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MILANO 1863

DIPARTIMENTO DI ELETTRONICA,
INFORMAZIONE E BIOINGEGNERIA

SID

MOS & Noise review

MEMS and Microsensors, a.a. 2017/2018, M.Sc. in Electronics Engineering

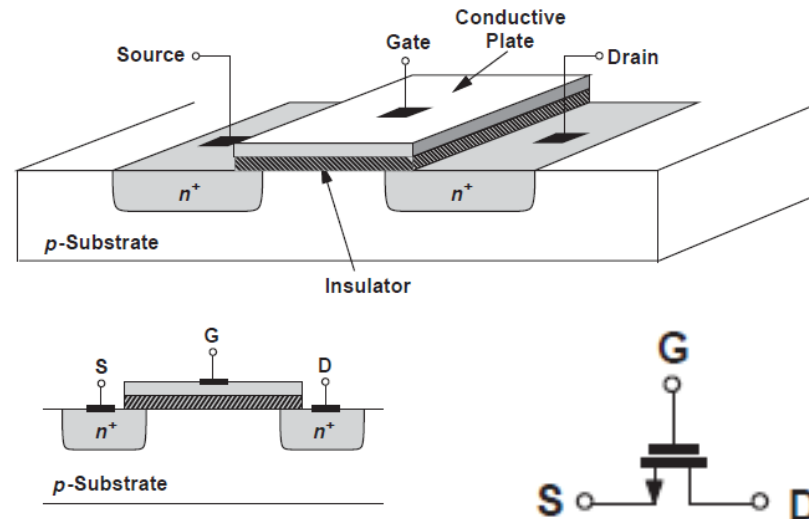




Outline

- MOSFET
 - Switch
 - Buffer
- Noise
 - Features
 - Sources
 - SNR

- A MOSFET (*metal-oxide semiconductor field-effect transistor*) is an electronic device with three terminals: **gate**, **drain** and **source**.
- By **applying a voltage at the gate**, it generates an electrical field under the gate that **controls the current flow through the channel between drain and source**.
- There is no current flow from the gate into the MOSFET.



- A MOSFET is a very versatile device.
- It can be used as:
 - **a switch;**
 - an amplifier;
 - **a buffer;**
 - a variable capacitor;
 - an elementary block for digital systems;
 - a photodetector;
 - ...

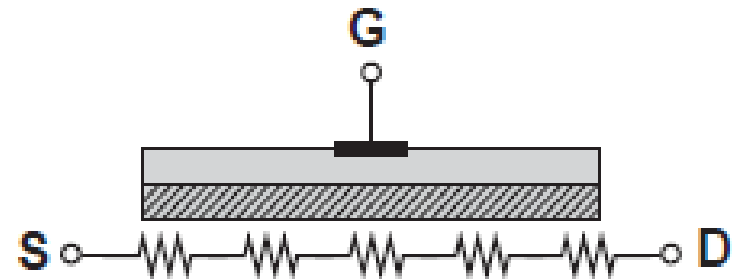
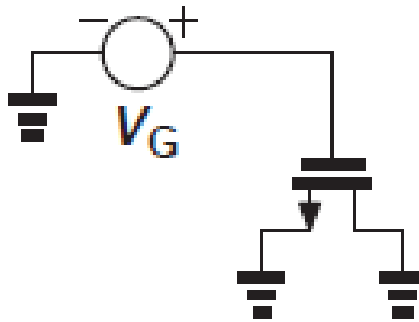


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MOSFET Switch

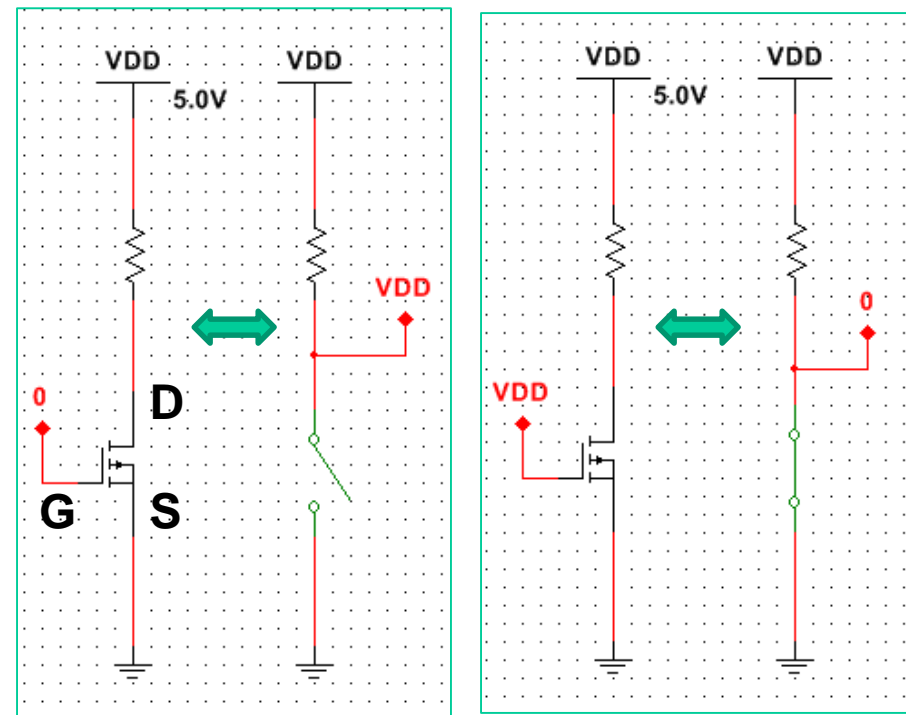
- A MOSFET *may* be thought of as a variable resistor, where the gate-source voltage difference V_{GS} controls the drain-source resistance R_{DS} .
- The drain-source resistance can be controlled by the gate-source voltage, like pinching or opening a valve and stopping or allowing water flowing through a pipe.
- We can make $R_{DS} \rightarrow \infty \rightarrow$ **open circuit**
- We can make $R_{DS} \rightarrow 0 \rightarrow$ **short circuit**
- Because of this property, a MOSFET *can* be used as a **switch**.



MOSFET Switch

- For an N-Channel MOSFET (nMOS) switch, the source is connected to ground. To turn the MOSFET ON, we need to raise the voltage on the gate. To turn it OFF, we connect the gate to ground.
- When there is no voltage between the gate and the source, the drain-source resistance is very high, which is almost like a **open circuit**: no current flow between the drain and the source.
 - $V_{GS} = 0 \rightarrow R_{DS} \rightarrow \infty$
- When a gate-source voltage difference is applied, the drain-source resistance is low, and there will be current flowing between the drain and the source: **closed circuit**.
 - $V_{GS} > 0 (V_{GS} > V_{TH}) \rightarrow R_{DS}$
- When the gate-source voltage difference is very high: **short circuit**.
 - $V_{GS} \gg 0 \rightarrow R_{DS} \rightarrow 0$

