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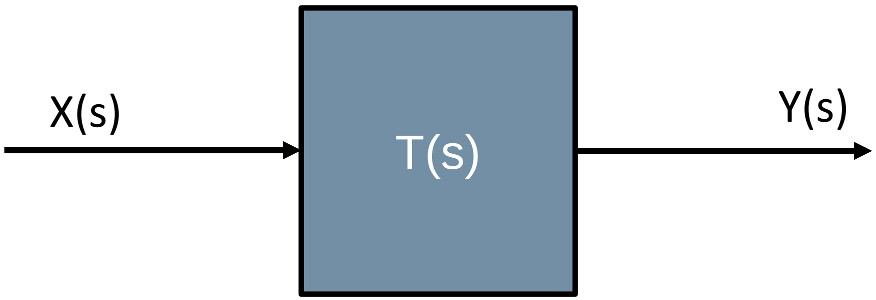
Electronics basics for MEMS and Microsensors course

MEMS and Microsensors, a.a. 2017/2018, M.Sc. in Electronics Engineering

Transfer function

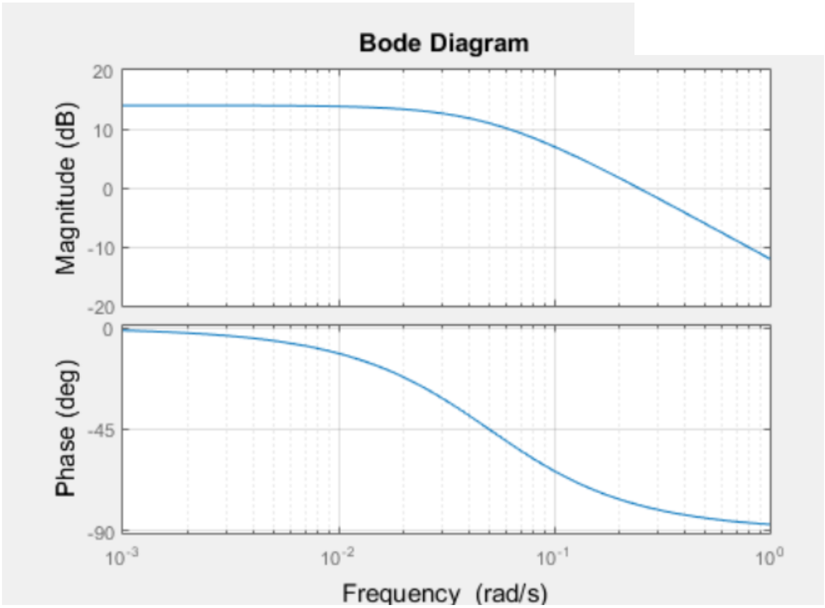


$$T(S) = \frac{Y(s)}{X(s)}$$

- The **transfer function** of a linear time-invariant (LTI) system is the function of complex variable *H(s)* that describes, in the **frequency domain**, the relationship between the input X and the output Y of the system.

- Typical representation: **Bode plots** of **modulus** (or magnitude) in dB units $|T(j\omega)|_{dB} = 20\log_{10}|T(j\omega)|$ and **phase** $\angle T(j\omega) = atan\left(\frac{Im(T(j\omega))}{Re(T(j\omega))}\right)$, obtained through a domain restriction of the complex function $T(s)$ (i.e. imposing $s = j\omega$).

- A certain sinusoidal input at a generic frequency will by **amplified/attenuated** at the output (described by the modulus diagram), and will have a certain **phase shift** (described by the phase diagram).



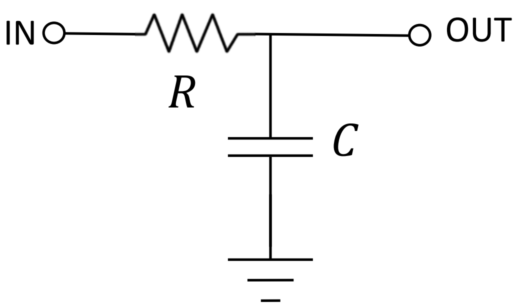
Poles/zeroes and Bode plot rules

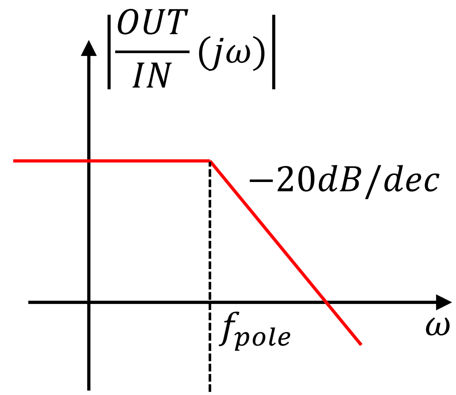
- Given the expression of a **generic transfer function**, one can find the solutions z_n and p_n that null the numerator and the denominator, respectively.

