \pi r_e^2}{g} = 101.97\, fF \qquad \rightarrow \qquad S_{b,out} = \left(0.14 \frac{nV}{\sqrt{Hz}}\right)^2$$

This contribution is negligible with respect to the amplifier noise: the system is limited by the electronics. The obtained noise power spectral density integrated over the bandwidth gives the rms output voltage noise:

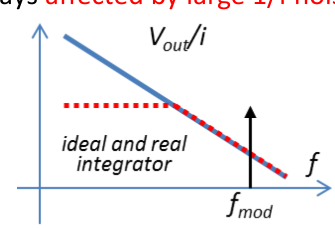
$$V_{n,out} = \sqrt{S_{v,out} \cdot BW} = \mathbf{298.9\, nV}$$

Dividing the term above by the sensitivity ($\Delta V_{out}/y$), we obtain a minimum measurable displacement lower than the resolution target (0.93 pm, as calculated above). The system thus matches the required resolution.

$$y_{min} = \frac{V_{n,out}}{\frac{2C_0}{g} \cdot \frac{V_{bias}}{C_F}} = \mathbf{0.28\, pm}$$

5) It is not practically possible to use this simple circuit in a real implementation, because even the smallest leakage current of the operational amplifier would be integrated in the feedback capacitance leading to a saturation of the readout chain. Further, a parasitic feedback resistance always exists. It is therefore good practice to put a physical feedback resistance, controlling its value. When a feedback resistance R_F is considered in parallel to C_f, the problem of the electronics chain saturation is solved. The DC gain is given by the resistance, with a pole at a time constant $\tau = R_F C_F$ (e.g. a few kHz), as depicted in the figure below.

However, as the system no longer behaves as an integrator for DC pressures, any constant pressure values result in a zero output of the amplifier. Further, consider that DC operation is always affected by large 1/f noise issues.



The solution of both issues is to modulate the stators voltage with a frequency higher than the noise corner frequency of the operational amplifier, and at least a decade after the frequency given by the feedback pole (50-200 kHz are reasonable values for noise corners up to 5-20 kHz). In this way the signal is shifted out from the 1/f noise, and the system operates again in the integrating region of the transfer function.