nMOS

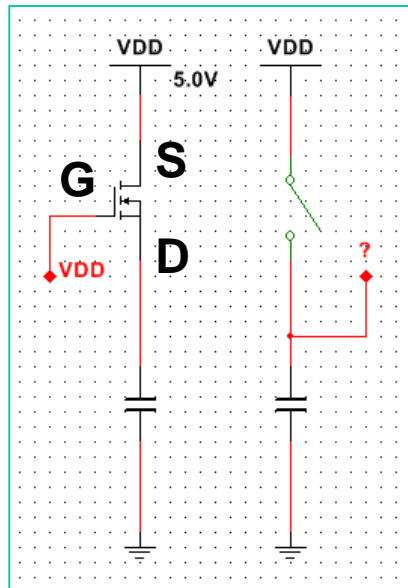


$$R_{DS} = \frac{1}{k' \frac{W}{L} (V_{GS} - V_{TH})}$$

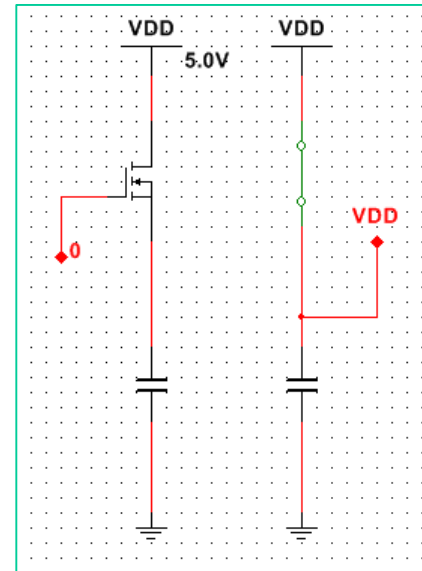
- Typical values: 1 – 10 k Ω

MOSFET Switch

- For a P-Channel MOSFET (pMOS) switch, the source is connected to the power rail. In order to allow current to flow, the gate needs to be pulled to ground. To turn it OFF, the gate needs to be pulled to the power rail.



pMOS



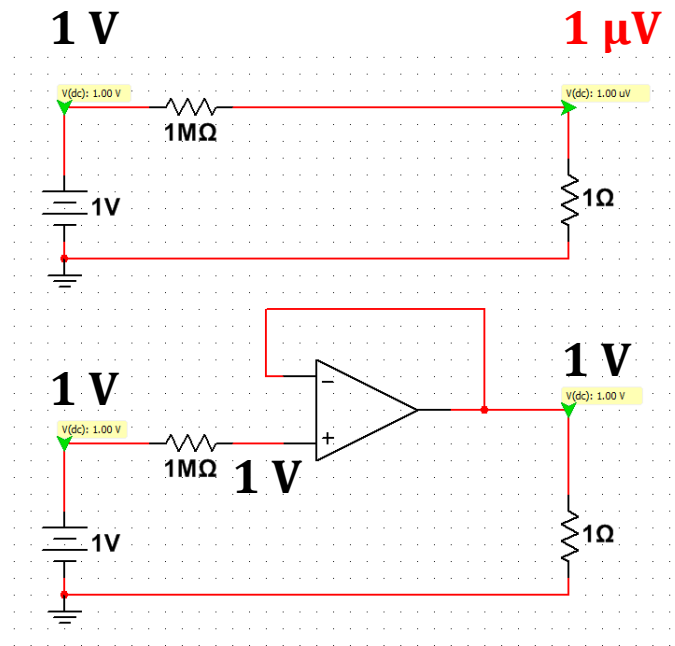


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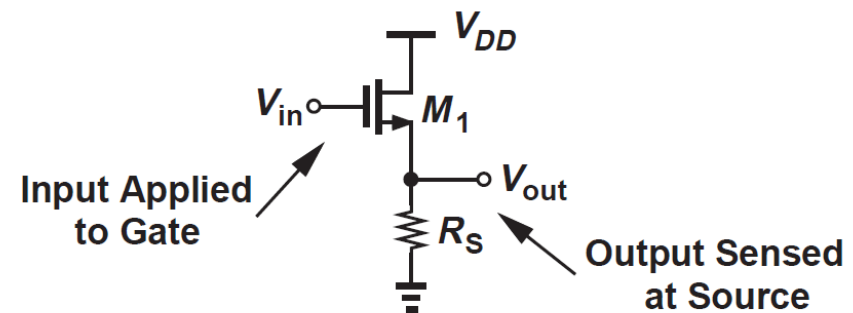
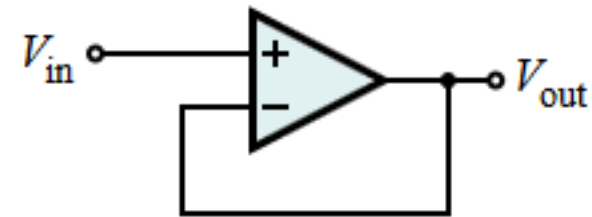
Buffer

- A voltage **buffer** amplifier is used to transfer a voltage from a first circuit, having a **high output impedance**, to a second circuit with a **low input impedance**.
- The interposed buffer amplifier prevents the second circuit from loading the first circuit unacceptably and interfering with its desired operation: decoupling function.
- In the ideal voltage buffer has
 - infinite input resistance
 - zero output resistance
- If the voltage is transferred unchanged (i.e. if the voltage gain is 1), the amplifier is a **unity gain buffer**, also known as a **voltage follower**, because the output voltage follows or tracks the input voltage.



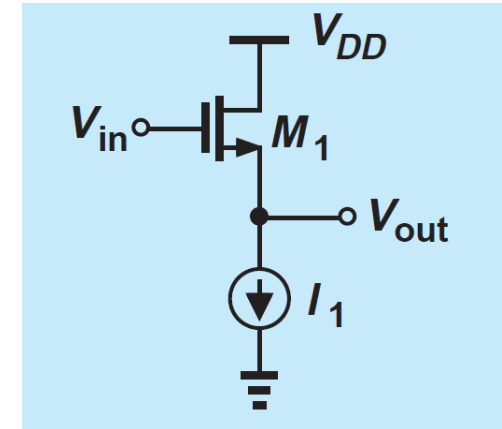
Voltage Follower

- You can use Op Amps to implement a voltage follower.
 - Big area
- You can use a single transistor circuit: for example a MOSFET in the *common-drain configuration* (called **source follower**)
 - Small area (suitable for pixels in digital imaging)



Source Follower

- How does the source follower work?
- Since I_1 is an ideal current source, the MOS cannot change its current. Hence, its V_{GS} must remain **constant**.
- Writing $V_{out} = V_{in} - V_{GS}$, we recognize that V_{out} **exactly follows** V_{in} , since V_{GS} is constant.
- If V_{in} rises by a small amount ΔV_{in} , also V_{out} rises by the **same** amount.



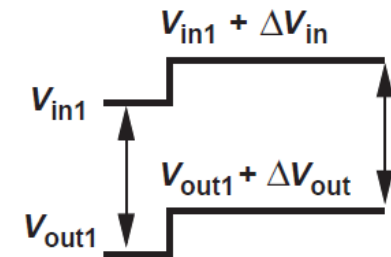
- Small signal model: $V_{in} = V_{IN} + v_{in}$ and $V_{out} = V_{OUT} + v_{out}$.

$$A_v = \frac{v_{out}}{v_{in}} = 1$$

- Small signal **unity gain!**

$$V_{OUT} = V_{IN} - V_{GS}$$

- V_{out} is always lower than V_{in} by an amount equal to V_{GS} , and the circuit is said to provide a **level shift**.



$$I_D = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_{TH})^2$$

Source Follower

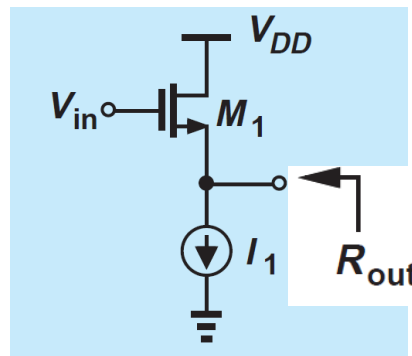
- Has an amplifier with a unity gain any practical value?
- In order to appreciate the usefulness of source followers, let us compute their input and output impedances.
- **The input impedance is very high**, making the circuit a good *voltmeter*; i.e., the follower can sense a voltage without disturbing it:

$$R_{in} = \infty$$

- The output impedance consists of the resistance seen looking up into the source (assuming an ideal current source):

$$R_{out} = \frac{1}{g_m}$$

- The circuit thus presents a **relatively low output impedance**. Followers can serve as good buffers, e.g., *between a... photodiode and a low-impedance load*.