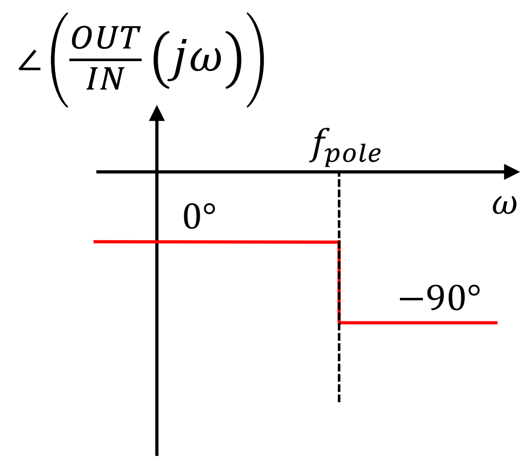
- z_n elements are named **zeroes** and p_n elements are named **poles**;

- Some **basic rules** to plot a **Bode graph**:
  - Zeroes give ***+20dB/decade*** slope in the modulus plot;
  - Zeroes give a ***+90°*** **shift** in the phase diagram;
  - Poles give ***-20dB/decade*** slope in the modulus plot;
  - Poles give a ***-90°*** **shift** in the phase diagram;

- Basic example: a simple **RC**.





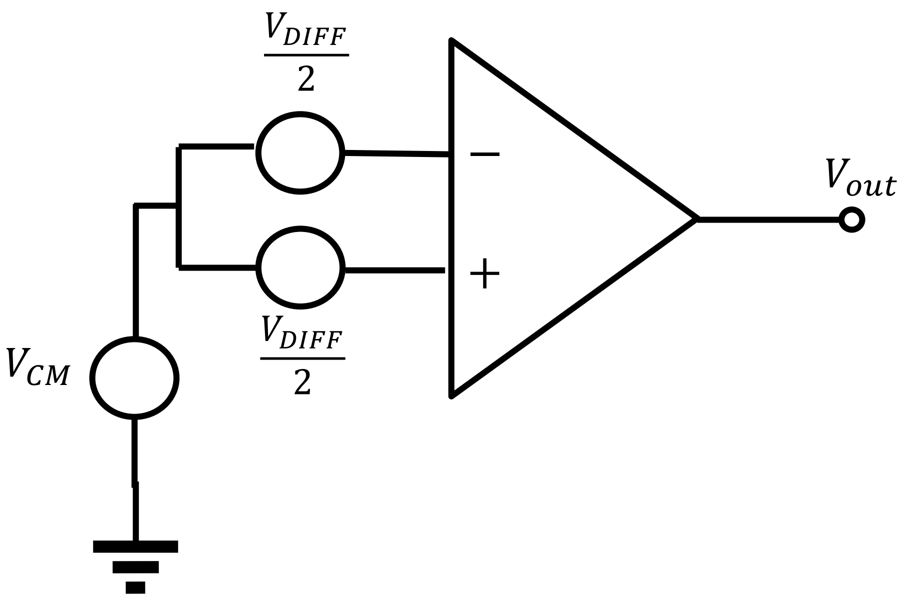


$$\frac{OUT}{IN} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

$$\tau = RC,$$

$$f_{pole} = \frac{1}{2\pi RC}$$

Operational amplifier



- Typically, an op-amp has 3 signal-related pins: the **non inverting (+)** and the **inverting (-) input** pins and the **output**.

- An ideal operational amplifier responds only to **differential** signals V_{DIFF}, and rejects **common-mode** signals V_{CM}. Thus, writing the output voltage expression:

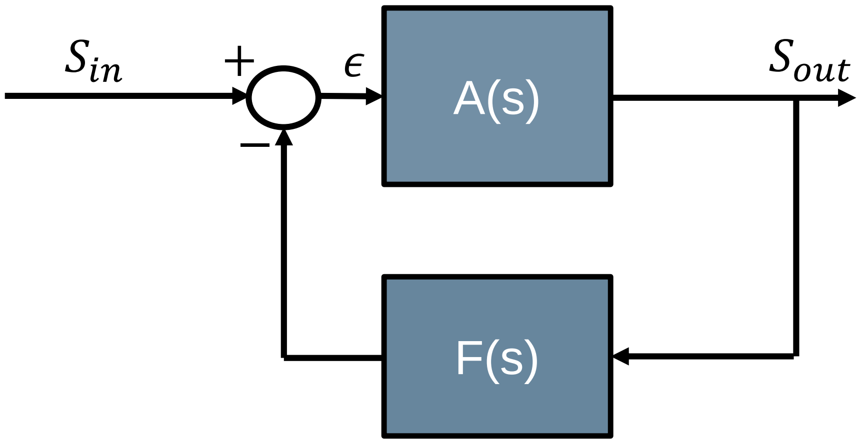
$$\boldsymbol{V_{out} = A_0 \cdot V_{DIFF} + A_{CM} \cdot V_{CM}}$$

it can be said that $\boldsymbol{A_0}$, the differential open-loop gain, should be very **high**, ideally infinite, and the common-mode gain $\boldsymbol{A_{CM}}$ should be ideally **null**.

- The **input impedance** of an **ideal** amplifier is, by definition, **infinite**. The ideal op-amp does not absorb any input current: signal currents of inverting and non-inverting terminals are null.

- The **output impedance** of an **ideal** amplifier is, by definition, **null**. So, the output terminal acts as an ideal voltage generator: $\boldsymbol{V_{out}}$ will be always equal to $\boldsymbol{A_0 \cdot V_{DIFF}}$, regardless of the amount of current that has to flow towards the output load.

Negative feedback



- The **negative feedback** concept relies on the scheme shown in the figure, where *A(s)* is typically a very high gain block (e.g. an operational amplifier!) and F(s) can be a passive component (e.g. a resistor or a capacitor).
- Elaborating signal expressions in the represented loop:

$$\epsilon = s_{in} + s_{out} \cdot F(s)$$

$$\downarrow$$

$$s_{out} = \epsilon \cdot A(s) = \left(s_{in} + s_{out} \cdot F(s)\right) \cdot A(s)$$

$$\downarrow$$

$$\epsilon = \frac{s_{in}}{1 - A(s)F(s)} = \frac{s_{in}}{1 - G_{loop}(s)}$$

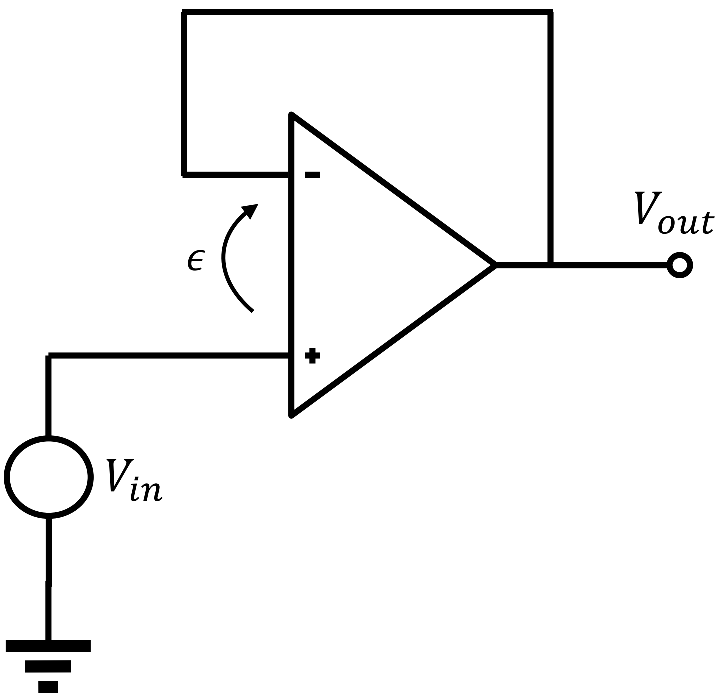
- The $\boldsymbol{\epsilon}$ signal driving the high-gain amplifier **is reduced** by the effect of the feedback. *A(s)F(s)* is called the **loop gain**, and determines the ''strength'' of the feedback.

$$G(s) = \frac{s_{out}}{s_{in}} = \frac{A(s)}{1 - A(s)F(s)} = \frac{A(s)}{1 - G_{loop}(s)}, if\ |G_{loop}| \gg 1 \rightarrow G(s) = \frac{1}{F(s)}$$

- For a **high loop gain**, the transfer function of the entire block is only determined by the **feedback components**.

- But…**why** do we use negative feedback?
  - High differential gains of operational amplifier are inaccurate, they can't be used as standalone reliable differential gain blocks. But they represent the main building block of a robust negative feedback loop, in which the signal gain is only determined by feedback passive components, usually more precise and reliable.

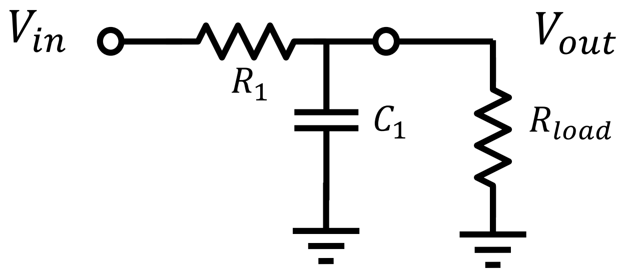
Basic configurations: voltage buffer

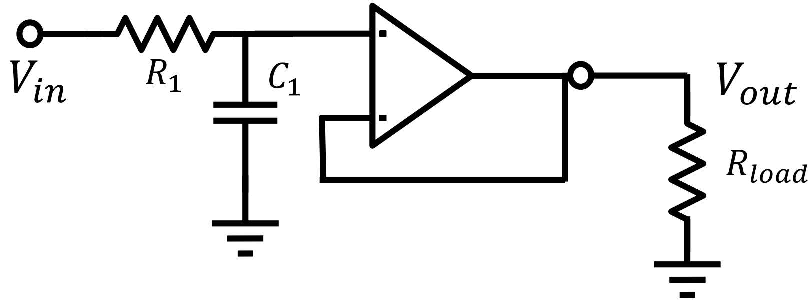


- You can easily recognize the scheme of the **negative feedback** in the schematic in figure. The **A(s)** block is represented by the **operational amplifier**, and the **F(s)** is simply equal to **1** (as the output is simply shorted to the negative pin).
- You can study negative feedback remembering that ϵ, i.e. the signal driving the amplifier, **is lowered** by the effect of the loop, and it's **ideally zero**. Consequently, the voltage at the inverting pin is equal to *Vin,* order to keep $\epsilon = \mathbf{0}$. This node is shorted to the output, so:

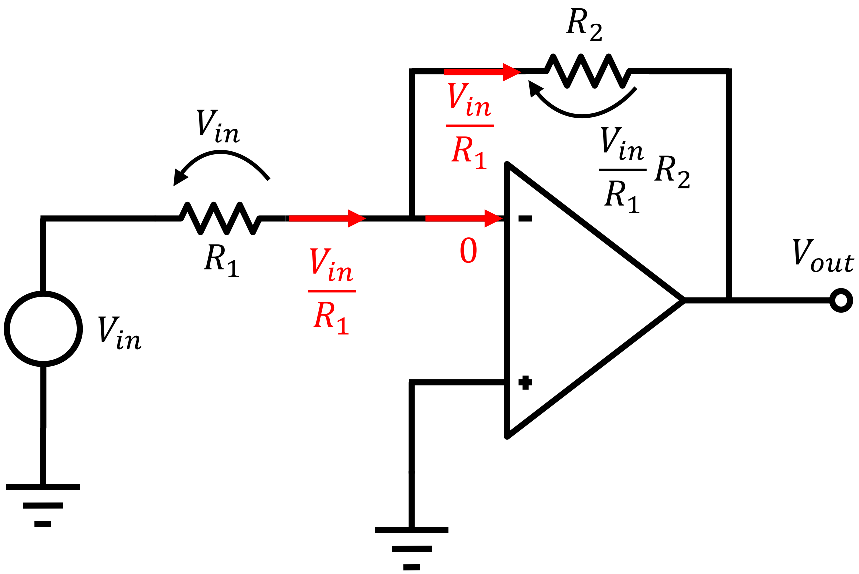
$$V_{out} = V_{in}$$

- But…**why** do we need a unitary gain? To **decouple** analog stages avoiding load effects. Try to compute the transfer functions of the two represented schematics…In the second case the load resistor is not influencing the RC stage.



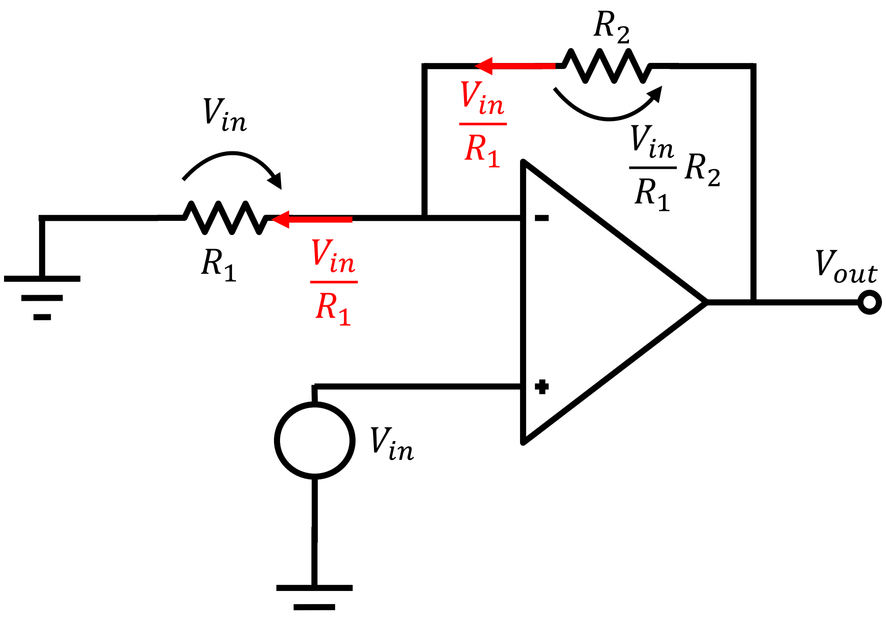


Basic configurations: inverting and non-inverting stages



- Again, simply remembering to **null the ϵ signal** and knowing that **no current flows** into the opamp **input pins** (high impedance input), you can evaluate the transfer function of the **inverting** configuration:

$$V_{out} = -V_{in} \cdot \frac{R_2}{R_1}$$



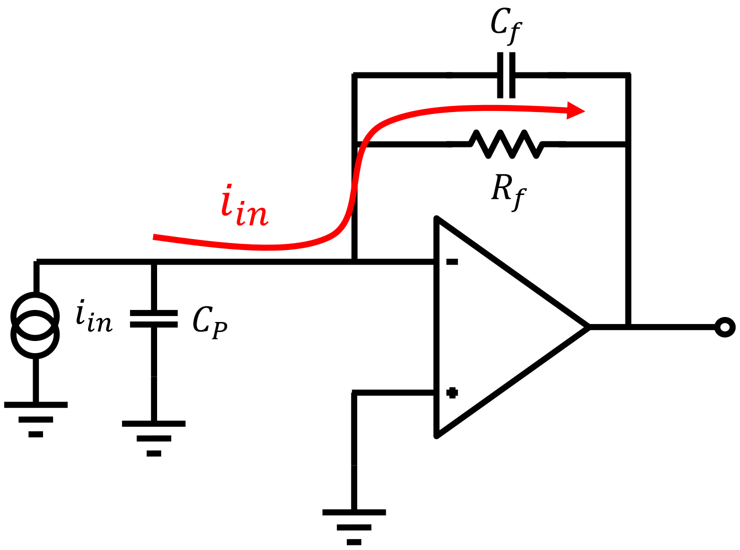
- And you can follow the same steps to evaluate the gain of the **non-inverting** configuration:

$$V_{out} = V_{in} \cdot \left(1 + \frac{R_2}{R_1}\right)$$

- You can use this configuration to obtain an amplification of your input signal. The amount of the amplification can be fixed selecting the resistance ratio.

Ohm's law: V =I/R

TCA and TRA amplifiers



- Essentially, this stage is an **inverting amplifier,** with a **capacitor** in parallel to the feedback resistor. Differently from the situation of the previous slide, we are now considering a **current** as an **input** (given e.g. by the MEMS capacitance variation in time).
- C_p doesn't take part in the signal transfer function. Indeed, if ϵ is null, the voltage difference between the two terminals of this capacitor is null, and then no signal will flow into it.
- We can calculate the gain in a similar way with respect to the inverting amplifier, this time multiplying the current by the feedback impedance given by: $\frac{1}{sC_f} \parallel R_f = \frac{R_f}{1+sR_fC_f}$.

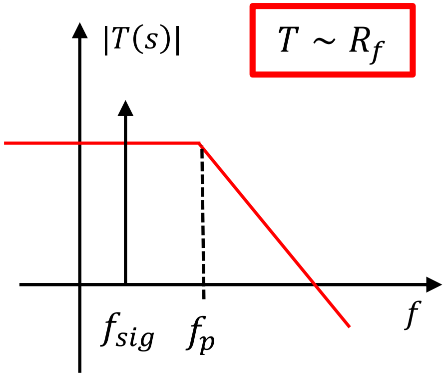
$$V_{out} = i_{in} \cdot \frac{R_f}{1 + sR_fC_f} \rightarrow T(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{R_f}{1 + sR_fC_f}$$

- So, the behavior of this stage is **frequency-dependent**.

TRANS-RESISTANCE AMPLIFIER (TRA)

- The signal frequency is **lower than the pole** $f_p = \frac{1}{2\pi R_f C_f}$. So we can neglect the capacitor and my output will be simply the input current **multiplied** by the R_f.
- This solution is rarely adopted in our cases of interest. Typically, the resistor thermal noise dominates… (see the slides of the course)

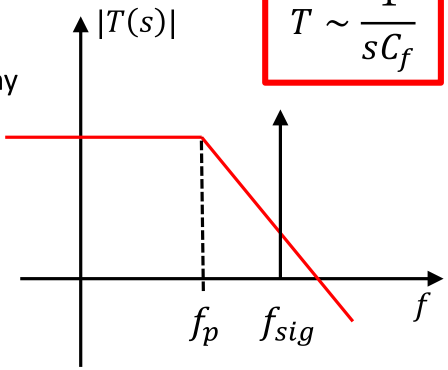
$$T \sim R_f$$



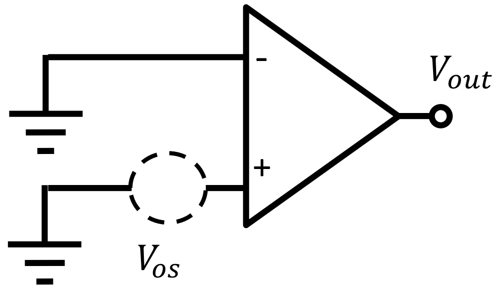
TRANS-CAPACITANCE AMPLIFIER (TCA)

- The signal frequency **is higher than the pole** $f_p = \frac{1}{2\pi R_f C_f}$. So I can neglect the resistance and my output will be given by the input current **integrated** on the C_f.
- This is the typical solution used in most of the case studies during the course…