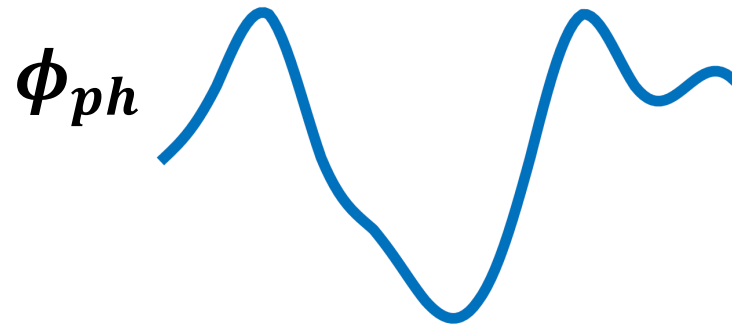
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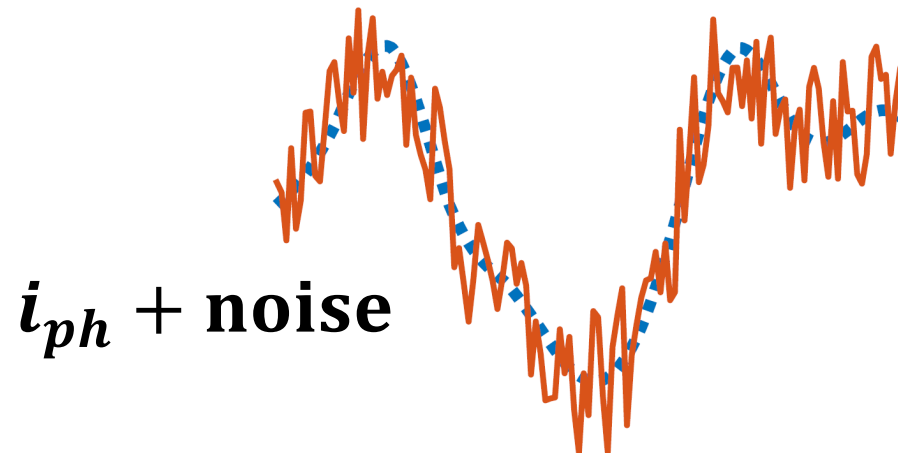
Noise

- What is noise?
- We want to measure the photon flux $\phi_{ph}(t)$

- Photon flux



- Photocurrent



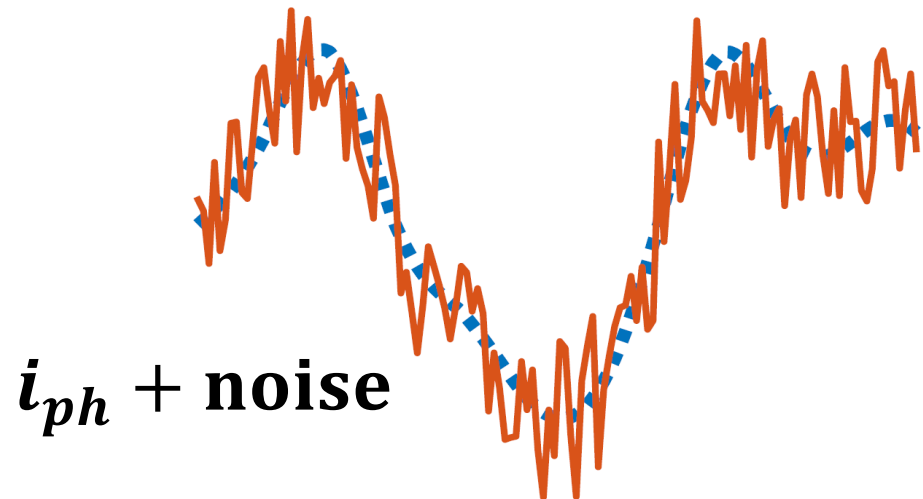
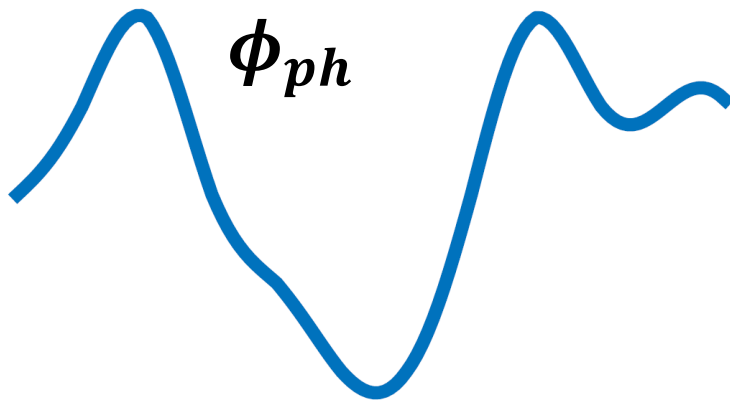
Noise

- Ideally, the photocurrent has a deterministic value

$$i_{ph}(t) = q\phi_{ph}(t)$$

- Due to noise:

$$i_{ph}(t) = q\phi_{ph}(t) + \mathbf{i}_n(t)$$





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Statistics

- Noise is **not** a deterministic signal, but it is randomly fluctuating.
- It should be described with statistical parameters:
 - Mean value \bar{i}_n
 - Variance σ_n^2 , that measures how far noise is spread out from its mean value
 - Standard deviation σ_n , that is the square root of the variance

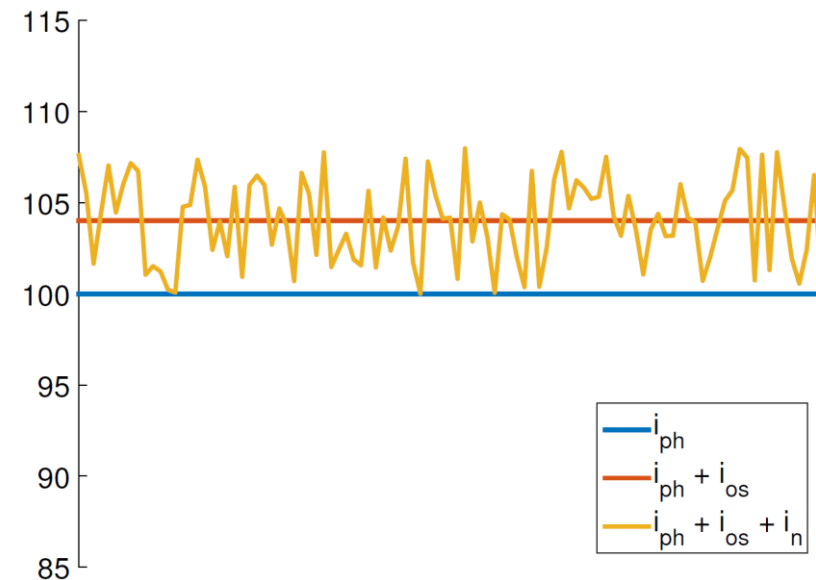
Noise and Offset

- The mean value of noise is, by definition, **0**.
- If there's a signal, with a non-zero component, in addition to the *useful* signal (photocurrent), we talk about **offset**.