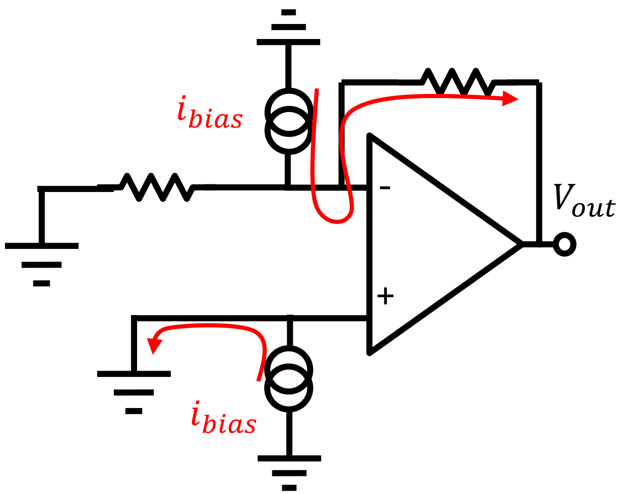
$$T \sim \frac{1}{sC_f}$$



Some op-amps non-idealities -1

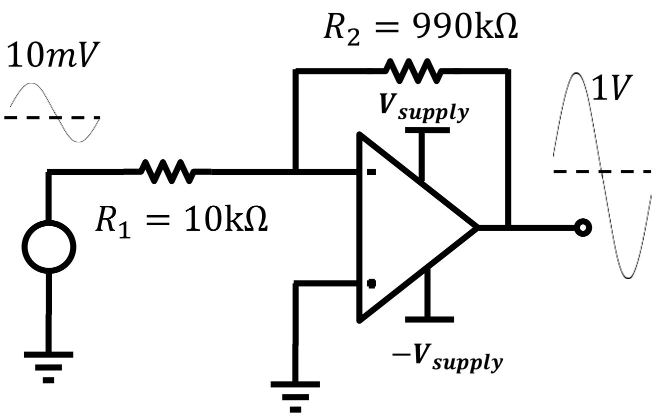


- When the pins of an opamp are at the same voltage, the output should be null. This is not true in a **real opamp**: a differential voltage V_{os} should be applied to the pins in order to keep the output at ground.
- Message to take home: when you design a opamp stage, keep in mind that even without input signal, you will have a **non-null DC output**.
- Typically, if your signal is at a frequency higher than DC, you can operate a **frequency selection** using an high-pass filter (see next slides), cutting out offset and keeping the useful signal.

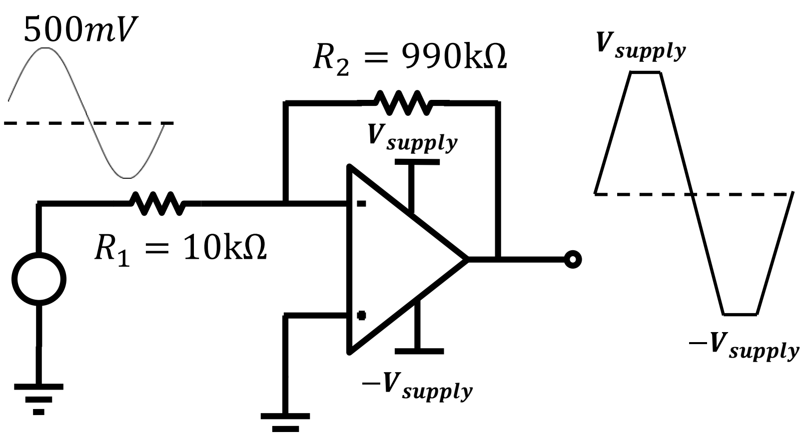


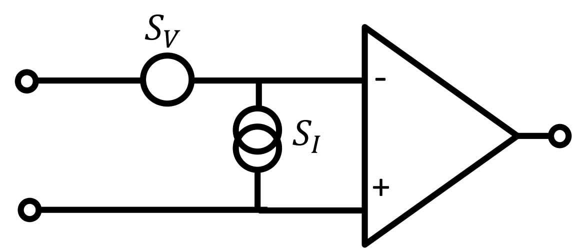
- We said that no current flows into either input terminal. This is a key concept for analyzing an opamp stage signal gain. However, in reality, a **small current** flows into **both inputs pins**. You can model this effect with **DC current** generators and find the contribution of this currents in terms of output voltage.
- Check exercise 2 about accelerometers in order to understand issues given by bias currents and common solutions

Some op-amps non-idealities -2



- The opamp is supplied through **DC** suitable **voltage** sources, called V_{supply}. The output of an opamp cannot be higher than this voltages. An opamp stage with a $\pm 3V$ supply, a 100 gain and a too high input signal will **clamp** at the supply voltage!





- The **noise** of an opamp can be modeled through equivalent **input-referred** voltage and current noise generators.

- For a typical MOS differential-pair-input opamp, the dominant contribution is the **voltage noise** given by the **MOS couple**:

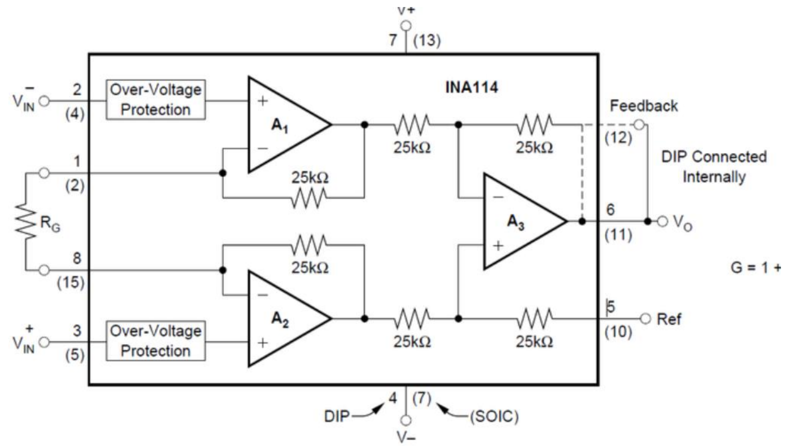
$$S_V = 2 \cdot \frac{4K_BT\gamma}{g_m}\left[\frac{V^2}{Hz}\right],\ \gamma = \frac{2}{3}$$

- To calculate the contribution given by this noise sources to the stage output, you can consider this sources as signal sources and calculate their transfer function to the output **squared** (because we are dealing with **noise power**, not amplitude).

- Check the exercise classes for noise transfer functions in typical MEMS readout circuital schemes.

Advanced configurations: INA

- The shown schematic represents an Instrumentation amplifier (INA).



- Regardless of the circuital complexity, what you need to know is that this block implements an **high-precision differential amplifier.** Thus, the output of this stage can be written as:

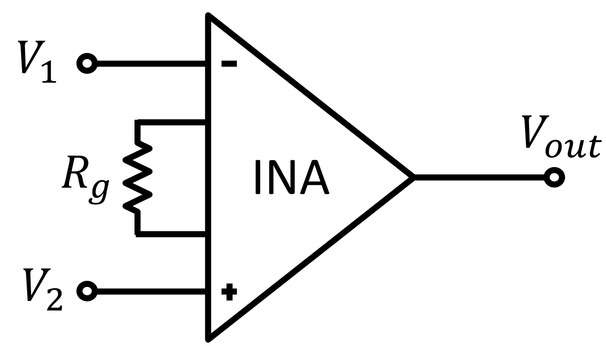
$$V_{out,INA} = G_{INA}(V_2 - V_1)$$

- The INA gain G_{INA} is fixed by internal parameters of the component and by a user-selectable external resistance:

Provided by the manufacturer

$$G_{INA} = 1 + \frac{49{,}5k\Omega}{Rg}$$

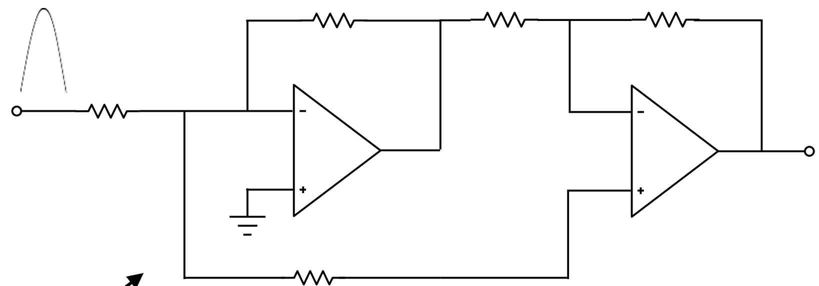
user-selectable



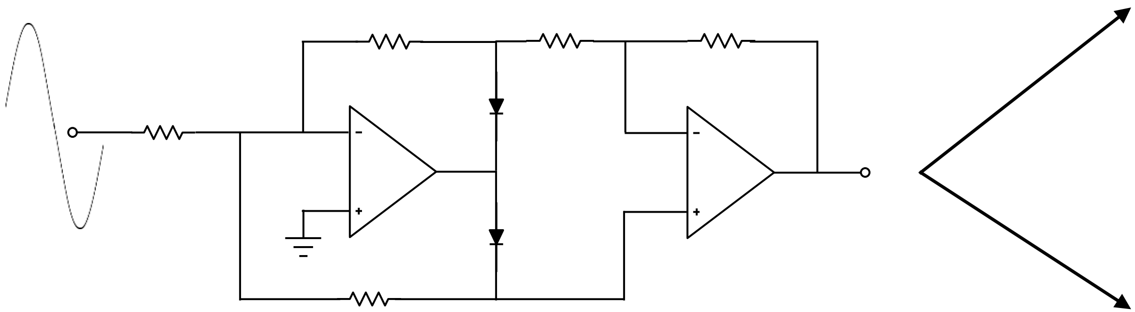
Advanced configurations: rectifier

- In order to **rectify** a sinusoidal wave, the circuit behavior should be **non-linear**: it should have a positive gain for the positive semi-period of the sine wave, and an inverting gain for the rest of the period. This is possible using non-linear components as **diodes**.

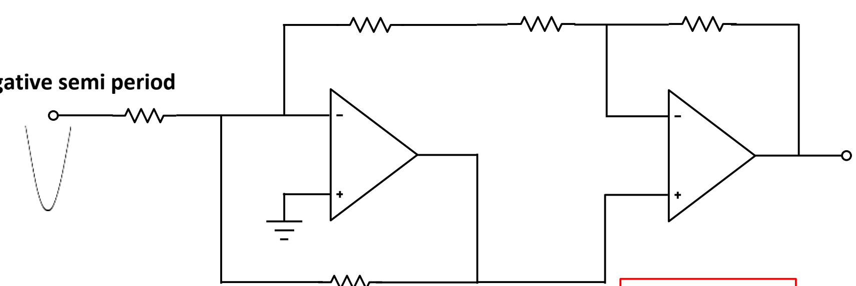
Positive semi period



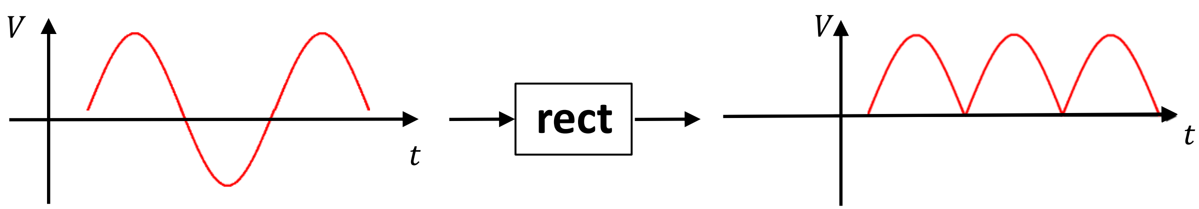
$G > 0$



Negative semi period

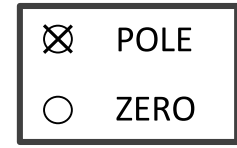


$G < 0$

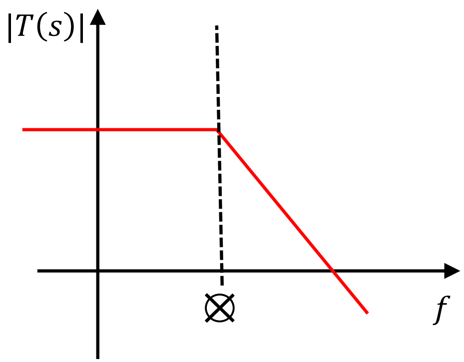


Filters

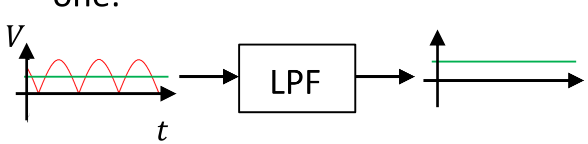
- Filters are **frequency-selective** elements: they amplify with a gain $G \geq 1$ frequency components in a specific range, attenuating components outside this range.
- Filters are typically used to isolate the signal bandwidth, **cutting off noise** at higher (or lower) frequencies. More in general, filters are used when a frequency-selective operation is needed.



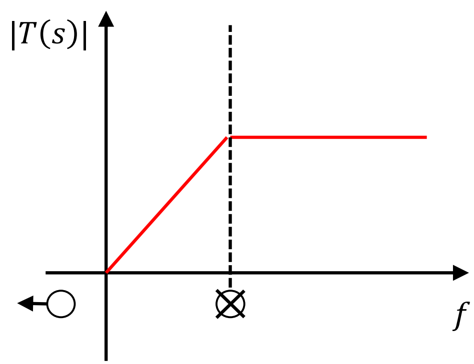
LOW-PASS-FILTERS



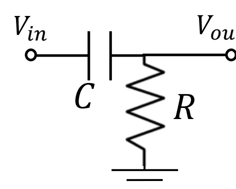
- A simple **RC** is a low pass filter, as seen in slide 2
- An example of an application is the **mean value extraction** from a rectified sinusoidal wave. This signal is composed by a sinusoidal component and a DC value: the LPF filter attenuates the AC component and let pass the DC one:



HIGH-PASS-FILTERS



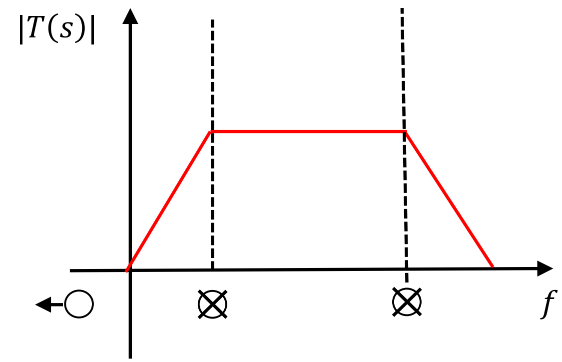
- This **CR** implements an high pass filter:



$$T(s) = \frac{sCR}{1 + sRC}$$

- This kind of filter can be used to **cancel** unwanted **low-frequency contributions** keeping the AC signal untouched. (e.g. I can erase the **DC offset** at the opamp output…)

BAND-PASS-FILTERS



- A **band pass** can be realized thorugh passive and active networks, as the other kind of filters.
- Also the **MEMS resonant peak** is a band-pass filter! It selectively amplifies only a range of frequencies near the peak! This is why, if we drive the MEMS with a square-wave, we obtain a sinusoidal current as an output…