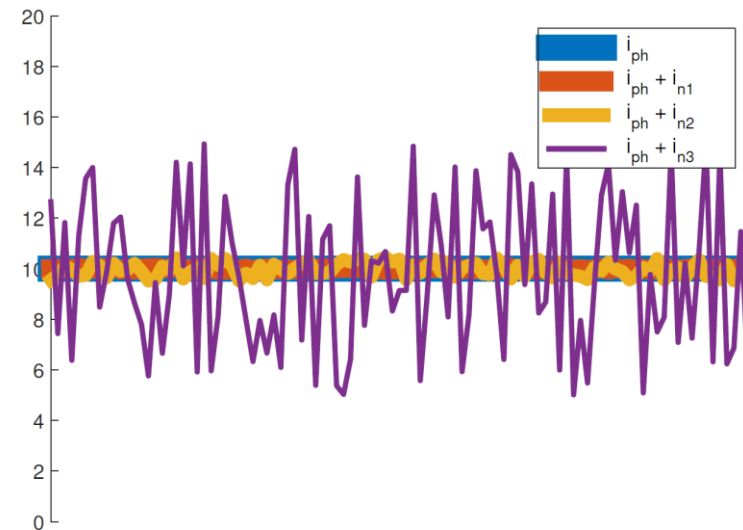
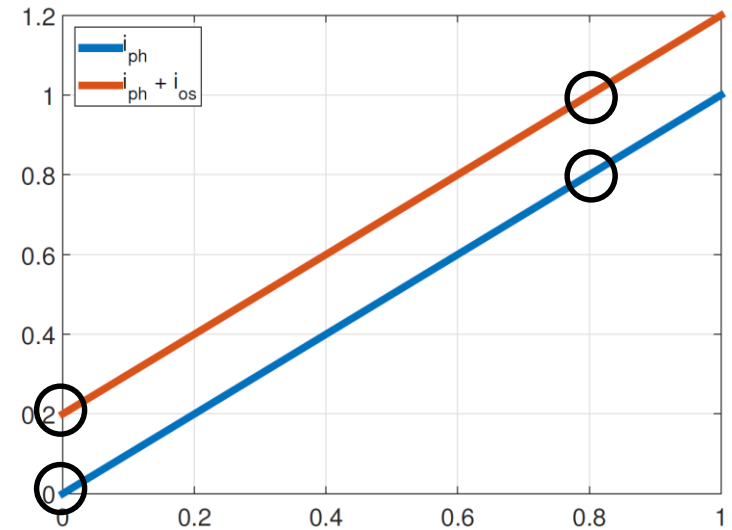
Signal: 100

Measurement: $104 + \text{noise} = 100 + 4 + \text{noise}$

- Signal
- Offset
- Noise



- Offset can be **compensated!**
 - Calibration
 - Double sampling
- Noise can be **reduced!**
 - Proper filtering
 - Good design



Offset in Photodiodes

- With no photon flux (i.e. dark), we would like to have no current flow in the photodiode: $I = 0 \text{ A}$
- But... reverse-biased photodiodes have a reverse bias current:
 $I(\phi_{ph} = 0) \neq 0 \text{ A}$
- In digital imaging, this *offset current* is called **dark current**.

Dark Current

- Two sources of dark current:

- Diffusion of minority carriers** in bulk regions

$$I_{sat} = A_d \left(\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} \right) = A_d q n_i^2 \left(\frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n} \right) = 0.04 \text{ fA}$$

$$I_{sat} = I_{sat}(T)$$

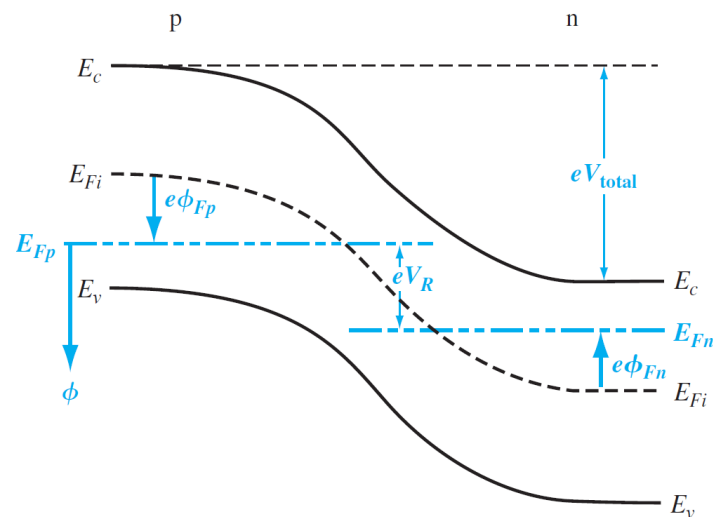
- Thermal generation** of EHPs in the depletion region

$$I_{gen} = A_d \frac{q n_i W}{2\tau_0} = 1.4 \text{ fA}$$

$$I_{gen} = I_{gen}(T, V_R)$$

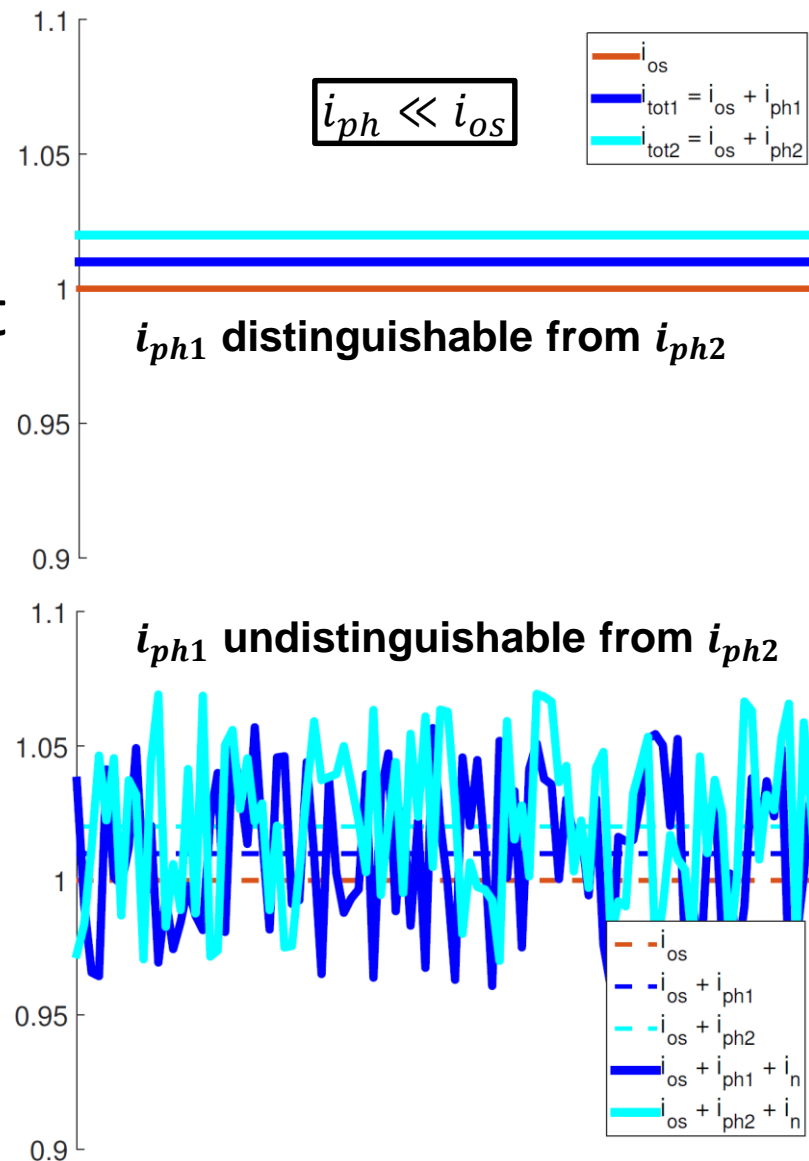
- For a silicon photodiode at room temperature, the generation current gives the dominant dark current contribution.

$$I_d = I_{sat} + I_{gen} \approx I_{gen}$$



Noise and Offset

- In noiseless systems, variations lower than i_{os} can be measured/appreciated, because i_{os} is simply an offset and can be compensated.
- In noisy systems, any variation **lower** than noise cannot be measured/appreciated.



Signal-to-Noise Ratio

- The **minimum detectable signal** is set by the amplitude of signal (current, voltage, charge) **fluctuations (noise)** of both the sensor and the electronics

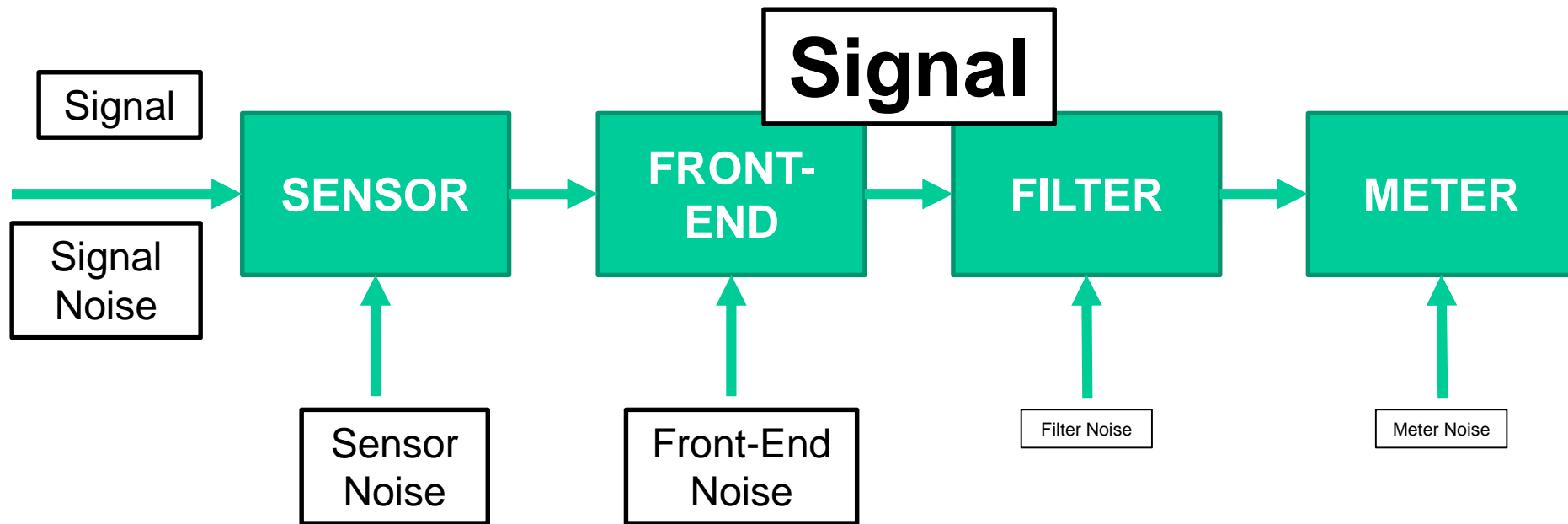
$$SNR = \frac{\text{signal}}{\text{noise}} = \frac{i_s}{\sigma_n}$$

- If $SNR = 1$, then

$$\text{signal} = \sigma_n$$

**Minimum detectable
signal**

System point-of-view



- ‘Noise war’ happens at **first** stages!
- Then... no problems... if you don't make mistakes!

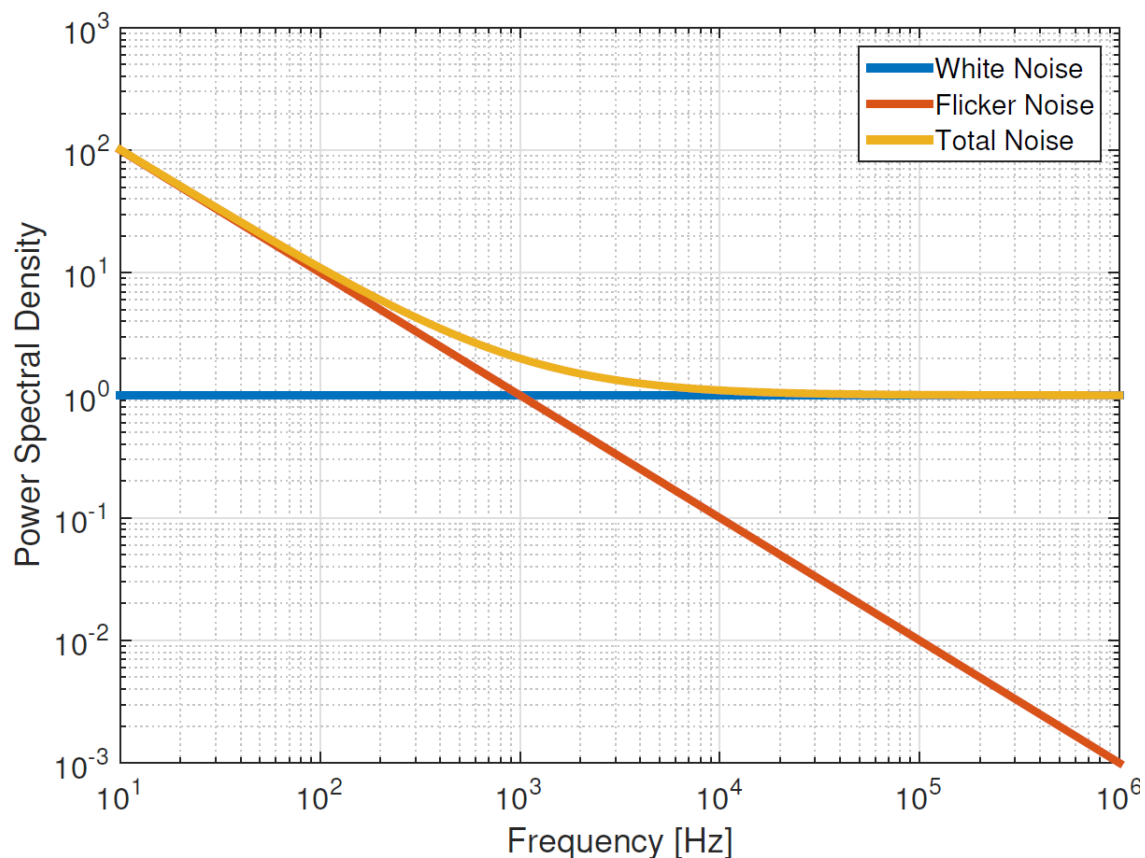
Frequency Domain

- Noise is generally expressed in terms of power spectral density $S_n^2(f)$, i.e. in the frequency domain:
 - V^2/Hz
 - A^2/Hz
 - C^2/Hz
 - LSB^2/Hz
- Given a certain noise power spectral density $S_n^2(f)$ associated with a noise source, the **variance** of the noise can be evaluated as

$$\sigma_n^2 = \int_0^{\infty} S_n^2(f) df$$

Frequency Domain

- White Noise
 - $S_n^2(f) = S_{n,w}^2$
- Flicker Noise
 - $S_n^2(f) = \frac{A_n^2}{f}$



White Noise

- Which is the variance of white noise?

$$\sigma_n^2 = \int_0^{\infty} S_{n,w}^2 df = S_{n,w}^2 \int_0^{\infty} 1 df = \infty!$$

$$\sigma_n^2 = \int_0^{\infty} S_{n,w}^2 |T(f)|^2 df = S_{n,w}^2 \int_0^{\infty} |T(f)|^2 df$$

- We define the **noise equivalent bandwidth** of a filter:

$$BW_n = \int_0^{\infty} |T(f)|^2 df$$

$$\sigma_n^2 = S_{n,w}^2 BW_n$$

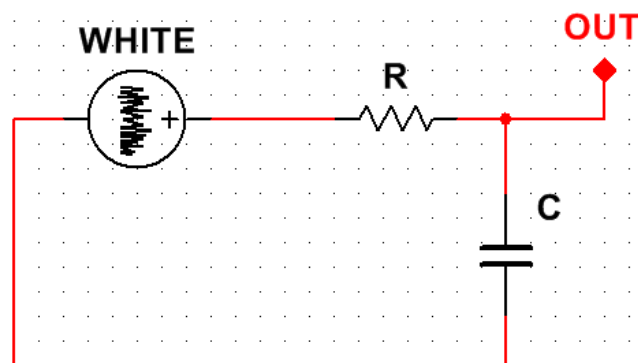
Noise Equivalent Bandwidth

- RC Low-Pass Filter

$$T(s) = \frac{1}{1 + sRC}$$

$$\int_0^{\infty} \left| \frac{1}{1 + (j2\pi f)RC} \right|^2 df = \frac{1}{4RC}$$

$$BW_n = \frac{1}{4RC}$$

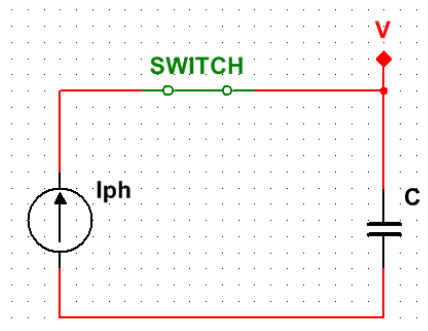
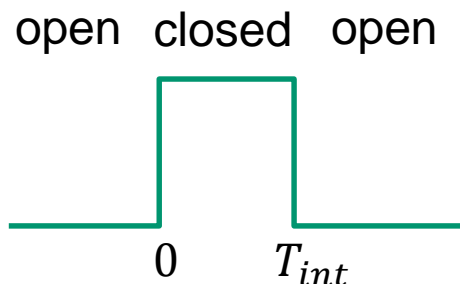
 $S_{n,w}^2$


$$\sigma_n = \sqrt{S_{n,w}^2 \frac{1}{4RC}}$$

Noise Equivalent Bandwidth

- Discrete time integrator

$$BW_n = \frac{1}{2T_{int}}$$



$$V = \frac{1}{C} \int_0^{T_{int}} I_{ph}(t) dt$$

$$S_{n,w}^2 \implies \sigma_n = \sqrt{S_{n,w}^2 \frac{1}{2T_{int}}}$$

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Noise in Digital Imaging

- **Shot noise**
- **Thermal noise**
- **Flicker noise**
- There are also several **other noise** sources related with electronics, that depend on the readout architecture (**kTC**, **quantization**,...)

Shot Noise

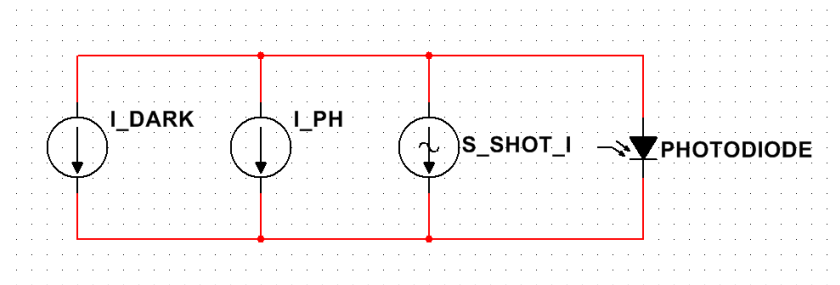
- Shot noise results from unavoidable random statistical fluctuations of the electric current when charge carriers traverse a gap.
- Current is a flow of discrete charges, and the fluctuation in the arrivals of those charges creates shot noise.
- In a reverse biased photodetector there are two shot current sources:

- signal current due to photon absorption and conversion in the depletion region $I_s = q\phi_{ph}$

$$S_{I,s}^2 = 2qI_s \text{ [A}^2\text{/Hz]}$$

- dark current I_d

$$S_{I,d}^2 = 2qI_d \text{ [A}^2\text{/Hz]}$$



Shot Noise: SNR

- The total contribution is given by the sum of the two:

$$S_{shot,I}^2 = 2qI_s + 2qI_d$$

$$SNR = \frac{I_s}{\sigma_{shot,I}} = \frac{I_s}{\sqrt{S_{shot,I}^2 BW_n}} = \frac{I_s}{\sqrt{2q(I_s + I_d)BW_n}}$$

- Large signals**

- dominant shot noise contribution is due to signal current:

$$SNR \approx \frac{\sqrt{I_s}}{\sqrt{2qBW_n}}$$

- Small signals**

- dominant shot noise contribution is due to dark current:

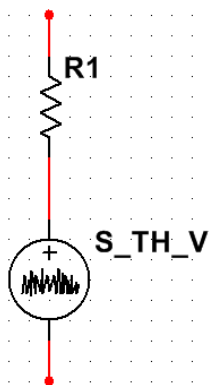
$$SNR \approx \frac{I_s}{\sqrt{2qI_d BW_n}}$$

Thermal Noise: Resistor

- Consider a semiconductor (e.g. a resistor) with no connections at its electrodes (i.e. floating).
- Its average current is null because it is an open circuit.
- But... electrons can move randomly due to thermal agitation, leading to a statistical fluctuation of the voltage/current.
- This thermal agitation of electrons creates a random voltage which can be measured at resistor electrodes resulting in voltage/current noise.

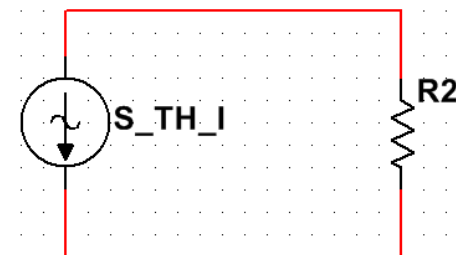
$$S_{th,V}^2 = 4kTR_1$$

[V²/Hz]



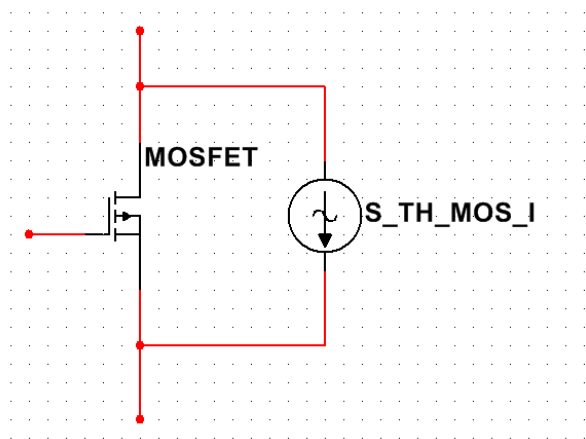
$$S_{th,I}^2 = \frac{4kT}{R_2}$$

[A²/Hz]



Thermal Noise: MOSFET

- A MOSFET channel (the path between the source and the drain) is a resistive path.
- It has its own thermal noise.
- If the MOS is biased in its **saturation** region (e.g. a source follower):



$$S_{th,I}^2 = 4kT\gamma g_m$$

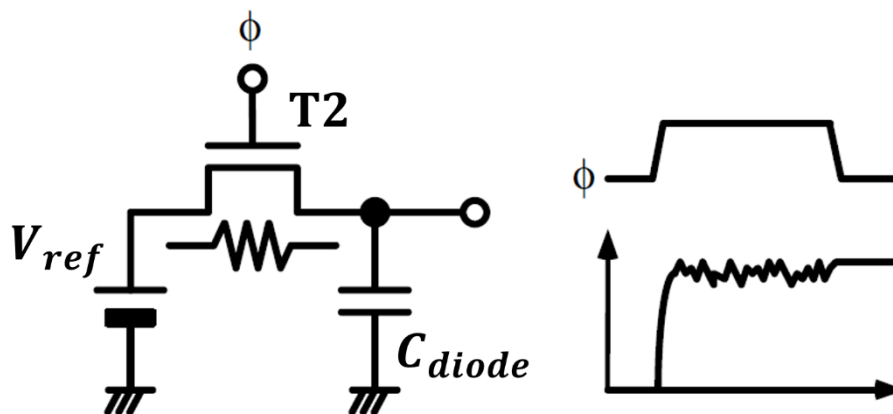
[A²/Hz]

- g_m : (small-signal) transconductance of the MOS
- $\gamma = 2/3$

kTC Noise

- When a capacitance is reset, noise called *reset* or *kTC* noise appears at the capacitance node when the switch is turned OFF. This noise comes from the thermal noise of the MOS switch.
- A MOS transistor (T2) is considered a resistance R_{on} during the ON period, and thermal noise appears. When the switch opens, this noise is then sampled and held by the capacitor.
- Thermal noise voltage spectral density is $S_{reset,V}^2 = 4kTR_{on}$.
- Since, during ON, $BW_n = 1/4\tau = 1/4R_{on}C$, we have

$$\sigma_{reset,V} = \sqrt{4kTR_{on} \frac{1}{4R_{on}C}} = \sqrt{\frac{kT}{C}} \text{ [V]}$$



Quantization Noise

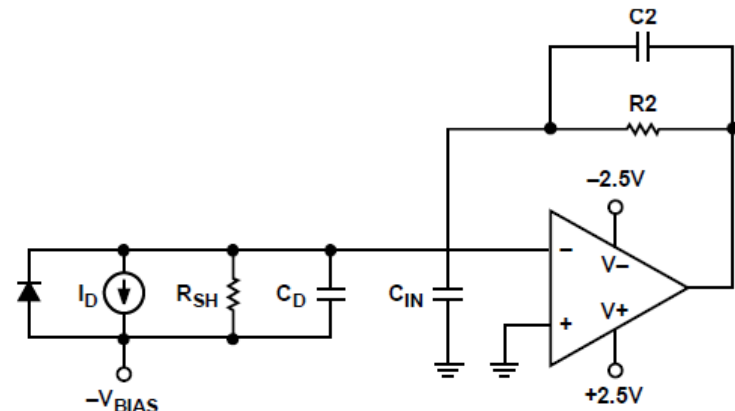
- In an ideal analog-to-digital converter, the quantization error is uniformly distributed between $-1/2$ LSB and $+1/2$ LSB, and the signal has a uniform distribution covering all quantization levels.
- The mean squared error produced by quantization is

$$\sigma_V^2 = \frac{LSB^2}{12} = \frac{\left(\frac{FSR}{2^n}\right)^2}{12}$$

- FSR of the ADC
- n : number of bits

Op Amp Noise

- A photodetector is usually readout with transimpedance stage, using an operational amplifier.
- Operational amplifiers manufacturers always provide a datasheet on which many features are provided.
- Noise performance is provided **referred to the inputs** of the amplifier. Usually two contributes are given: input-referred current and voltage noise spectral densities.
- In matrices of pixels this readout approach is not implemented due to area constraints.

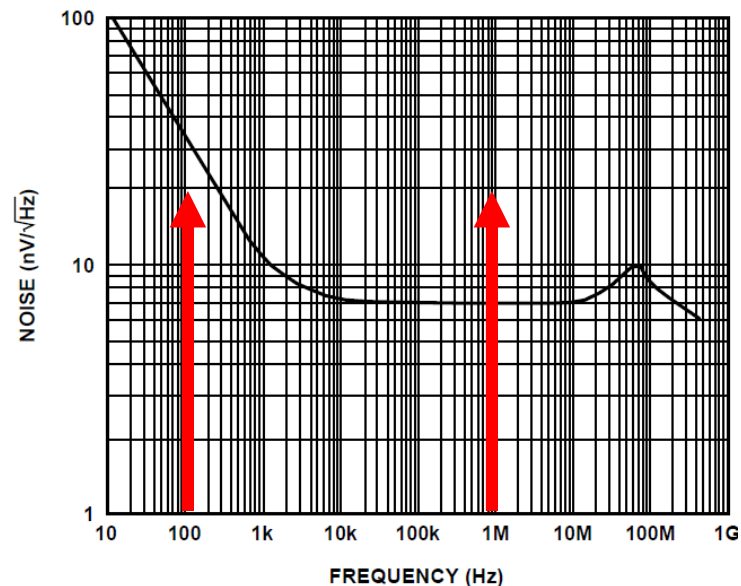


$$v_{out} = I_D * R2$$

Data Sheet		AD8615/AD8616/AD8618				
SPECIFICATIONS						
$V_S = 5\text{ V}$, $V_{CM} = V_S/2$, $T_A = 25^\circ\text{C}$, unless otherwise noted.						
Table 1.						
Parameter	Symbol	Conditions	Min	Typ	Max	Unit
INPUT CHARACTERISTICS						
Offset Voltage, AD8616/AD8618	V_{OS}	$V_S = 3.5\text{ V}$ at $V_{CM} = 0.5\text{ V}$ and 3.0 V		23	60	μV
Offset Voltage, AD8615		$V_{FW} = 0\text{ V}$ to 5 V		23	100	μV
				80	500	μV
Phase Margin	ϕ_m			63		Degrees
NOISE PERFORMANCE						
Peak-to-Peak Noise	e_n p-p	0.1 Hz to 10 Hz		2.4		μV
Voltage Noise Density	e_n	$f = 1\text{ kHz}$		10		$\text{nV}/\sqrt{\text{Hz}}$
		$f = 10\text{ kHz}$		7		$\text{nV}/\sqrt{\text{Hz}}$
Current Noise Density	i_n	$f = 1\text{ kHz}$		0.05		$\text{pA}/\sqrt{\text{Hz}}$
Channel Separation	C_S	$f = 10\text{ kHz}$		-115		dB
		$f = 100\text{ kHz}$		-110		dB

Flicker Noise

- Up to now, all noise sources were considered white.
- In reality other kinds of noise exist, generated from different physical phenomena (e.g. electron trapping in a transistor channel), which have a $1/f$ behavior. Typically the contribution of these noise sources are dominant over white noise until ≈ 1 kHz.
- A direct consequence is that a small signal of a certain amplitude:
 - might be covered in noise ($SNR < 1$), if it is at low frequency
 - it could be clearly visible ($SNR \gg 1$) if modulated at higher frequencies

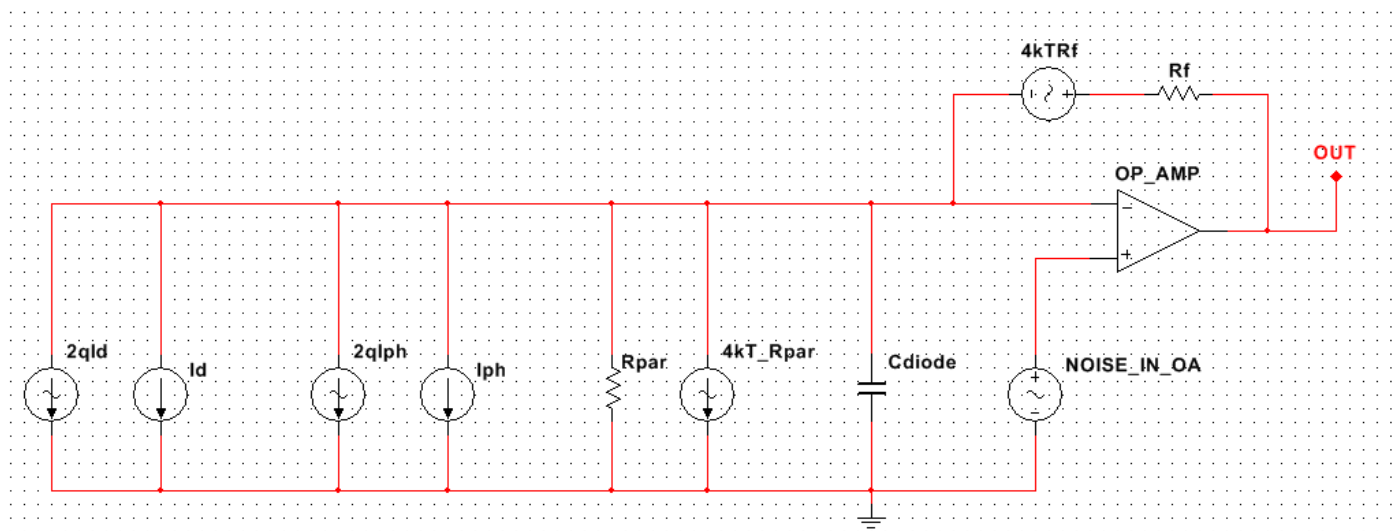


Outline

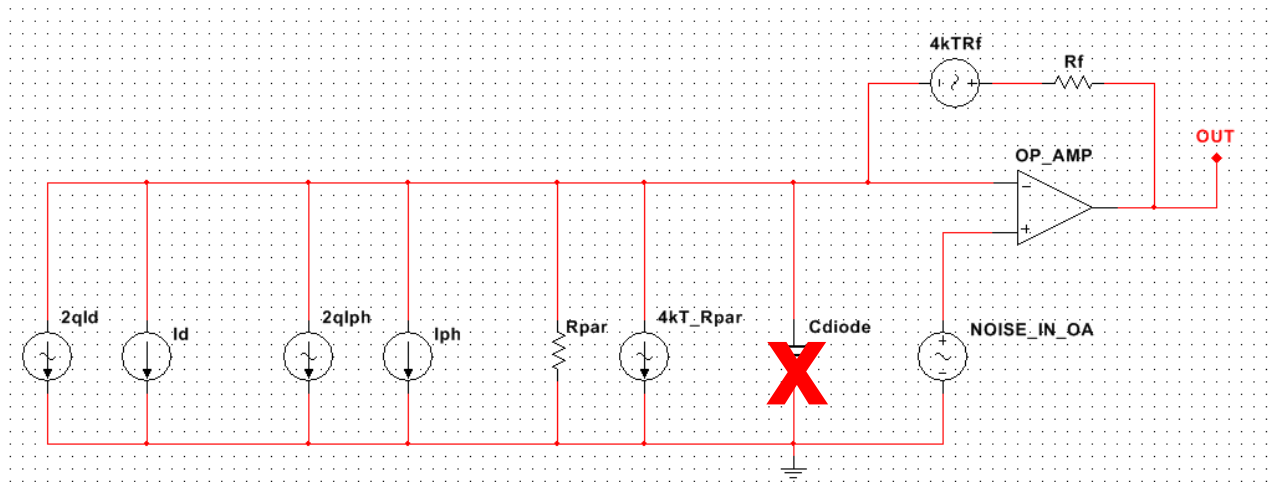
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Total Noise

- Squares of the magnitude of the transfer function should be used to 'move' noise from one *point* to the other.
- Squares of the magnitude of the transfer function should be used to sum different noise sources.
- Small signal model should be used.
- To compare signal and noise, they all have to be evaluated at the same *point*.



Total Noise



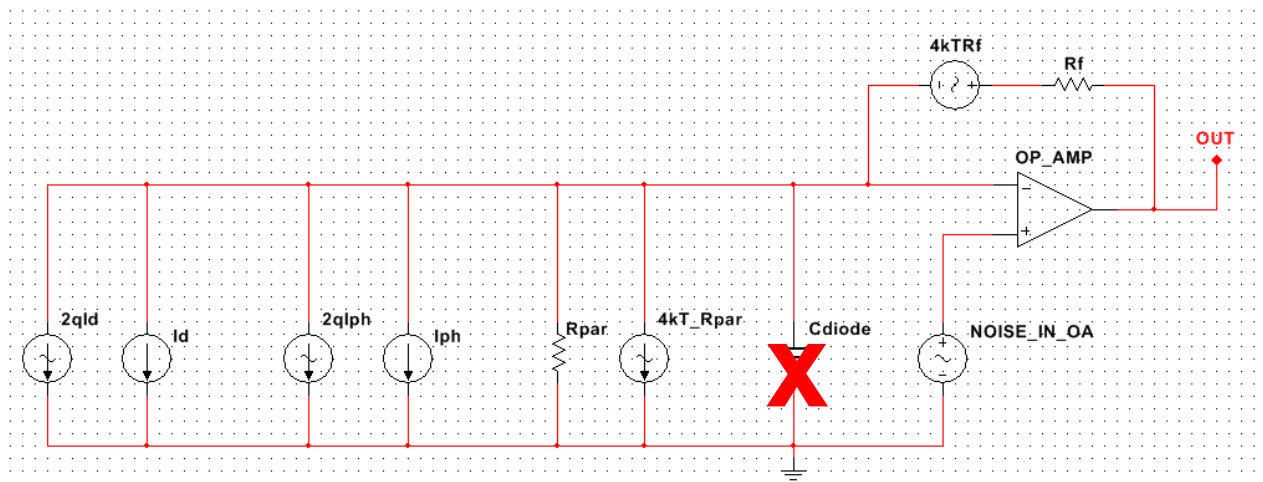
- Let's evaluate the signal and the noise at the output.

- $V_{out} = I_{ph} * R_f + I_d * R_f$

- $S_{n,OUT,V}^2 = 2qI_d * R_f^2 + 2qI_{ph} * R_f^2 + \frac{4kT}{R_{par}} R_f^2 + 4kTR_f + S_{n,IN,OA}^2 \left(1 + \frac{R_f}{R_{par}}\right)^2$

$$SNR = \frac{V_{out,signal}}{\sigma_{n,OUT,V}} = \frac{I_{ph}R_f}{\sqrt{\left[\left(2q(I_d + I_{ph}) + \frac{4kT}{R_{par}} \right) R_f^2 + 4kTR_f + S_{n,IN,OA}^2 \left(1 + \frac{R_f}{R_{par}} \right)^2 \right] BW_n}}$$

Total Noise



- Let's evaluate the SNR at the **input**.

$$SNR = \frac{I_{signal}}{\sigma_{n,I}} = \frac{I_{ph}}{\sqrt{\left[\left(2q(I_d + I_{ph}) + \frac{4kT}{R_{par}} \right) + 4kTR_f \frac{1}{R_f^2} + S_{n,IN,OA}^2 \left(1 + \frac{R_f}{R_{par}} \right)^2 \frac{1}{R_f^2} \right] BW_n}}$$

$$SNR = \frac{I_{signal}}{\sigma_{n,I}} = \frac{I_{ph}}{\sqrt{\left[2q(I_d + I_{ph}) + \frac{4kT}{R_{par}} + \frac{4kT}{R_f} + S_{n,IN,OA}^2 \left(1 + \frac{R_f}{R_{par}} \right)^2 \frac{1}{R_f^2} \right] BW_n}}$$

- SNR does not change in relation to the point where it is calculated!